Physics 1B-1 Winter 2021
MidTerm 1

January 28th

Solutions

Due to the extraordinary circumstances caused by the COVID-19 pandemic, the MidTerm exam is online opened-textbook opened-notes exam.

You are allowed to use textbook (and/or text e-book), your notes, your home assignments and home assignment solutions; please do not consult any other materials. This exam is an attempt to evaluate the individual proficiency of the students in the subject of this class. Group work of students on the problems is strictly prohibited. Please respect this guidance to avoid conflict with the UCLA registrar regulations.

Exam proctoring will be administered during the originally scheduled for the 1B-1 lectures time period from 14:00 to 15:50 (Pacific) using Zoom (help can be found at https://ucla.zoom.us). Any questions and requests for clarification about the problems in the exam must be asked via Zoom private chat window to proctor to avoid disruption of the work of other students taking exam. If needed, the answers to the questions and requests will be provided to the whole class via Zoom.

Please use blank paper (or blank graph paper) to write your solutions. You can also use iPads, but in all cases, you are responsible to generate PDF file for the submission of your work for grading. Write your name, ID and problem number on each page to minimize possible misunderstanding.

The PDF files of your work must be uploaded to the Gradescope site no later than 16:00 (Pacific) on January 28th, 2021. Only PDF format is allowed to be uploaded.

Problems are not ordered in the order of complexity. Please look at all problems and start with those, which seem the most familiar to you. It is not expected that the majority of the class will complete all problems (including **Extra Credit** questions) in a given time. The hints are provided in each problem to help you to save time.

Problem	Points	Extra Credit
Problem 1	20	10
Problem 2	20	NA
Problem 3	NA	50
Problem 4	20	20
Problem 5	20	NA
Problem 6	30	NA

Name	ID	Total	Extra Credit	Percentage
		110	80	

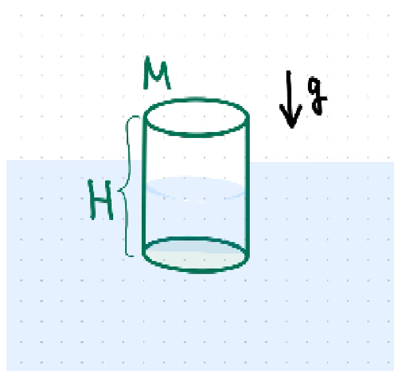
1. An empty cylindrical vessel with thin walls (inner and outer diameter are the same) has mass M and height H. The vessel is half full of water and is floating upright on the surface of the water pond. The mass of the water in the vessel, if it is filled up entirely, is m.

   a How deep below the surface of the water is the base of the vessel? (5 points)

   b What is the maximum mass of the vessel, M_{max}, which would allow vessel to still float on the surface of the pond given that it is half full of water? (5 points)

   c Suppose the vessel floats in equilibrium, and you push it down by a small distance $A \ll H$ and then let it oscillate freely (neglect viscosity and radiation of waves). What is the restoring force, F, on the vessel at the moment when you let it go? (10 points)

   d **[Extra Credit]** Use the restoring force that you found, explain why the motion of the vessel is simple harmonic motion (SHM) and find the angular frequency of this SHM. (10 points)



Solution

a $Mg + \frac{1}{2}mg = \frac{h}{H}mg$ Buoyant force. Hence, $h = H(\frac{1}{2} + \frac{M}{m})$.

b $d = H \rightarrow M = \frac{1}{2}m$.

c The net force upward $F = \frac{h+A}{H}mg - \left(Mg + \frac{1}{2}mg\right) = A\frac{mg}{H}$, where A is amplitude of the downward displacement.

d **[Extra Credit:]** The force obeys to Hooke's law and therefore the motion is SHM: $\left(M + \frac{1}{2}m\right)\ddot{z} = -z\frac{mg}{H}$. The frequency of the oscillations $\omega = \sqrt{\frac{g}{H}}\sqrt{\frac{2}{1+2\frac{M}{m}}}$.

