

# Physics 1B Quiz 1

NATHAN LIN

TOTAL POINTS

**50 / 50**

QUESTION 1

1 Honor Pledge 0 / 0

✓ + 0 pts Correct

QUESTION 2

Problem 1 25 pts

2.1 part a 5 / 5

✓ + 5 pts Correct

+ 3 pts Bernoulli's

+ 2 pts Solved for vx

+ 4 pts Minor algebra error

2.2 part b 5 / 5

✓ + 5 pts Correct

+ 5 pts Correct by above

+ 4 pts Minor algebra error

+ 2 pts Solve for t

+ 1 pts Solve for v

+ 0 pts No credit

2.3 part c 5 / 5

✓ + 5 pts Correct

+ 5 pts correct by above

+ 1 pts Equated terms

+ 3 pts Solve for y

+ 1 pts Pick + solution

+ 0 pts No credit

+ 4 pts Minor algebra error

+ 3 pts Significant algebra error

2.4 part d 5 / 5

✓ + 5 pts Correct

+ 4 pts Plugged in wrong

+ 2 pts Mass flow rate

+ 2 pts Volume flow rate

+ 0 pts No credit

2.5 part e 5 / 5

✓ + 5 pts Correct

+ 5 pts Correct by above

+ 4 pts Error plugging in

+ 2 pts dV/dt

+ 2 pts Solve for r

+ 0 pts No credit

+ 1 pts Plug in correct

QUESTION 3

Problem 2 25 pts

3.1 part a 6 / 6

✓ + 6 pts Correct

+ 4 pts Incomplete

+ 3 pts wrong

+ 0 pts no work

+ 4 pts mistake

3.2 part b 6 / 6

✓ + 6 pts Correct

+ 5 pts mistake

+ 4 pts wrong

+ 3 pts incomplete

+ 0 pts no work

3.3 part c 7 / 7

✓ + 7 pts Correct

+ 6 pts mistake

+ 4 pts wrong

+ 3 pts incomplete

+ 0 pts no work

3.4 part d 6 / 6

- ✓ + 6 pts Correct
- + 4 pts mistake
- + 3 pts wrong
- + 2 pts incomplete
- + 0 pts no work

3.5 part e 0 / 0

- ✓ + 0 pts Correct

## Quiz 1

Physics 1B, Fall 2020

October 19, 2020

Dr. Alec Vinson

Students are allowed open notes and open book for this quiz. Besides online materials specifically for this course (e.g. postings on CCLE and the eText), all other online resources are considered unauthorized material access for this quiz.

To receive full credit, the student must **show all work**.

Answers may be typed using typesetting software that can utilize mathematical symbols (e.g.  $\LaTeX$ , or Microsoft Word in conjunction with the use of its "equation" tool, etc.), or may be written with note-taking software (e.g. Microsoft OneNote, etc.) using a stylus + touchscreen, or may be written on physical paper. If writing on physical paper, the student must save their work by taking pictures, preferably converting those pictures to PDF format.

Students are to submit their work on Gradescope to the appropriate assignment labeled "Quiz 1" before the deadline of 2pm PT on October 19, 2020.

Students are provided a 4 hour window to complete the quiz, though it should only take approximately one hour to complete.

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**Important:** Please print your name on the provided line within the statement below, and then sign your name next to the line marked with an 'x' below it. You may do this by signing this paper via note-taking or PDF-editing software, or printing the page, signing it physically, and taking a picture. Alternatively, you may write out the statement on your own piece of paper, exactly as written, sign it, and take a picture. The signed pledge should be the first page of your Quiz 1 submission on Gradescope.

I, Nathan Lin, affirm that I will not give or receive any unauthorized help on this quiz, that all work will be my own, and that I will not share or disseminate the quiz or my solutions in any manner, online, physically, or otherwise.

Sign: x



1 Honor Pledge 0 / 0

✓ + 0 pts Correct

## Quiz 1

$$1. a. P_{atm} + \rho gh + \frac{1}{2} \rho v_1^2 = P_{atm} + \rho gy + \frac{1}{2} \rho v_x^2$$

$$\rho gh = \rho gy + \frac{1}{2} \rho v_x^2$$

$$v_x = \sqrt{2g(h-y)}$$

$$b. y = v_{0y}t + \frac{1}{2}gt^2$$

$$y = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2y}{g}}$$

$$x = v_x t, v_x = \frac{x}{t}$$

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$$c. x \sqrt{\frac{g}{2y}} = \sqrt{2g(h-y)}$$

