

Name: _____

Student ID#: _____

Instructions

- Do not open the exam until told to do so.
- You are permitted a calculator, 3" by 5" note card, and pens/pencils.
- Please turn off your cell phone ringer and do not access the phone during the exam.
- Showing your work is required for full credit.
- Strike out any of your work you believe to be wrong, you will not be graded on that.
- Please circle your final answer.
- If you require additional paper, I will have some with me.
- Good luck, and **Don't Panic**.

$$\text{Density of water} = 1000 \frac{\text{kg}}{\text{m}^3}$$

$$\text{Density of iron or steel} = 7800 \frac{\text{kg}}{\text{m}^3}$$

$$g = 9.81 \text{m/s}^2$$

$$G = 6.67 \times 10^{-11} \frac{\text{Nm}^2}{\text{kg}^2}$$

$$1 \text{ hour} = 3600 \text{ seconds}$$

$$\sin 2\theta = 2 \sin \theta \cos \theta$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\text{Radius of earth} = 6370 \text{km} = 3960 \text{mi}$$

$$\text{Mass of the earth} = 5.98 \times 10^{24} \text{kg}$$

$$\text{Mass of the moon} = 7.35 \times 10^{22} \text{kg}$$

$$\text{Earth-Moon distance (mean)} = 3.84 \times 10^5 \text{km}$$

$$\text{Mass of the electron} = 9.11 \times 10^{-31} \text{kg} = 0.000549 \text{u}$$

$$\text{Mass of the proton} = 1.6726 \times 10^{-27} \text{kg} = 1.00728 \text{u}$$

$$\text{Moment of inertia of a solid cylinder} = \frac{1}{2} MR^2$$

$$\text{Moment of inertia of a solid sphere} = \frac{2}{5} MR^2$$

Questions [100 Total Points]

1. (20 points) A 0.76 m diameter water main transports water with a flow rate of $2 \frac{\text{m}^3}{\text{s}}$ at a pressure of $1.5 \times 10^5 \text{ Pa}$. At a particular joint, the pipe diameter is reduced to 0.55 m. Assume there is no height change between the two pipe sections. If we treat water as an ideal fluid with no turbulence or viscosity,

- (a) (8 points) What is the flow **velocity** in the **wider** portion of pipe?

$$Av = \frac{2 \text{ m}^3}{\text{s}}$$

$$v = \frac{2 \text{ m}^3/\text{s}}{\pi r^2} = \frac{2 \text{ m}^3/\text{s}}{\pi \left(\frac{0.76 \text{ m}}{2}\right)^2} = 4.409 \text{ m/s}$$

+8

- (b) (12 points) What is the **pressure** and flow **velocity** in the smaller diameter section of pipe?

$$A_1 v_1 = A_2 v_2$$

$$v_2 = \frac{A_1}{A_2} v_1 = \frac{\pi r_1^2}{\pi r_2^2} v_1 = \frac{\left(\frac{0.76}{2}\right)^2}{\left(\frac{0.55}{2}\right)^2} 4.409 \text{ m/s}$$

$$= 8.419 \text{ m/s}$$

+6

$$P_1 + \frac{1}{2} \rho v_1^2 = P_2 + \frac{1}{2} \rho v_2^2$$

$$P_2 = P_1 + \frac{1}{2} \rho (v_1^2 - v_2^2) = 1.5 \times 10^5 \text{ Pa} + \frac{1}{2} (1000 \text{ kg/m}^3) \left((4.409)^2 - (8.419)^2 \right)$$

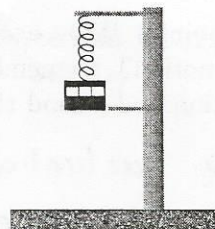
$$= 1.2428 \times 10^5 \text{ Pa}$$

+6

98
100

28

2. (30 points) A somewhat questionable amusement park decides to open a new ride consisting of a 4-person pod (unloaded mass of 1014 kg) suspended by a long spring of negligible mass and damping.



(a) (6 points) During testing, the (unpopulated) pod is raised a distance $h=5.3$ m above its equilibrium position and released. It takes 1.5 seconds to first pass the platform located at the equilibrium position. What is the period and frequency of oscillation?

$T = 4 \cdot 1.5 \text{ s} = 6 \text{ s}$

$f = \frac{1}{T} = \frac{1}{6} \text{ Hz}$



(b) (6 points) Amplitude of oscillation?

$A = 5.3 \text{ m}$



(c) (6 points) Spring constant k ?

