20W-PHYSICS1B-1 Midterm 2

TOTAL POINTS

102 / 105

QUESTION 1

Problem 1 15 pts

1.1 (A) 5 / 5

✓ + **5 pts** **Correct**

+ **0 pts** Incorrect

1.2 (B) 5 / 5

✓ + **5 pts** **Correct**

+ **0 pts** Incorrect

1.3 (C) 5 / 5

✓ + **5 pts** **Correct**

+ **0 pts** Incorrect

QUESTION 2

Problem 2 30 pts

2.1 (A) 7 / 10

✓ + **2 pts** **Gauss Law**

✓ + **2 pts** **Correct formula for E**

✓ + **2 pts** **Correct calculation for E**

+ **2 pts** Correct formula for Q

+ **2 pts** Correct calculation for Q

+ **0 pts** Wrong

+ 1 Point adjustment

2.2 (B) 10 / 10

✓ + **2.5 pts** **Correct formula**

✓ + **2.5 pts** **Correct interpretation as a point charge**

✓ + **2.5 pts** **Correct total charge enclosed for the system**

✓ + **2.5 pts** **Correct calculation**

+ **0 pts** wrong

2.3 (C) 10 / 10

✓ + **2 pts** **Correct Initial energy**

✓ + **1 pts** **Correct initial energy calculation**

✓ + **2 pts** **Correct final energy**

✓ + **1 pts** **Correct final energy calculation**

✓ + **2 pts** **Energy conservation equation**

✓ + **2 pts** **Correct calculation**

+ **0 pts** Wrong

QUESTION 3

Problem 3 30 pts

3.1 (A) 10 / 10

✓ + **2 pts** **Correct electric field on left**

✓ + **2 pts** **Correct electric field on right**

✓ + **2 pts** **Correct formula of electric field**

✓ + **1 pts** **Correct field direction between plates**

✓ + **2 pts** **Correct calculation of sigma**

✓ + **1 pts** **Correct calculation of electric field between plates**

+ **0 pts** Wrong

3.2 (B) 10 / 10

✓ + **3 pts** **V = Ed or integral**

✓ + **4 pts** **Correct Expression for V**

✓ + **3 pts** **Correct Numerical Value**

+ **0 pts** Incorrect

3.3 (C) 10 / 10

✓ + **3 pts** **E_{i} = E_{f}**

✓ + **4 pts** **Correct Energies**

✓ + **3 pts** **Final Answer**

+ **0 pts** Incorrect

QUESTION 4

Problem 4 30 pts

4.1 (A) 10 / 10

✓ + 2 pts Potential Integral

✓ + 2 pts Potential Calculations

✓ + 2 pts Correct Potential

✓ + 2 pts E = - grad(V)

✓ + 2 pts Correct E given V

+ 0 pts Incorrect

4.2 (B) 10 / 10

✓ + 2 pts Potential Integral

✓ + 2 pts Potential Calculations

✓ + 2 pts Correct Potential

✓ + 2 pts E = -grad(V)

✓ + 2 pts Correct E given V

+ 0 pts Incorrect

4.3 (C) 10 / 10

✓ + 5 pts E_{x} is non-zero

✓ + 5 pts Reasoning

+ 0 pts Incorrect

+ 3 pts Mention symmetry (only if incorrect answer)

Write your name here:

Write your UCLA ID here

Midterm #2, Physics 1B, Winter 2020

Section 1 – Thomas Dumitrescu

- Please write your name and UID in the boxes on the front page and your name in the boxes at the top of the odd numbered pages.

- Please write your answers within the margins outlined by the boxes on each page.

- if you are using the "additional space" pages, please label them carefully and refer to them within the answer box for the original problem.

- Closed book, **one** 5x3in note card (both sides) allowed.

- Scientific Calculators allowed, no computers or smartphones, please put books and notebooks in your backpacks.

- If a problem is ambiguous, notify the instructor. Clarifications will be written on the blackboard. Check the board occasionally.