$$x^2 \left(\frac{g}{2y}\right) = 2g(h-y)$$

$$x^2 = 4y(h-y)$$

$$4y^2 - 4yh + x^2 = 0$$

$$y^2 - yh + \frac{x^2}{4} = 0$$

$$y = \frac{h + \sqrt{h^2 - x^2}}{2}$$

$$y = \frac{h \pm \sqrt{h^2 - 4\left(\frac{x^2}{4}\right)}}{2}$$

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$$d. \frac{dV}{dt} = \frac{m}{\rho t}$$

$$= \frac{5 \text{ kg}}{(1000 \text{ kg/m}^3)(60 \text{ s})}$$

$$\frac{dV}{dt} = 8.3 \times 10^{-5} \text{ m}^3/\text{s}$$

$$e. \frac{dV}{dt} = Av$$

$$\frac{dV}{dt} = \pi r^2 v_x$$

$$r = \sqrt{\frac{dV}{\pi v_x dt}}$$

$$r = \sqrt{\frac{(8.3 \times 10^{-5} \text{ m}^3/\text{s})}{\pi (9.899 \text{ m/s})}}$$

$$r = 1.637 \times 10^{-3} \text{ m}$$

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+ 1 pts Solve for v

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+ **1 pts** Equated terms

+ **3 pts** Solve for y

+ **1 pts** Pick + solution

+ **0 pts** No credit

+ **4 pts** Minor algebra error

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✓ + 5 pts Correct

+ 4 pts Plugged in wrong

+ 2 pts Mass flow rate

+ 2 pts Volume flow rate

+ 0 pts No credit

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2.5 part e 5 / 5

✓ + 5 pts Correct

+ 5 pts Correct by above

+ 4 pts Error plugging in

+ 2 pts  $dV/dt$

+ 2 pts Solve for r

+ 0 pts No credit

+ 1 pts Plug in correct



2. a. At  $x_{eq} = 0$ ,  $m_1$  and  $m_2$  both reach their maximum speed since there is zero elastic potential energy. However, the restoring force  $F_s = -kx$  acts on  $m_1$  to reduce its speed, while  $m_2$  continues at  $v_{max}$  on a frictionless surface. The masses separate at  $x_{eq}$  therefore.

$$t = \frac{1}{4} T, \quad T = 2\pi \sqrt{\frac{m_1 + m_2}{k}} = \frac{2\pi}{\omega}$$

$$t = \frac{\pi}{2} \sqrt{\frac{m_1 + m_2}{k}}$$

@  $x = 0 \text{ m}$

b.  $\frac{1}{2} k A^2 = \frac{1}{2} m_1 v^2 + \frac{1}{2} k x^2$  @  $x = 0 \text{ m}$

$$v = \sqrt{\frac{k A^2}{m_1 + m_2}}$$

$$v = A \sqrt{\frac{k}{m_1 + m_2}}$$

c.  $\frac{1}{2} k A^2 = \frac{1}{2} m_2 v^2 + \frac{1}{2} k x_{max}^2$ , KE of mass 2 + U of mass 1

$$x_{max}^2 = \frac{k A^2 - m_2 v^2}{k}$$

$$x_{max}^2 = \frac{k A^2 - m_2 A^2 \left(\frac{k}{m_1 + m_2}\right)}{k}$$

$$x_{max}^2 = A^2 \left(1 - \frac{m_2}{m_1 + m_2}\right)$$

$$x_{max} = A \sqrt{\frac{m_1}{m_1 + m_2}}$$

d.  $t_{total} = t_a + t'$ ,  $t_a = \frac{\pi}{2} \sqrt{\frac{m_1 + m_2}{k}}$

$$t' = \frac{\pi}{2} \sqrt{\frac{m_1}{k}}$$

$$t_{total} = \frac{\pi}{2} \left( \sqrt{\frac{m_1 + m_2}{k}} + \sqrt{\frac{m_1}{k}} \right)$$

e.  $x(t) = A_0 e^{-t/\tau} \cos(\omega t + \phi)$

@  $x_{max}$ :  $\omega t + \phi = 0$ ,  $\cos(0) = 1$

$$\frac{k}{m} \gg \frac{b^2}{4m^2}, \text{ so } t = t_{total} \text{ (from part d)}$$

$$x_{max} = A \sqrt{\frac{m_1}{m_1 + m_2}} e^{-\frac{b\pi}{4m_1} \left( \sqrt{\frac{m_1 + m_2}{k}} + \sqrt{\frac{m_1}{k}} \right)}$$



3.1 part a 6 / 6

- ✓ + 6 pts Correct
- + 4 pts Incomplete
- + 3 pts wrong
- + 0 pts no work
- + 4 pts mistake

2. a. At  $x_{eq} = 0$ ,  $m_1$  and  $m_2$  both reach their maximum speed since there is zero elastic potential energy. However, the restoring force  $F_s = -kx$  acts on  $m_1$  to reduce its speed, while  $m_2$  continues at  $v_{max}$  on a frictionless surface. The masses separate at  $x_{eq}$  therefore.