2. In 1993 the radius of hurricane Emily was about 350 km. The wind speed near the center ("eye") of the hurricane, whose radius was about 30 km, reached about 200 $\frac{\text{km}}{\text{h}}$. As air swirled in from the rim of the hurricane toward the eye, its angular momentum remained roughly constant. Assume that the density of the air is $\rho_{air} = 1.2\frac{\text{kg}}{\text{m}^3}$ and it remains the same everywhere within hurricane morphology. Estimate

   a the wind speed at the rim of the hurricane; (5 points)

   b the pressure difference at the Earth's surface between the eye and the rim in units of atmospheric pressure $P_{atm} = 10^5 \frac{\text{N}}{\text{m}^2}$. (10 points)

   c If the kinetic energy of the swirling air in the eye could be converted completely to gravitational potential energy, how high would the air go? (5 points)

Solution

a Angular momentum conservation law implies that $rv = \text{const}$. Hence, $30\text{km} \cdot 200\frac{\text{km}}{\text{h}} = 350\text{km} \cdot v_{rim}$. The wind speed at the rim of the hurricane is $v_{rim} = 17\ \frac{\text{km}}{\text{h}}$.

b Per Bernoulli's equation $P_{eye} + \frac{1}{2}\rho_{air}v_{eye}^2 = P_{atm} + \frac{1}{2}\rho_{air}v_{rim}^2$. Therefore $P_{atm} - P_{eye} = \frac{1}{2}\rho_{air}\left(v_{eye}^2 - v_{rim}^2\right) = \frac{1}{2}1.2\left(\left(200 \times \frac{1000}{3600}\right)^2 - \left(17 \times \frac{1000}{3600}\right)^2\right) = 1838\frac{\text{N}}{\text{m}^2} = 0.018P_{atm}$.

c $\rho_{air}gh = \frac{1}{2}\rho_{air}v_{eye}^2 \rightarrow h = \frac{1}{2}\frac{v_{eye}^2}{g} = \frac{1}{2}\frac{\left(200\times\frac{1000}{3600}\right)^2}{9.8} = 157.5\ \text{m}$

3. **[Extra Credit]** A spherical elastically stretchable balloon is filled with helium on the ground. The nylon stretchable fabric of the balloon exerts additional pressure on the helium inside the balloon equal to $\frac{2\sigma}{R}$, where R is balloon radius and σ is fabric surface tension constant. The pressure of the helium inside exceeds the atmospheric pressure, P_0, by a small fraction ε to maintain the radius of the balloon R_0 on the ground. The total mass of the helium balloon with the payload is M and the total mass of the air displaced by the balloon is $m = \frac{4}{3}\pi R_0^3 \rho_0$, where ρ_0 is the density of air on the ground. Given that the m is significantly larger than M, the buoyant force lifts the balloon to a very high altitude, H, where it comes to an equilibrium. The density and the pressure of the air at this altitude are dramatically reduced so that the balloon expands to a sphere of radius R_H.

   a Find an expression for the surface tension of the fabric σ, given R_0 and overpressure of the helium εP_0. (5 points)

   b Assume that the helium in the balloon is an ideal gas in which case the ratio of the pressure to the density and temperature remain constant and independent of altitude i.e. $\frac{P_{\mathrm{He}}(H)}{\rho_{\mathrm{He}}(H)T_H} = \frac{(1+\varepsilon)P_0}{\rho_{\mathrm{He}}(0)T_0}$, where T_0 and T_H are temperatures of the helium at the ground and altitude H. Find the equation for the pressure of the helium $P_{\mathrm{He}}(H)$ inside the balloon, if the ratio of radii $\frac{R_0}{R_H}$ of the balloon and the ratio of temperatures $\frac{T_H}{T_0}$ are known. (10 points) *Hint: the mass of the helium in the balloon remains the same at all altitudes.*

   c The relationship $\frac{P_{\mathrm{air}}(H)}{\rho_{\mathrm{air}}(H)T_H} = \frac{P_0}{\rho_0 T_0}$ between the air pressure and air density and its temperature as a function of altitude is also nearly exact for the atmosphere. Find the air pressure $P_{\mathrm{air}}(H)$ as a function of the balloon radius R_H given that at this altitude the buoyant force is in equilibrium with the balloon weight. Express your result in terms of the ratios $\frac{M}{m}$, $\frac{R_0}{R_H}$ and $\frac{T_H}{T_0}$. (10 points)

