Physics 1B - Winter 2017

MIDTERM EXAM #1

READ THIS BEFORE YOU BEGIN

- You are allowed to use only yourself and a writing instrument on this exam.
- If you finish more than 5 minutes before the end of the exam period, please raise your hand and a proctor will collect your exam. Otherwise, please stay in your seat until the end of time is called.
- When the exam is finished, please remain in your seat, pass your exam to the aisle, and the proctor(s) will come around and collect your exam. Once your exam is collected, you may leave the room.
- **Show all work.** The purpose of this exam is primarily to test how you think; you will get more partial credit for a logical, well-thought-out response, and ***you will get little or no credit for an answer without convincing reasoning.*** Points will be given specifically for the quality of your reasoning which includes clarity and conciseness.
- Please **box all of your final answers** to computational problems.
- You may use the back of any exam paper as room for extra work.

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Discussion Section # 1D(?) wednesday 5pm

+11

Problem 1.

Experiments show that sound travels roughly four times faster in water than in air despite water being about 1000 times denser.

a. **(5 points)** How is this possible? Isn't the speed of sound in a medium supposed to *decrease* with increasing density? Give a physical explanation by using the microscopic model of a longitudinal mechanical wave consisting of many tiny masses connected by springs.

b. **(3 points)** Give a mathematical explanation as well by appealing to the relationship between the speed of sound in a fluid and whatever relevant parameters the speed of sound depends on.

c. **(3 points)** Approximately what must be the ratio of the bulk modulus of air to that of water?

a. If a longitudinal wave is modeled by tiny masses connected by springs, this means that there are more masses and springs due to higher density, but the spring constant k is much larger for each spring. As a result, the wave propagates much quicker. +5

b. The speed of sound in a fluid is $v = \sqrt{\frac{B}{\rho}}$ where B is the bulk modulus and ρ is the density. The bulk modulus in water is much larger than the corresponding quantity in gases. +3

c. $V_{air} = \frac{1}{4} V_{water}$

$$\sqrt{\frac{B_{air}}{1}} = \frac{1}{4}\sqrt{\frac{B_{water}}{1000}}$$

$$B_{air} = \frac{1}{16}\frac{B_{water}}{1000}$$

+3

$B_{air} : B_{water} \rightarrow \boxed{1 : 16000}$

+22

Problem 2.

A small marble of mass m is released from rest near the bottom of a cylindrically symmetrical bowl having depth d, and it subsequently oscillates back and forth. If the bowl is placed on a table whose surface we take to be at $y = 0$, then the surface of the bowl on which the marble rolls is described in cross-section by the function

$$y(x) = d\left[1 - \cos\left(\frac{\pi x}{2R}\right)\right] \tag{1}$$

where $y(x)$ is the height of a point on the surface that is a displacement x from the bottom point of contact of the bowl with the table.

a. **(6 points)** Draw the bowl in cross-section in each of the following cases:

   i. $d/R \ll 1$

   ii. $d/R = 1$

   iii. $d/R \gg 1$

   where "$\ll$" means "much less than" and "$\gg$" means "much greater than." Be sure to indicate the variables d and R on your diagrams so that it's clear what they mean physically.

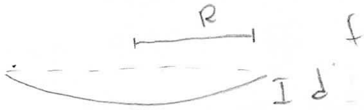
b. **(3 points)** Suppose that the marble is executing small oscillations about the bottom of the bowl, then should its frequency of small oscillations increase or decrease as a function of the ratio d/R? Explain in an intuitive, physical way why you expect this behavior.

c. **(7 points)** Determine the frequency f of small oscillations of the marble in terms of the variables given. For simplicity, you can treat the marble is if it were sliding without friction in the bowl instead of rolling.

d. **(3 points)** Use your answer from c. to estimate the value f would have for a real bowl.