- Time for exam: 60 minutes

- There are 4 questions, check that your exam has all 13 pages.

- Useful quantities:

$$\begin{aligned} \epsilon_0 &= 8.85 \times 10^{-12}\,\mathrm{C}^2\mathrm{m}^{-2}\mathrm{N}^{-1} \\ m_{\text{electron}} &= 9.11 \times 10^{-31}\,\mathrm{kg} \\ m_{\text{proton}} &= 1.67 \times 10^{-27}\,\mathrm{kg} \\ q_{\text{electron}} = -q_{\text{proton}} &= -1.602 \times 10^{-19}\,\mathrm{C} \end{aligned}$$

Good Luck !!

-additional space for calculation- Please denote exactly which problem you are working on

Write your name here:

Problem 1: [15pts] Concept questions

a) **[5pts]** A ficticious Gaussian surface (dotted) encloses three identical negative charges. A fourth, positive charge (equal in magnitude to each of the three negative charges inside) is then placed outside the surface as shown. How do the electric flux Φ through the surface and the magnitude $E = |\vec{E}|$ of the electric field at the point P change once the positive charge is introduced?

(1) They both stay the same.

(2) Φ increases, but E stays the same. ✗

(3) Φ decreases, but E stays the same. ✗

(4) Φ stays the same, but E increases. ✗

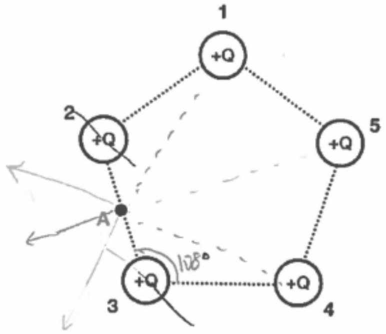
[✓ **(5)**] Φ stays the same, but E decreases.

(6) Φ and E both increase. ✗

(7) Φ and E both decrease. ✗

Φ same

b) [5pts] Consider a regular pentagon, whose corners are labeled 1, 2, 3, 4, 5 (see figure below). A positive charge $+Q$ is placed at every corner. What is the direction of the electric field at the point A, which is exactly halfway between corners 2 and 3.



(1) Towards corner 2 X

(2) Towards corner 3 X

(3) Towards corner 4 X

(4) Away from corner 4

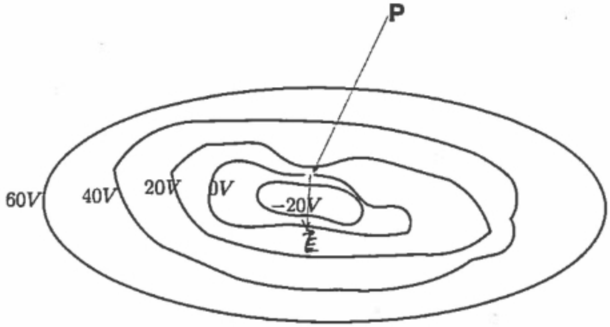
(5) Towards corner 5 X

[✓ (6)] Away from corner 5

(7) The electric field at point A is zero

Write your name here:

c) **[5pts]** The figure below shows equipotential surfaces with their potential given in Volts. If a negative point charge is placed at point P (indicated by the dot on the $0V$ equipotential) so that it is initially at rest, in which direction would it move?



[✓ **(1)** Up]

(2) Down

(3) Left

(4) Right

(5) Since the charge is placed on the equipotential with $V = 0$, it will remain at rest.

$\vec{E}$ points in dir of steepest descent

e^- will move opposite to proton

$\Rightarrow$ low V to high V

Problem 2: [30pts] An atomic nucleus can be modeled as a spherically symmetric charge distribution with the following charge density,

$$\rho(r) = \begin{cases} \rho_0(1 - \frac{r}{R}) & (r \leq R) \\ 0 & (r > R) \end{cases}$$

Here R is the radius of the nucleus and ρ_0 denotes the charge density at the center of the nucleus (at $r = 0$).

a) [10pts] Find the magnitude and the direction of the electric field everywhere as a function of position. Express your answer in terms of the total charge Q of the nucleus.