   d At high altitude the inside helium pressure and the surface tension pressure of the balloon are in equilibrium with the outside pressure. Use this condition to derive the equation for the ratio of $\frac{R_H}{R_0}$ in terms of initial helium overpressure ε and the ratios of $\frac{M}{m}$ and $\frac{T_H}{T_0}$. (10 points) Estimate R_H, if $R_0 = 10\mathrm{m}$, $\frac{M}{m} = \frac{1}{4}$, $\frac{T_H}{T_0} = \frac{4}{5}$ and $\varepsilon = 0.07$. (5 points) *Note: Atmospheric temperature in the range of altitudes between* 10 *and* 35 km *is nearly constant and is around* $-50°C$ *(*223K*).*

   e Use parameters given in d to estimate by how much the air density is diluted at this high altitude, $\frac{\rho_{\mathrm{air}}(H)}{\rho_0} = ?$. (10 points)

Solution

a $P_0 + \frac{2\sigma}{R_0} = (1+\varepsilon)P_0 \rightarrow 2\sigma = R_0 \varepsilon P_0$

b $\frac{4}{3}\pi R_0^3 \rho_{\mathrm{He}}(0) = \frac{4}{3}\pi R_H^3 \rho_{\mathrm{He}}(H) \rightarrow \frac{\rho_{\mathrm{He}}(H)}{\rho_{\mathrm{He}}(0)} = \left(\frac{R_0}{R_H}\right)^3$

$\frac{P_{\mathrm{He}}(H)}{\rho_{\mathrm{He}}(H)T_H} = \frac{(1+\varepsilon)P_0}{\rho_{\mathrm{He}}(0)T_0}. \rightarrow P_{\mathrm{He}}(H) = (1+\varepsilon)P_0 \frac{T_H}{T_0}\frac{\rho_{\mathrm{He}}(H)}{\rho_{\mathrm{He}}(0)} = (1+\varepsilon)P_0 \frac{T_H}{T_0}\left(\frac{R_0}{R_H}\right)^3$

c $\frac{4}{3}\pi R_H^3 \rho_{\mathrm{air}}(H) g = Mg$ buoyant force in equilibrium with gravity. $\rho_{\mathrm{air}}(H) = \frac{M}{\frac{4}{3}\pi R_0^3}\left(\frac{R_0}{R_H}\right)^3$

$\frac{\rho_{\mathrm{air}}(H)}{\rho_0} = \frac{M}{m}\left(\frac{R_0}{R_H}\right)^3$, where $m = \frac{4}{3}\pi R_0^3 \rho_0$

$\frac{P_{\mathrm{air}}(H)}{\rho_{\mathrm{air}}(H)T_H} = \frac{P_0}{\rho_0 T_0} \rightarrow P_{\mathrm{air}}(H) = P_0 \frac{\rho_{\mathrm{air}}(H)}{\rho_0} = \frac{M}{m}P_0 \frac{T_H}{T_0}\left(\frac{R_0}{R_H}\right)^3$

d $P_{\mathrm{air}}(H) + \frac{2\sigma}{R_H} = P_{\mathrm{He}}(H)$

$\frac{M}{m}P_0 \frac{T_H}{T_0}\left(\frac{R_0}{R_H}\right)^3 + P_0\varepsilon\left(\frac{R_0}{R_H}\right) = P_0(1+\varepsilon)\frac{T_H}{T_0}\left(\frac{R_0}{R_H}\right)^3$

$\varepsilon = \left(1 - \frac{M}{m} + \varepsilon\right)\frac{T_H}{T_0}\left(\frac{R_0}{R_H}\right)^2 \rightarrow \frac{R_H}{R_0} = \sqrt{\frac{T_H}{T_0}\frac{\left(1-\frac{M}{m}+\varepsilon\right)}{\varepsilon}}$, $R_H = 10\mathrm{m}\sqrt{\frac{4}{5}\frac{\left(1-\frac{1}{4}+0.07\right)}{0.07}} = 30.6\mathrm{m}$

e $\frac{\rho_{\mathrm{air}}(H)}{\rho_0} = \frac{M}{m}\left(\frac{R_0}{R_H}\right)^3 = \frac{M}{m}\left(\frac{T_H}{T_0}\frac{\varepsilon}{1-\frac{M}{m}+\varepsilon}\right)^{\frac{3}{2}} = \frac{1}{4}\left(\frac{4}{5}\frac{0.07}{1-\frac{1}{4}+0.07}\right)^{\frac{3}{2}} = 4.5 \times 10^{-3}$