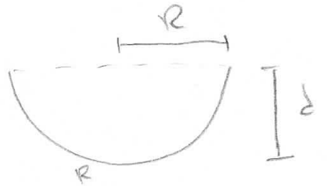
e. **(3 points)** Compare your expression from part c. to your expectation from part b.. Do these two answers agree? If they agree explain. If they don't agree, this means that either your intuition was wrong, or your math was wrong, or perhaps both. If your intuition was wrong, explain. If your math was wrong, you are encouraged to fix it.

a i. +2



$f \to 0$

ii. +2



$f \to c$

iii. +2

$f \to \infty$

+3

b. The frequency should increases as the ratio d/R increases. In the cross-section in ai, the restoring force for going a certain distance x away from the center, compared to aii and aiii. The frequency is proportional to the square root of the restoring force

c. $y(x) = d[1-\cos(\frac{\pi x}{2R})]$

$y'(x) = \frac{d\pi}{2R}\sin(\frac{\pi x}{2R})$

$W = \int F dx$

$F = ma$

$f = \frac{\omega}{2\pi}$

$U(x) = mgy$ +2

$U'(x) = mg\frac{d\pi}{2R}\sin(\frac{\pi x}{2R}) = F$

$a = g\frac{d\pi}{2R}\sin(\frac{\pi x}{2R})$ $\sin\frac{\pi x}{2R} \approx \frac{\pi x}{2R}$ when x is small

+2

$a \approx g\frac{d\pi^2}{4R^2}x$

$\omega^2 = \frac{gd\pi^2}{4R^2}$ +2

+1

$\omega = \frac{\pi}{2R}\sqrt{gd} \rightarrow \boxed{f = \frac{1}{4R}\sqrt{gd}}$

d. $R = 0.05$ m

$g = 10$ m/s^2 + 3

$d = 0.04$ m

$f = \frac{1}{4(0.05)}\sqrt{10 \cdot 0.04}$

$= \frac{1}{0.2}\sqrt{0.4} = \boxed{1.00 \text{ Hz}}$

e. Yes the two answers agree. Frequency is proportional to $\sqrt{\frac{R}{d}}$, but ther is an extra $\frac{1}{\sqrt{R}}$ multiplied to it. All the extremes of d/R ratios still have the same result in frequency

+3

+12

Problem 3.

Two tuning forks that each generate pure tones of frequency $f = 688\,\text{hz}$ are placed a distance $d = 5\,\text{m}$ apart in PAB 1-425 and are vibrating in phase. The air in the room is at standard temperature and pressure.

a. **(2 points)** There is an integer number of wavelengths that fit in the distance between the tuning forks, what is that number?

b. **(5 points)** Along the line joining the tuning forks, there is a certain number of displacement nodes. Use the fact that displacement nodes are points of destructive interference to determine how many such nodes there are along that line.

c. **(3 points)** Determine the distance between successive nodes.

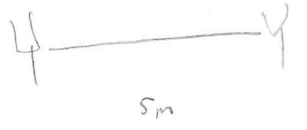
d. **(3 points)** Noting the result of part a. and noting that the locations of the forks are displacement anti-nodes, the displacement as a function of position and time, $y(x, t)$, of the air between the forks is the same as that of an open-open pipe of length d. In what harmonic would such a pipe have to vibrate so that the air inside would reproduce the wave profile $y(x, t)$ between the forks?

a. $v = \lambda f$

$\lambda = \frac{v}{f} = \frac{344}{688} = 0.5\ \text{m}$

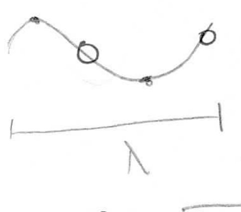
+ 2

$\frac{5}{0.5} = \boxed{10 \text{ wavelengths}}$ ✓

b. 

$10 \cdot 2 = \boxed{20 \text{ displacement nodes}}$

+4



c. $\frac{d}{20} = \frac{5}{20} = \boxed{0.25\ \text{m}}$ ✓ +3

d. $f_n = \frac{nv}{2L} = \frac{nv}{2d}$ $\quad \frac{2d}{n} = \lambda$ +3

$n = \frac{2d}{\lambda} = \frac{2 \cdot 5}{0.5} = 20$ ✓

$\boxed{20\text{th harmonic}}$

$f_L = \frac{V + V_L}{V + V_S} f_S$

Problem 4.

