Physics 1C – Midterm

Feb. 12th, 2016

Problem 1: 20/20

Problem 2: 20/20

Problem 3: 18/20

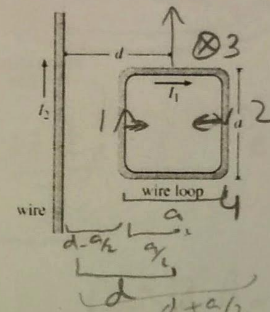
Total: 58 /60

20

(1) A square loop of wire with side length a carries a current I_1. The center of the loop is located a distance d from an infinite wire carrying a current I_2. The infinite wire and loop are in the same plane; two sides of the square loop are parallel to the wire and two are perpendicular as shown in figure below.

(a) What is the magnitude, F, of the net force on the loop? Express the force in terms of I_1, I_2, a, d, and μ_0.

(b) What is the magnitude of the magnetic moment, m? Express F from part (a) in terms of m.



b) magnetic moment (m)

$= IA$

$= I_1 \times a^2$

$F_{net} = \dfrac{2\mu_0 I_2 m}{(4d^2 - a^2)\pi}$

a) on segment 1, current in same direction as wire.

$B = \dfrac{\mu_0 I_2}{2\pi r}$

for 1, $r = d - a/2$

$B_1 = \dfrac{\mu_0 I_2}{2\pi(d - a/2)}$

$F_1 = I l B$

$= I_1 a \dfrac{\mu_0 I_2}{2\pi(d - a/2)}$

$= \dfrac{\mu_0 I_1 I_2 a}{2\pi(d - a/2)}$

similarly,

$B_2 = \dfrac{\mu_0 I_2}{2\pi(d + a/2)}$ $\quad F_2 = \dfrac{\mu_0 I_1 I_2 a}{2\pi(d + a/2)}$

We see from symmetry that F_3 & F_4 (Forces on segments 3 & 4) would cancel each other.

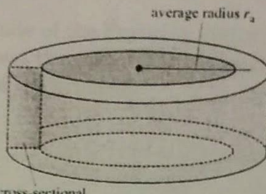
Thus

$F_{net} = F_1 - F_2 = \dfrac{\mu_0 I_1 I_2 a}{2\pi}\left[\dfrac{1}{(d - a/2)} - \dfrac{1}{(d + a/2)}\right]$

$= \dfrac{\mu_0 I_1 I_2 a}{2\pi}\left[\dfrac{\cancel{d} + a/2 - \cancel{d} + a/2}{d^2 - a^2/4}\right]$

$= \dfrac{2\mu_0 I_1 I_2 a^2}{(4d^2 - a^2)\pi}$

(2) An air-filled toroidal solenoid has a mean radius of r_a and a cross-sectional area of A_a (see the figure). The current flowing through it is I , and it is desired that the energy stored within the solenoid be at least U_0. What is the least number of turns that the winding must have?



$$B = \frac{\mu_0 N I}{2\pi r}$$

Energy $U = \frac{1}{2} L I^2$

$$L = \frac{N \Phi_B}{I}$$

$$\Phi_B = \int_S B \cdot dA = \frac{\mu_0 N I}{2\pi r} \cdot A_a$$

$$L = \frac{N \cdot \frac{\mu_0 N \cancel{I} \cdot A_a}{2\pi r}}{\cancel{I}} = \frac{N \mu_0 N A_a}{2\pi r_a} = \frac{N^2 \mu_0 A_a}{2\pi r_a}$$

$$\boxed{L = \frac{N^2 \mu_0 A_a}{2\pi r_a}}$$

$$U = \frac{1}{2} \frac{N^2 \mu_0 A_a}{2\pi r_a} \cdot I^2$$

$$U_0 = \frac{1}{4} \frac{N^2 \mu_0 A_a I^2}{\pi r_a}$$

$$N^2 = \frac{4 U_0 \pi r_a}{\mu_0 A_a I^2}$$

$$\boxed{N = \frac{2}{I} \sqrt{\frac{U_0 \pi r_a}{\mu_0 A_a}}}$$ ✓

20

$q = CV$

$\cancel{q = \epsilon E r}$

$\cancel{\frac{q}{\epsilon r} = E}$

(3) A capacitor is charged by a constant current i. At the beginning of this charging process ($t = 0$), there is no charge on the plates.

(a) Find the magnetic field inside the capacitor at radial distance r from the axis of the capacitor.

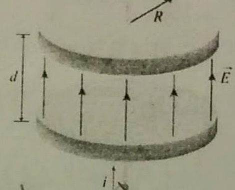
(b) Find an expression for the magnitude of the Poynting vector $|\mathbf{S}(t)|$ on the surface that connects the edges of the two circular plates. Express the magnitude of the Poynting vector in terms of t, i, R, π, ϵ_0, and other variables and parameters of the problem.

$$\oint B \cdot dl = \mu_0 \left[I + \epsilon_0 \frac{d\Phi_E}{dt} \right]_{encl}$$