4. A bungee jumper with mass 72.0 kg jumps from a high bridge. The spring constant of the bungee cord is equal to 150 $\frac{\mathrm{N}}{\mathrm{m}}$. After reaching his lowest point, he oscillates up and down many times until he finally comes to rest 25.0 m below the level of the bridge. When the amplitude of oscillations was reduced to 4 m, it was measured that 10% of the energy of oscillations is dissipated every cycle due to damping. Calculate

   a the unstretched length of the bungee cord; (10 points)

   b the lowest point reached by bungee jumper assuming that his initial velocity was equal to zero and neglecting the energy loss caused by damping during the first cycle of oscillations; (10 points)

   c **[Extra Credit:]** the damping ratio ($\gamma = \frac{b}{2m\omega}$, where b is damping constant and ω angular frequency of oscillations without damping), the period of the oscillations and the mean lifetime of the oscillation, $\tau = \frac{m}{b}$. (20 points)

Solution

Calculate

a the unstretched length of the bungee cord
$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{150\frac{\mathrm{N}}{\mathrm{m}}}{72.0\mathrm{kg}}} = 1.45\frac{\mathrm{rad}}{\mathrm{sec}}$
$k\Delta l_0 = mg$; $\Delta l_0 = \frac{g}{\omega^2} = 4.66$ m; $l_0 = 25.0 - 4.66 = 20.34$ m

b the lowest point reached by bungee jumper assuming that his initial velocity was equal to zero and neglecting the energy loss caused by damping during the first cycle of oscillations
The energy conservation law requires
$\frac{k\Delta L^2}{2} = mg\left(l_0 + \Delta L\right); \Delta L^2 = 2\Delta l_0\left(l_0 + \Delta L\right); \Delta L = \Delta l_0 + \sqrt{\left(l_0 + \Delta l_0\right)^2 - l_0^2} = 4.66 + \sqrt{25.0^2 - 20.34^2} =$ 19.19 m
$l_0 + \Delta L = 39.5$ m below the level of the bridge

c **[Extra Credit:]** The $\frac{2\pi}{T} = \omega\sqrt{1 - \frac{b^2}{4m^2\omega^2}} = \omega\sqrt{1-\gamma^2}$ and therefore $T = \frac{2\pi}{\omega}\frac{1}{\sqrt{1-\gamma^2}}$.
The percentage energy loss per cycle is $1 - 0.1 = e^{-2\frac{b}{2m}T} = e^{-\frac{b}{2m\omega}\frac{4\pi}{\sqrt{1-\gamma^2}}} = e^{-4\pi\frac{\gamma}{\sqrt{1-\gamma^2}}} = 0.9$ and $\frac{\gamma}{\sqrt{1-\gamma^2}} = -\frac{1}{4\pi}\ln(0.9) = 8.3843 \times 10^{-3}$. The damping ratio is $\gamma = 8.3840 \times 10^{-3} \ll 1$.
The period of the oscillations $T = \frac{2\pi}{\omega}\frac{1}{\sqrt{1-\gamma^2}} = 4.3531\,\mathrm{sec}\,\frac{1}{\sqrt{1-\gamma^2}} = 4.3533\,\mathrm{sec}$
The mean lifetime $\tau = \frac{m}{b} = \frac{1}{2\gamma\omega} = \frac{1}{2\times1.45\times8.3840\times10^{-3}}\,\mathrm{sec} = 41.129\,\mathrm{sec}$.