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### 3.2 part b 6 / 6

- ✓ + **6 pts** Correct
- + **5 pts** mistake
- + **4 pts** wrong
- + **3 pts** incomplete
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2. a. At  $x_{eq} = 0$ ,  $m_1$  and  $m_2$  both reach their maximum speed since there is zero elastic potential energy. However, the restoring force  $F_s = -kx$  acts on  $m_1$  to reduce its speed, while  $m_2$  continues at  $v_{max}$  on a frictionless surface. The masses separate at  $x_{eq}$  therefore.

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### 3.3 part c 7 / 7

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@  $x_{max}$ :  $\omega t + \phi = 0$ ,  $\cos(0) = 1$

$$\frac{k}{m} \gg \frac{b^2}{4m^2}, \text{ so } t = t_{total} \text{ (from part d)}$$

$$x_{max} = A \sqrt{\frac{m_1}{m_1 + m_2}} e^{-\frac{b\pi}{4m_1} \left( \sqrt{\frac{m_1 + m_2}{k}} + \sqrt{\frac{m_1}{k}} \right)}$$



3.4 part d 6 / 6

- ✓ + 6 pts Correct
- + 4 pts mistake
- + 3 pts wrong
- + 2 pts incomplete
- + 0 pts no work



2. a. At  $x_{eq} = 0$ ,  $m_1$  and  $m_2$  both reach their maximum speed since there is zero elastic potential energy. However, the restoring force  $F_s = -kx$  acts on  $m_1$  to reduce its speed, while  $m_2$  continues at  $v_{max}$  on a frictionless surface. The masses separate at  $x_{eq}$  therefore.

$$t = \frac{1}{4} T, \quad T = 2\pi \sqrt{\frac{m_1 + m_2}{k}} = \frac{2\pi}{\omega}$$

$$t = \frac{\pi}{2} \sqrt{\frac{m_1 + m_2}{k}}$$

@  $x = 0 \text{ m}$

b.  $\frac{1}{2} k A^2 = \frac{1}{2} m_1 v^2 + \frac{1}{2} k x^2$  @  $x = 0 \text{ m}$

$$v = \sqrt{\frac{k A^2}{m_1 + m_2}}$$

$$v = A \sqrt{\frac{k}{m_1 + m_2}}$$

c.  $\frac{1}{2} k A^2 = \frac{1}{2} m_2 v^2 + \frac{1}{2} k x_{max}^2$ , KE of mass 2 + U of mass 1

$$x_{max}^2 = \frac{k A^2 - m_2 v^2}{k}$$

$$x_{max}^2 = \frac{k A^2 - m_2 A^2 \left(\frac{k}{m_1 + m_2}\right)}{k}$$

$$x_{max}^2 = A^2 \left(1 - \frac{m_2}{m_1 + m_2}\right)$$

$$x_{max} = A \sqrt{\frac{m_1}{m_1 + m_2}}$$

d.  $t_{total} = t_a + t'$ ,  $t_a = \frac{\pi}{2} \sqrt{\frac{m_1 + m_2}{k}}$

$$t' = \frac{\pi}{2} \sqrt{\frac{m_1}{k}}$$

$$t_{total} = \frac{\pi}{2} \left( \sqrt{\frac{m_1 + m_2}{k}} + \sqrt{\frac{m_1}{k}} \right)$$

e.  $x(t) = A_0 e^{-t/\tau} \cos(\omega t + \phi)$

@  $x_{max}$ :  $\omega t + \phi = 0$ ,  $\cos(0) = 1$

$$\frac{k}{m} \gg \frac{b^2}{4m^2}, \text{ so } t = t_{total} \text{ (from part d)}$$

$$x_{max} = A \sqrt{\frac{m_1}{m_1 + m_2}} e^{-\frac{b\pi}{4m_1} \left( \sqrt{\frac{m_1 + m_2}{k}} + \sqrt{\frac{m_1}{k}} \right)}$$

3.5 part e 0 / 0

✓ + 0 pts Correct