Physics 1B – Midterm 2

July 17^{th}, 2015

Name: ____________________

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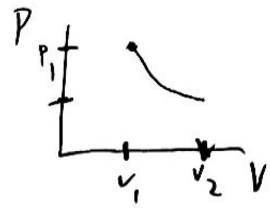
+20

(1) n moles of an ideal gas undergo a reversible isothermal expansion from V_1 to $V_2 > V_1$ at temperature T. What is the change in entropy ΔS of the gas?

$$\Delta S = \int \frac{dQ}{T}$$



$$pV = nRT$$

$$P_1 = \frac{nRT}{V_1}$$

$$dQ = dU + dW$$

$$\int dQ = \cancel{n C_V dT}^{0} + \int p\,dV$$

$$pV = nRT = \text{cte}$$

$$P_1 V_1 = P_2 V_2$$

$$p(V) = \frac{P_1 V_1}{V}$$

$$\Delta S = \frac{\int p\,dV}{T}$$

$$= \frac{\int_{V_1}^{V_2} \frac{P_1 V_1}{V}\,dV}{T}$$

$$\Delta S = \frac{P_1 V_1 \ln\left(\frac{V_2}{V_1}\right)}{T}$$

$$= \frac{\frac{nR\cancel{T}}{\cancel{V_1}} \cancel{V_1} \ln\left(\frac{V_2}{V_1}\right)}{\cancel{T}}$$

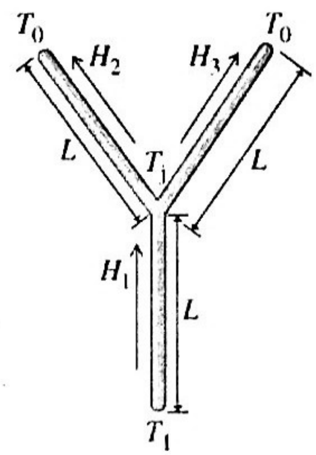
$$= nR\ln\left(\frac{V_2}{V_1}\right)$$

20

(2) Three identical rods are welded together to form a Y-shaped figure. The cross-sectional area of each rod is A, and they have length L and thermal conductivity k. See the figure below. The free end of rod 1 is maintained at T_1 and the free ends of rods 2 and 3 are maintained at a lower temperature T_0. **You may assume that there is no heat loss from the surfaces of the rods.**

(a) What is T_j, the temperature of the junction point? Express your answer in terms of T_1 and T_0. (Hint: Suppose you know the answer. Calculate the currents in terms of T_j. **Use the law of conservation of energy** to derive an equation with the unknown T_j. Solve for T_j.)

(b) Find the heat currents H_1, H_2 and H_3. Express the heat current in terms of any or all of k, L, A, and the temperatures T_1 and T_0.



a. $$\frac{kA}{L}(T_1 - T_j) = \frac{kA}{L}(T_j - T_0) + \frac{kA}{L}(T_j - T_0)$$

$$\cancel{\frac{kA}{L}} T_1 - \cancel{\frac{kA}{L}} T_j = \cancel{\frac{kA}{L}} T_j - \cancel{\frac{kA}{L}} T_0 + \cancel{\frac{kA}{L}} T_j - \cancel{\frac{kA}{L}} T_0$$

$$3T_j = T_1 + 2T_0$$

$$T_j = \frac{T_1 + 2T_0}{3}$$

b. $$H_1 = \frac{kA}{L}(T_1 - T_j) = \frac{kA}{L}\left(T_1 - \frac{T_1 + 2T_0}{3}\right)$$

$$= \frac{kA}{L}\left(\frac{2T_1 - 2T_0}{3}\right)$$

$$H_2 = H_3 = \frac{kA}{L}(T_j - T_0) = \frac{kA}{L}\left(\frac{T_1 + 2T_0}{3} - T_0\right) = \frac{kA}{L}\left(\frac{T_1 - T_0}{3}\right)$$

+20

(3) In this problem you are to consider an adiabatic expansion of an ideal diatomic gas, which means that the gas expands with no addition or subtraction of heat. Assume that a gas is initially at pressure p_0, volume V_0, and temperature T_0. In addition, assume that the temperature of the gas is such that you can neglect vibrational degrees of freedom. Thus, the ratio of heat capacities is $\gamma = C_p/C_V = 7/5$. Note that, unless explicitly stated, the variable γ should not appear in your answers–if needed use the fact that $\gamma = 7/5$ for an ideal diatomic gas.

(a) Find an analytic expression for $p(V)$, the pressure as a function of volume, during the adiabatic expansion. Express the pressure in terms of V and any or all of the given initial values p_0, T_0, and V_0.

(b) At the end of the adiabatic expansion, the gas fills a new volume V_1, where $V_1 > V_0$. Find W, the work done by the gas on the container during the expansion. Express the work in terms of p_0, V_0, and V_1. Your answer should not depend on temperature.

(c) Find ΔU, the change of internal energy of the gas during the adiabatic expansion from volume V_0 to volume V_1. Express the change of internal energy in terms of p_0, V_0, and/or V_1.