~~capacitor~~

a) $B(2\pi r) = \mu_0 i \cdot \frac{r^2}{R^2}$

$\boxed{B = \frac{\mu_0 i r}{2\pi R^2}}$ +10



$\Phi_E = A E$

$= E \times \pi r^2$

$\Phi_{E_{max}} = E \times \pi R^2$

$\frac{\Phi_{E_r}}{\Phi_{E_{max}}} = \frac{r^2}{R^2}$

a) $B \cdot 2\pi r = \mu_0 \epsilon_0 \frac{d\Phi_E}{dt} \cdot \frac{r^2}{R^2}$

$B = \frac{\mu_0 i R^2}{2\pi r^3}$

$B = \frac{\mu_0 \epsilon_0}{2\pi r} \frac{d\Phi_E}{dt} \frac{r}{R^2}$

$B = \frac{\mu_0 \epsilon_0 r}{2\pi r R^2} \cdot \pi r^2 \times \frac{dE}{dt} = \frac{\mu_0 \epsilon_0 r}{2\pi R^2} \cdot \pi r^2 \cdot \frac{i}{2\pi r \epsilon_0} = \frac{\mu_0 i r^2}{4\pi R^2}$

$\oint E \cdot \cancel{dt} = \frac{q}{\epsilon_0}$

+8

$E \cdot 2\pi R = \frac{q}{\epsilon_0}$

$E = \frac{q}{2\pi R \epsilon_0}$

$\frac{dE}{dt} = \frac{i}{2\pi R \epsilon_0}$

b) $\mu_0 \vec{S} = \vec{E} \times \vec{B}$

$B = \frac{\mu_0 i}{2\pi R}$

$\vec{E} = \frac{}{2\pi R \epsilon_0}$

$|S| = \frac{|E||B|}{\mu_0}$

$|S| = \frac{\frac{\cancel{\mu_0} I}{2\pi R} \cdot \frac{q}{2\pi R \epsilon_0}}{\cancel{\mu_0}} = \cancel{\frac{i E}{2\pi R}}$

$= \frac{i}{2\pi R} \cdot \frac{q}{2\pi R \epsilon_0} = \frac{i \cdot i t}{4\pi R^2 \epsilon_0}$

$\boxed{|S| = \frac{i^2 t}{4\pi R^2 \epsilon_0}}$

Formula Sheet

$$\mathbf{F} = q\mathbf{v}\times\mathbf{B}, \qquad \mathbf{F} = I\vec{\ell}\times\mathbf{B} \tag{1}$$

$$\Phi_B = \int \mathbf{B}.d\mathbf{A}, \qquad \oint \mathbf{B}.d\mathbf{A} = 0, \qquad \oint \mathbf{E}.d\mathbf{A} = \frac{Q_{\rm enc}}{\epsilon_0} \tag{2}$$

$$\tau = \vec{\mu}\times\mathbf{B} \tag{3}$$

$$\mathbf{B} = \frac{\mu_0}{4\pi}\frac{q\mathbf{v}\times\hat{\mathbf{r}}}{r^2}, \qquad d\mathbf{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\hat{\mathbf{r}}}{r^2} \tag{4}$$

$$\oint \mathbf{B}.d\vec{\ell} = \mu_0\left(I_{\rm enc} + \epsilon_0\frac{d\Phi_E}{dt}\right), \qquad i_D = \epsilon_0\frac{d\Phi_E}{dt} \tag{5}$$

$$\mathcal{E} = \oint \mathbf{E}.d\vec{\ell} = -\frac{d\Phi_B}{dt} \tag{6}$$

$$\mathcal{E} = -L\frac{di}{dt}, \qquad L = N\Phi_B/i \tag{7}$$

$$U = \frac{1}{2}Li^2, \qquad u = \frac{B^2}{2\mu_0} \tag{8}$$

$$\tau_{RL} = L/R, \qquad \omega_{LC} = \sqrt{1/LC}, \qquad \omega'_{LRC} = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}} \tag{9}$$

$$I_{rav} = 2I/\pi, \qquad I_{rms} = I/\sqrt{2}, \qquad V_{rms} = V/\sqrt{2} \tag{10}$$

$$i = I\cos\omega t, \qquad v = V\cos(\omega t + \phi), \qquad V = IZ, \tag{11}$$

$$Z_{LRC} = \sqrt{R^2 + [\omega L - 1/\omega C]^2}, \qquad \tan\phi = \frac{\omega L - 1/\omega C}{R} \tag{12}$$

$$P_{avg} = \frac{1}{2}VI\cos\phi = V_{rms}I_{rms}\cos\phi \tag{13}$$

$$E = cB, \qquad c = 1/\sqrt{\epsilon_0\mu_0} \tag{14}$$

$$\mathbf{E} = E_{max}\cos(kx - \omega t)\hat{j}, \qquad \mathbf{B} = B_{max}\cos(kx - \omega t)\hat{k} \tag{15}$$

$$\mathbf{S} = \frac{1}{\mu_0}\mathbf{E}\times\mathbf{B}, \qquad I = S_{avg} = E_{max}B_{max}/2\mu_0, \qquad P_{rad} = I/c \quad \text{or} \quad 2I/c \tag{16}$$

$$kx = 0, \pi, 2\pi\ldots \quad (\text{nodal planes of } \mathbf{E}), \qquad kx = \pi/2, 3\pi/2, 5\pi/2\ldots \quad (\text{nodal planes of } \mathbf{B}) \tag{17}$$