$r \leq R$: $\int \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$

$$EA = \frac{q_{enc}}{\varepsilon_0}$$

$$E \cdot \cancel{4\pi} r^2 = \frac{\cancel{4\pi}\rho_0\left(\frac{r^3}{3} - \frac{r^4}{4R}\right)}{\varepsilon_0}$$

$r \leq R$: $E(r) = \frac{\rho_0\left(\frac{r}{3} - \frac{r^2}{4R}\right)}{\varepsilon_0}$

($\vec{E} \parallel d\vec{A}$ by spherical symmetry) and E same for same radial dist from center

$$q_{enc} = \int \rho(r)\, dV$$

$$= \int_0^r \rho_0\left(1 - \frac{r}{R}\right) \cdot 4\pi r^2\, dr$$

$$= 4\pi\rho_0 \int_0^r r^2 - \frac{r^3}{R}\, dr$$

$$= 4\pi\rho_0 \left[\frac{r^3}{3} - \frac{r^4}{4R}\right]_0^r$$

$$= 4\pi\rho_0\left(\frac{r^3}{3} - \frac{r^4}{4R}\right)$$

$r > R$: $\rho(r) = 0$

$$\Rightarrow EA = \frac{q_{enc}}{\varepsilon_0} = \frac{Q}{\varepsilon_0}$$

$$E = \frac{Q}{4\pi r^2 \varepsilon_0}$$

$$\boxed{E(r) = \begin{cases} \frac{\rho_0}{\varepsilon_0}\left(\frac{r^3}{3} - \frac{r^4}{4R}\right) & r \leq R \\ \frac{Q}{4\pi r^2 \varepsilon_0} & r > R \end{cases}}$$

Write your name here:

b) **[10pts]** Find the potential difference $V(r) - V(\infty)$ for any point $r > R$ (i.e. outside the nucleus). Explain your reasoning.

For point outside nucleus $(r > R)$, the nucleus acts like a point charge with charge Q, as demonstrated by part (a). Therefore, $V(r) - V(\infty)$ is like the potential difference for point charge, which is $\boxed{\frac{kQ}{r}}$.

c) **[10pts]** A nucleus can decay by emitting an alpha particle, which consists of two protons and two neutrons (i.e. its mass is $4m_{\text{proton}}$ and its charge is $2q_{\text{proton}}$). Quantum physics makes it possible for the alpha particle to jump to a finite distance $r_0 > R$ outside the nucleus. Assuming that the alpha particle starts from rest at $r_0 = 10^{-14}$m, what is its escape speed far away from the nucleus? Assume that the charge of the nucleus after it has emitted the alpha particle is $Q = 90q_{\text{proton}}$, and that the nucleus is very heavy, so it remains at rest.

$$U_i + \cancelto{0}{K_i} + \cancelto{0}{W_{ext}} = K_f + U_f$$

treat nucleus as point charge outside

$$V_{\text{point}} = \frac{kQ}{r}$$

$$K_f = U_i - U_f = q_\alpha (V_i - V_f)$$

$$\frac{1}{2} m v^2 = q_\alpha \left(\frac{kQ}{r_0} - 0 \right)$$

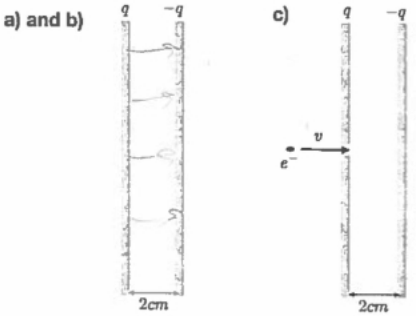
$$\begin{bmatrix} q_{\text{proton}} = q \\ m_{\text{proton}} = m \end{bmatrix}$$