5. The shape of a transverse wave is described by the wave function $y(x,t) = 0.45\text{mm}\cdot\cos\left(\frac{3.0}{\text{cm}}\cdot x - \frac{1.2}{\text{sec}}\cdot t\right) - 0.22\text{mm}\cdot\sin\left(\frac{3.0}{\text{cm}}\cdot x - \frac{1.2}{\text{sec}}\cdot t\right)$. Find

   a the wave number and the angular frequency; (2+2 points)

   b the wavelength and the period; (3+3 points)

   c the wave velocity; (4 points)

   d the wave amplitude. (6 points)

Solution

a the wave number and angular frequency; $k = 3.0\ \frac{\text{rad}}{\text{cm}}$; $\omega = 1.2\ \frac{\text{rad}}{\text{sec}}$

b the wavelength and the period; $\lambda = \frac{2\pi}{k} = 2.1$ cm; $T = \frac{2\pi}{\omega} = 5.2$ sec

c the wave velocity; $v = \frac{\omega}{k} = \frac{\lambda}{T} = \frac{2.1\text{cm}}{5.2\text{sec}} = 0.4\ \frac{\text{cm}}{\text{sec}}$

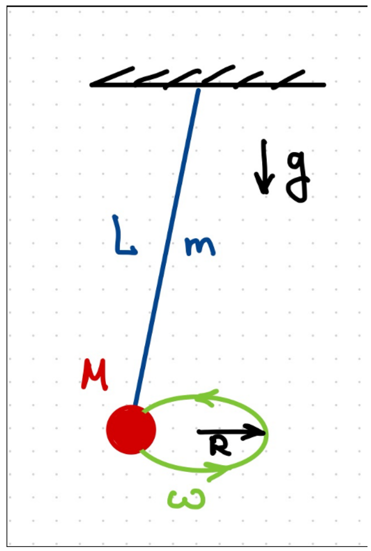
d the wave amplitude and its phase. $A = \sqrt{0.45^2 + 0.22^2}\text{mm} = 0.5\text{mm}$, $\cos\theta = \frac{0.45\text{mm}}{0.5\text{mm}} = 0.9$, $\sin\theta = \frac{0.22\text{mm}}{0.5\text{mm}} = 0.44$, hence $\theta = 0.456$ rad (26.1°) in $y(x,t) = A\cos\left(\frac{3.0}{\text{cm}}\cdot x - \frac{1.2}{\text{sec}}\cdot t + \theta\right)$.

6. A metal sphere of mass $M = 50$kg swings at the end of a thin wire of mass $m = 104$g in a circular motion in the field of gravity (see Figure).

   a Given that $\frac{m}{M} \ll 1$, neglect the mass of the wire in your calculations and write out Newton's force equations for the system. (10 points)

   b How many times can a wave pulse travel the length of the wire L while the metal sphere completes one circular rotation? (10 points)

   c The wire has gauge 8 (diameter 4.1mm) and it is made from steel (density $7.85 \times 10^3 \frac{\text{kg}}{\text{m}^3}$). If the radius of the circular motion, R, of the metal sphere is much smaller than the length of the wire L ($\frac{R}{L} \ll 1$), what is the period of rotation of the sphere and the travel time of the wave pulse through the length of the wire? (10 points)



Solution

a The balance of forces on the metal sphere
$M\omega^2 R = F\sin\theta = F\frac{R}{L} \rightarrow \frac{F}{\frac{m}{L}} = \frac{M}{m}\omega^2 L^2 = v^2$

$Mg = F\cos\theta = F\sqrt{1-\frac{R^2}{L^2}} = M\omega^2 L\sqrt{1-\frac{R^2}{L^2}} \approx M\omega^2 L$

b Thus $\frac{vT}{L} = N = 2\pi\sqrt{\frac{M}{m}} = 2\pi\sqrt{\frac{50}{0.104}} \approx 138$

c $L = \frac{m}{\rho\cdot\frac{\pi}{4}d^2} = \frac{104\text{g}}{7.85\frac{\text{g}}{\text{cm}^3}\frac{\pi}{4}0.41^2\text{cm}^2} = 100.35\text{cm}$ and $\omega = \sqrt{\frac{g}{L}} = \sqrt{\frac{9.8\frac{\text{m}}{\text{sec}^2}}{1.0\text{m}}} = 3.13\frac{\text{rad}}{\text{sec}}$, $T = \frac{2\pi}{\omega} = \frac{2\pi}{3.13\frac{\text{rad}}{\text{sec}}} = 2.00\,\text{sec}$. Hence the travel time through the wire $\frac{2\,\text{sec}}{138} = 1.45 \times 10^{-2}$ sec.