

# Physics 1B - Midterm 1

*July 2, 2015*

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Problem 2: /20

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Problem 4: /15

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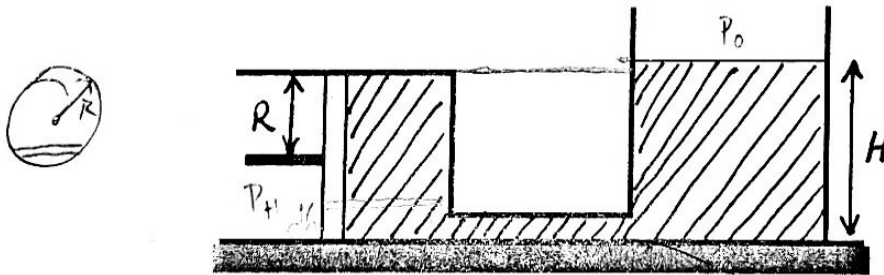
Total: 28 /75

76

**20 points**

1) Two cylinders with horizontal and vertical axes respectively, rest on a horizontal surface. The cylinders are connected from the bottom through a thin tube. The **horizontal** cylinder of radius  $R$ , is open at one end and has a piston in it. The **vertical** cylinder is open at the top. The cylinders contain water which completely fills the part of horizontal cylinder behind the piston and is at a certain level in the vertical cylinder. Determine the level  $H$  of water in the vertical cylinder at which the piston is in equilibrium. Neglect the friction.

**Hint:** You may need to carry out a simple integration, even though it is not necessary. Integration makes your analysis straightforward and easy!



$$P = \frac{F}{A}$$

$$P(y+h) - P(y) = \rho gh$$

$$F = AP$$

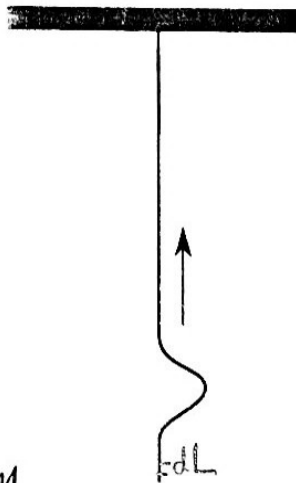
$$F_H = \pi R^2 \int_0^{2R} \rho g dh = \pi R^2 \rho gh \Big|_0^{2R} = 2\pi R^3 \rho g$$

$$F_v = P_0 A$$

20 points

- 2) A string of mass  $M$  and length  $L$  is hanging from the ceiling. You introduce a disturbance from the free end. The pulse travels up towards the ceiling. How long does it take for the pulse to reach the ceiling? Assume that the spatial extension of the pulse is negligible compared to  $L$ .

**Hint:** In order to calculate the total time, you need to find the velocity first!



$$v = \sqrt{\frac{T}{\mu}}$$

$$\mu = \frac{M}{L}$$

$$T - mg = 0$$

$T = mg$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{mg}{\frac{M}{L}}} = \sqrt{Lg}$$

$$vt = L$$

$$t = \frac{L}{v} = \frac{L}{\sqrt{Lg}} = \sqrt{\frac{L}{g}}$$

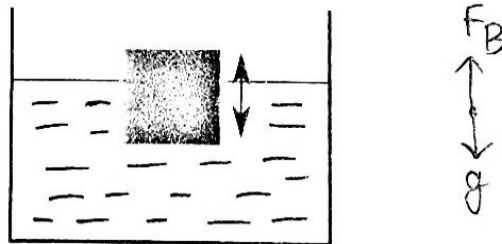
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20 points

3) The density of ice at the freezing point is less than that of the water at the same point. This is counterintuitive as most materials become denser as they solidify. Assume that the density of ice is given by  $\rho_i$  and that of water is  $\rho_w$ . We learned in the class that if the density of the object is larger than the liquid, the object completely submerges. Ice is different though:

- a) An ice cube of volume  $V$  is gently placed in water. Calculate the ratio of the submerged volume of the ice cube to its total volume.
- b) If you slightly push the ice cube down, the buoyancy force would bring it back towards its equilibrium position. Due to the initial energy that is given by your hand, the ice cube continues oscillating along the vertical direction. Find the frequency of small oscillations of the ice cube.

**Hint:** In order to find the frequency of small oscillations, try to derive the equation of motion. Then recast it in the form of the equation of the motion of a spring-mass system, for which we know how to obtain the frequency of oscillations.



a.

b.  $F_B - g = 0$  at eq.  
 $f(x) - g = 0$

$$\frac{d^2 y}{dt^2} + \omega^2 y = 0 \rightarrow \omega$$

$$F = m \frac{d^2 y}{dt^2}$$

$$\frac{d^2 y}{dt^2} - \frac{f(x) - g}{m} = 0$$

x2

15 points

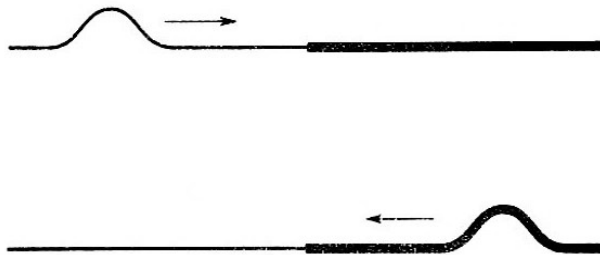
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4) Two strings are connected to one another. The left string has smaller mass per unit length. Describe in words:

a) what happens to the signal when it reaches the connection point, if you send a signal (disturbance) from the lighter string towards the heavier one?

b) what happens if you send the signal from the opposite end (from heavy to light string).

No parameters are needed. Simply explain the fate of a single signal which enters from a lower density to a higher density medium, and vice versa.



a. Some of the signal reflects back on the lighter string, and a slower pulse continues on the heavier string. <sup>+2 inverted?</sup>

+2

b. ~~The signal will continue down the lighter string with no reflection on the heavier string.~~ The signal will have a greater velocity as it moves to the lighter string.

+2

**Formula Sheet**

$$v = \sqrt{T/\mu}$$

$$P(y+h) - P(y) = \rho gh$$

$$\frac{d^2x}{dt^2} + \omega^2x = 0$$

$$P + \rho gy + \frac{1}{2}\rho v^2 = \text{const.}$$

$$y(x,t) = A_{sw} \sin \omega t \sin kx$$

$$f_n = \frac{nv}{2L}$$