+20

Nancy is a notorious gangster with many enemies, and she has commissioned a corrupt physicist to fit her car with a device that can detect at what speed cars are coming up behind her. If the speed of the car behind her is high, she receives a beeping alert over her speakers which allows her to recognize and escape from potential threats. The device works by shooting sound at the car behind her and comparing the frequency f_E of emitted sounds to the frequency f_R of received sound after the sound has bounced off of the other car and returned. Let v_N denote nancy's *speed*, and let $v_O > v_N$ denote the other car's *speed.* The other car is driving toward Nancy's car.

a. **(10 points)** Let α_N denote the ratio of the speed of Nancy's car to the speed of sound in air, and let α_O denote the ratio of the other car's speed to the speed of sound in air. Determine an expression for the ratio of the received frequency f_R to the emitted f_E in terms of α_N and α_O.

b. **(5 points)** Using your result from part a., estimate the ratio of f_R to f_E for cars traveling at reasonable highway speeds in California at an average temperature. It may help you to know that there are roughly 1610 meters in a mile.

c. **(8 points)** If α_N and α_O are both small ~~compared the speed of sound in air,~~ then your result from part a. can be approximated by individually Taylor expanding it in α_N and α_O and only retaining terms up to first order in these variables. Determine this Taylor expanded version of the ratio f_R/f_E, and use it to show that for small car speeds, the ration f_R/f_E only depends on the relative speed of Nancy's car and the other car.

a, $\alpha_N = \frac{V_N}{V}$ $\quad \alpha_O = \frac{V_O}{V}$

$\frac{1-\alpha_N}{1+\alpha_N}$

$f_1 = \frac{V + V_O}{V + V_N} f_E = \frac{1+\alpha_O}{1+\alpha_N} f_E$

+10

$f_R = \frac{V - V_N}{V - V_O} f_1 = \boxed{\frac{1-\alpha_N}{1-\alpha_O} \frac{1+\alpha_O}{1+\alpha_N} f_E}$ ✓

b, $V_N = 60$ mph ≈ 24 m/s ✓

$V_O = 70$ mph ≈ 28 m/s

+3

10 mph $\cdot \frac{1610 \text{ m}}{1 \text{ mi}} \frac{1 \text{ hour}}{3600 \text{ s}}$

$\frac{16100}{3600} \approx 4$ m/s

12

$\alpha_N = \frac{24}{344} \approx 0.104$

$\alpha_O = \frac{28}{344} \approx 0.122$

896

878

$\frac{f_R}{f_E} = \frac{0.896}{0.878} \frac{1.122}{1.104} \approx \frac{0.90}{0.88} \frac{1.12}{1.10}$

$\approx \boxed{1.003}$ +2

104 / 24) 344 / 24 / 104 / 96 / 80

122 / 28) 344 / 28 / 64 / 56 / 80

112 / 90 / 1008 / 1.008

88 / 11 / 88 / 88 / .968

1003 / 0.968) 1.00 / 0.968 / 3200

c. ~~$\frac{1-\alpha_N}{1+\alpha_N} \frac{d}{d\alpha_N} = \frac{-(1+\alpha_N)-(1-\alpha_N)}{(1+\alpha_N)^2} = \frac{-2}{(1+\alpha_N)^2}$~~

$$\frac{1+\alpha_0}{1-\alpha_0} \frac{d}{d\alpha_0} = \frac{(1-\alpha_0)+(1+\alpha_0)}{(1-\alpha_0)^2} = \frac{2}{(1-\alpha_0)^2}.$$

since α_0 is small, $(1-\alpha_0)^2 \to 1$ $\qquad \frac{1+\alpha_0}{1-\alpha_0} \approx 2\alpha_0$

thus

$$\frac{1-\alpha_N}{1+\alpha_N} \approx \frac{1}{2\alpha_N}$$

$$\boxed{\frac{f_R}{f_I} \approx \frac{\alpha_0}{\alpha_N}}$$

X

$$\boxed{\frac{f_R}{f_E} \approx \frac{\frac{v_0}{v}}{\frac{v_N}{v}} = \frac{v_0}{v_N}}$$

Thus, the ratio of the frequencies can be estimated using the relative speeds v_0 and v_N.

+~~1~~ 5

Problem	Score
1	11 / 11
2	22 / 22
3	12 / 13
4	20 / 23
Total	65 **/ 69**