$p_0 \quad V_0 \quad T_0$

a. $pV^{\gamma} = cte$

+6

$$p_0 V_0^{\gamma} = p(V) V^{\gamma}$$

$$p(v) = \frac{p_0 V_0^{\gamma}}{V^{\gamma}}$$

$$p(v) = \frac{p_0 V_0^{7/5}}{V^{7/5}}$$

b.

$$W = \int p\,dV$$

$$= \int_{V_0}^{V_1} \frac{p_0 V_0^{7/5}}{V^{7/5}}\,dv = -\frac{5}{2} p_0 V_0^{7/5} V^{-2/5}\Big|_{V_0}^{V_1}$$

+8

$$= -\frac{5}{2} p_0 \left[V_1^{-2/5} V_0^{7/5} - V_0 \right]$$

adiabatic → no change in heat

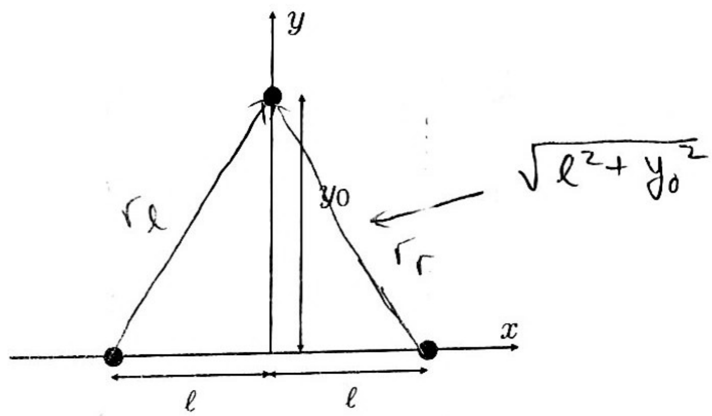
c. $\cancel{Q}^{0} = \Delta U + W$ +6

$$\Delta U = -W$$

$$\Delta U = \frac{5}{2} P_0 \left[V_1^{-2/5} V_0^{7/5} - V_0 \right]$$

15

(4) Two **identical**, positively charged particles $+q$, are located at $x = \pm\ell$. A third positive charge $+Q$ is sitting at $y = y_0$. For what value of y_0 does the third particle experience the largest electric force? Calculate the magnitude of this force.



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$$\vec{F} = \frac{kQq}{\ell^2 + y_0^2}\hat{r}_l + \frac{kQq}{\ell^2 + y_0^2}\hat{r}_r = m\vec{a}$$

$$\vec{r}_l = [\ell, y_0] \qquad \hat{r}_l = \left[\frac{\ell}{\sqrt{\ell^2+y_0^2}}, \frac{y_0}{\sqrt{\ell^2+y_0^2}}\right]$$

$$\vec{r}_r = [-\ell, y_0] \qquad \hat{r}_r = \left[\frac{-\ell}{\sqrt{\ell^2+y_0^2}}, \frac{y_0}{\sqrt{\ell^2+y_0^2}}\right]$$

$$\vec{F} = \frac{kQq}{\ell^2+y_0^2}\left[0, \frac{2y_0}{\sqrt{\ell^2+y_0^2}}\right] = m\vec{a}$$

$$\vec{F} = \frac{kQq\,2y_0}{(\ell^2+y_0^2)^{3/2}}\hat{j} \uparrow$$

$$0 = \frac{dF}{dy_0} = kQq2\left[\frac{(\ell^2+y_0^2)^{3/2} - y_0\frac{3}{2}(\ell^2+y_0^2)^{1/2}2y_0}{(\ell^2+y_0^2)^3}\right]$$

$$3y_0^2(\ell^2+y_0^2)^{1/2} = (\ell^2+y_0^2)^{3/2}$$

$$3y_0^2 = \ell^2 + y_0^2 \quad +1$$

$$2y_0^2 = \ell^2$$

$$y_0^2 = \frac{\ell^2}{2}$$

$$\boxed{y_0 = \frac{\ell}{\sqrt{2}}}$$

at $\frac{\ell}{\sqrt{2}}$, the third particle experiences the largest electric force,

$$F = \frac{kQq\,2y_0}{(\ell^2 + y_0^2)^{3/2}} = \frac{kQq\,\frac{2}{\sqrt{2}}\ell}{\left(\ell^2 + \frac{\ell^2}{2}\right)^{3/2}}$$

$$= \frac{kQq\,\sqrt{2}\ell}{\left(\frac{3\ell^2}{2}\right)^{3/2}}$$

$$= \frac{kQq\,\sqrt{2}\ell}{\frac{\sqrt{27}}{2\sqrt{2}}\ell^3} = \frac{4kQq}{3\sqrt{3}\ell^2}$$

Formula Sheet

$$\Delta U = Q - W, \qquad \text{First law of thermodynamics} \tag{1}$$

$$\Delta S = \int \frac{dQ}{T}, \qquad \text{reversible process} \tag{2}$$

$$pV = nRT, \qquad \text{equation of state of an ideal gas} \tag{3}$$

$$H = \frac{dQ}{dt} = \frac{kA}{L}(T_H - T_C), \qquad \text{heat current} \tag{4}$$

$$pV = \text{const.}, \qquad \text{isothermal process} \tag{5}$$

$$pV^{\gamma} = \text{const.}, \qquad \text{adiabatic process} \tag{6}$$

$$\mathbf{F} = \frac{kq_1q_2}{|r_{12}|^2}\hat{r}_{12}, \qquad \text{Coulomb's law} \tag{7}$$

$$\mathbf{F} = q\mathbf{E}, \qquad \text{electric force on charge } q \text{ in electric field } \mathbf{E} \tag{8}$$