$\frac{1}{r} \rightarrow 0$ far away from nucleus, so $V_f = 0$

$$\frac{1}{2} m_\alpha v^2 = q_\alpha \left(\frac{kQ}{r_0} \right)$$

$$v = \sqrt{\frac{2 q_\alpha kQ}{m_\alpha r_0}} = \sqrt{\frac{2 \cdot 2q \cdot k \cdot 90q}{4 m r_0}} = \boxed{3.53 \times 10^7 \text{ m/s}}$$

Problem 3: **[30pts]** Two very thin conducting sheets are parallel disks with radius $r = 0.4\text{m}$ and they are 2cm apart. The left sheet has charge $q = +1.0 \times 10^{-9}\text{C}$, and the right sheet has charge $-q$. You can model the electric fields as if the sheets were of infinite extent.

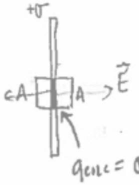


a) **[10pts]** What are the electric fields (magnitudes and directions) in the three regions, i.e. to the left of the left sheet, to the right of the right sheet, and in between the sheets?

$E_{\text{right sheet}} = E_r$

$$\int \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\varepsilon_0}$$

$q_{enc} = \sigma A$



For gaussian surface: $E_r 2\cancel{A} = \frac{q_{enc}}{\varepsilon_0}$

$$= \frac{-q}{\varepsilon_0} \cdot \underset{\sigma}{\frac{1}{\pi r^2}} \cdot \cancel{A}$$

$$E_l \cdot 2\cancel{A} = \frac{q_{enc}}{\varepsilon_0} = \frac{q}{\varepsilon_0} \cdot \frac{1}{\pi r^2} \cdot \cancel{A}$$

$$E_l = \frac{q}{2\pi\varepsilon_0 r^2} \qquad E_r = \frac{-q}{2\pi\varepsilon_0 r^2}$$

In between sheets: $E_{net} = E_l - E_r \Rightarrow \vec{E} = -\frac{q}{\pi\varepsilon_0 r^2}\hat{x} \Rightarrow \boxed{\vec{E} = 225\hat{x}\ \text{N/C}}$

Outside sheets: $E_{net} = E_l + E_r \rightarrow \boxed{\vec{E} = 0\ \text{N/C}}$
(by superposition)

Write your name here:

b) **[10pts]** What is the potential difference $V_{\text{left}} - V_{\text{right}}$ between the two sheets?

$$V_\ell - V_r = -\int_{\text{right}}^{\text{left}} \vec{E} \cdot d\vec{x} = -\int_{0.02}^{0} \frac{q}{\pi \varepsilon_0 r^2} \hat{x} \cdot d\vec{x} = -\int_{0.02}^{0} \frac{q}{\pi \varepsilon_0 r^2}\, dx$$

$$= \frac{q}{\pi \varepsilon_0 r^2} \left[x\right]_0^{0.02} = \frac{0.02\, q}{\pi \varepsilon_0 r^2} = \boxed{4.50\ V}$$

c) **[10pts]** An electron e^- is shot through a very small hole in the left sheet with speed v, toward the right. (You can neglect the effect of the hole on the electric fields.) What is the smallest value of v for which the electron manages to reach the negatively charged plate?

$$U_i + K_i + \cancelto{0}{W_{ext}} = U_f + \cancelto{0}{K_f}$$

0 because electron will stop @ min v

$$U_i + K_i = U_f$$

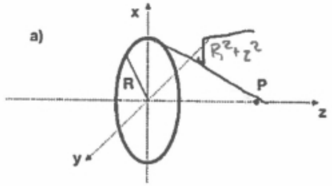
$$K_i = U_f - U_i = q\Delta V = q(V_r - V_\ell)$$

$$\frac{1}{2} m v^2 = q(V_r - V_\ell) \qquad q = -1.602 \times 10^{-19}\ C$$

$$v = \sqrt{\frac{2q(V_r - V_\ell)}{m}} \approx \boxed{1.26 \times 10^6\ m/s}$$

$$V_\ell - V_r \approx 4.50\ V$$

Problem4: [30pts] a) [10pts] Consider a thin ring of charge. The ring has radius R and lies in the xy-plane, centered at the origin (see figure). Assume that the ring has a uniform linear charge density λ. Compute the electrostatic potential $V(z)$ at a point P on the z-axis, as a function of z. Deduce the magnitude and direction of the electric field on the z-axis.



$$V = \int dV = \int \frac{k\,dq}{r} \qquad dq = \lambda\,ds$$

$$= \int \frac{k\lambda\,ds}{\sqrt{R^2+z^2}} = \frac{k\lambda}{\sqrt{R^2+z^2}} \int ds = \frac{2\pi R k\lambda}{\sqrt{R^2+z^2}}$$

$$\boxed{V(z) = \frac{2\pi R\,k\lambda}{\sqrt{R^2+z^2}}}$$

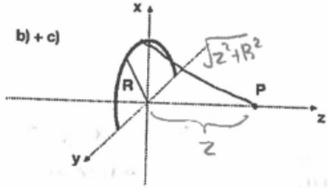
$$\|E\| = -\frac{\partial}{\partial z} V(z) = \frac{2\pi R k\lambda}{(R^2+z^2)^{3/2}} \cdot \cancel{2z \cdot \frac{1}{2}}$$

$$\boxed{E(z) = \frac{2\pi R k\lambda z}{(R^2+z^2)^{3/2}}}$$

Direction of $\vec{E}$ points in +z direction on z-axis

Write your name here:

b) [10pts] Now remove the bottom half of the ring in the previous problem, i.e. consider a charged semicircle lying in the upper half of the xy-plane (where $x > 0$, see figure), centered at the origin, with radius R and linear charge density λ. Find the electrostatic potential $V(z)$ at a point P on the z-axis, as a function of z, for this new situation. Use your answer to determine the z-component $E_z(z)$ of the electric field on the z-axis.



$$V = \int dV = \int \frac{k\,dq}{r} \quad \int_0^R \frac{k\lambda\,ds}{\sqrt{z^2+R^2}} \qquad dq = \lambda\,ds$$

$$= \frac{k\lambda}{\sqrt{z^2+R^2}} \int ds = \boxed{\frac{\pi R k\lambda}{\sqrt{z^2+R^2}} = V(z)}$$

$$E_z(z) = -\frac{\partial}{\partial z}\left(V(z)\right) = -1\,\frac{\pi R k\lambda}{(z^2+R^2)^{3/2}} \cdot 2z \cdot \frac{1}{2} = \frac{\pi R k\lambda z}{(z^2+R^2)^{3/2}}$$

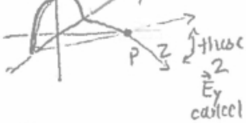
$$\boxed{E_z(z) = \frac{\pi R k\lambda z}{(z^2+R^2)^{3/2}}}$$

$$E_z = \int \frac{k\lambda z\,ds}{(z^2+R^2)^{3/2}} = \frac{k\lambda z}{(z^2+R^2)^{3/2}} \int ds$$

$$= \frac{k\lambda z}{(z^2+R^2)^{3/2}} \cdot \pi R \checkmark$$

c) **[10pts]** Consider the other components $E_x(z)$, $E_y(z)$ at the point P on the z-axis. Explain why one of these components vanishes while the other does not, and which one is which.

$E_y(z)$ vanishes because the ring of charge is symmetrical about the x-axis. Therefore, the electric field in the y-direction cancel each other out at point P since at each point along the ring of charge, there is another point whose E_y is in the opposite direction.



$E_x(z)$ does not vanish because only the ring of charge lies in the + x-axis side. Thus, the $\vec{E}_x$ all point in the (-) x-axis direction or along the z-axis at point P for all points on the ring of charge.

Write your name here:

-additional space for calculation- Please denote exactly which problem you are working on