~~$mg = kx$~~
 ~~$k = \frac{mg}{x} = \frac{(1014 \text{ kg})(9.81 \text{ m/s}^2)}{5.3 \text{ m}} = 1876.86 \text{ N/m}$~~

$\omega = \sqrt{\frac{k}{m}}$

$\omega = 2\pi f = \frac{\pi}{3} \text{ Hz}$

$\omega^2 = \frac{k}{m}$

$k = \omega^2 m = \left(\frac{\pi}{3}\right)^2 \cdot 1014 \text{ kg} = 1111.975 \text{ N/m}$



- 2 (d) (12 points) What acceleration would a passenger feel at the top and the bottom of the motion? (remember to include gravitation) Ignore the change in equilibrium position and period that happens when a passenger is added to the pod.

top acceleration

$$a = \frac{F}{m} = \frac{mg + kx}{m} = g + \frac{k}{m}x = g + \frac{1111.978(5.3)}{1014 \text{ kg}}$$

$$= 15.62 \text{ m/s}^2$$

bottom acceleration

$$a = \frac{F}{m} = \frac{mg + kx}{m} = g + \frac{k}{m}x = g + \frac{1111.978(5.3)}{1014}$$

$$= 3.998 \text{ m/s}^2$$

Try FBD for passenger.

3. (25 points) In the middle of a large field, an open-ended (both ends) organ pipe is producing a steady tone at its fundamental (first) mode of 392 Hz (G4). Assume the velocity of sound in air is $344 \frac{\text{m}}{\text{s}}$.

- (a) (5 points) What is the length of the pipe?

$$f_n = \frac{nv}{2L}$$

$$f_1 = \frac{v}{2L}$$

$$L = \frac{v}{2f} = \frac{344}{2(392)} = 0.4388 \text{ m}$$

25

- (b) (10 points) How fast would you need to flee the area to hear the pitch a whole step down? (a whole step down from G4 would be F4 = 349 Hz)

$$f_L = \frac{v + v_L}{v + v_S} f_S$$

$$\frac{f_L (v + v_S)}{f_S} = v + v_L$$

$$v_L = \frac{f_L (v + v_S)}{f_S} - v = \frac{349 - (344 + 0)}{392} - 344 = -37.73 \text{ m/s}$$

- (c) (10 points) If the intensity of the sound was 105 dB at a distance of 5.5 m, how far away would you need to be to carry on a normal conversation with a friend (~65 dB)?

$$B_1 - B_2 = 10 \left(\log_{10} \frac{I_1}{I_0} - \log_{10} \frac{I_2}{I_0} \right)$$

$$B_1 - B_2 = 10 \left(\log_{10} \frac{I_1}{I_2} \right) = 10 \log_{10} \left(\frac{r_2^2}{r_1^2} \right) = 20 \log_{10} \left(\frac{r_2}{r_1} \right)$$

$$\log_{10} \left(\frac{r_2}{r_1} \right) = \frac{B_1 - B_2}{20}$$

$$\frac{r_2}{r_1} = 10^{\frac{B_1 - B_2}{20}}$$

$$\frac{r_2^2}{r_1^2} = 10^{\frac{B_2 - B_1}{20}}$$

$$r_2 = \sqrt{r_1^2 \cdot 10^{\frac{B_2 - B_1}{20}}} = 5.5 \times 10^{\frac{105 - 65}{20}} = 550 \text{ m}$$

≥ 550 m

25

4. (25 points) A particular electric guitar has a string length of 0.65 m. The B string has a typical diameter of 0.33 mm and is made of solid steel ($\rho = 7.8 \times 10^3 \frac{\text{kg}}{\text{m}^3}$).

(a) (6 points) What is the linear mass density (mass per unit length) μ of the string?

$$\begin{aligned}\mu &= \frac{\text{mass}}{\text{length}} = \frac{\pi r^2 l \rho}{l} = \frac{\pi \left(\frac{0.33}{1000 \times 2}\right)^2 l (7.8 \times 10^3 \text{ kg/m}^3)}{l} \\ &= 6.6713 \times 10^{-4} \text{ kg/m}\end{aligned}$$

(b) (6 points) The fundamental frequency of the string is 247 Hz. What is the tension in the string?

$$\begin{aligned}f_n &= \frac{nv}{2L} = \frac{n}{2L} \sqrt{\frac{F}{\mu}} \\ f_1 &= \frac{1}{2L} \sqrt{\frac{F}{\mu}} \\ f_1 \cdot 2L &= \sqrt{\frac{F}{\mu}} \\ F &= (f_1 \cdot 2L)^2 \mu = (247 \cdot 2 \cdot 0.65 \text{ m})^2 \cdot 6.6713 \cdot 10^{-4} \\ &= 68.78 \text{ N}\end{aligned}$$

- (c) (8 points) If instead we wished to play an F# (370 Hz), at what distance from the bridge (where the strings are anchored to the body of the guitar) would we fret the string?

$$f_n = \frac{n}{2L} \sqrt{\frac{F}{\mu}}$$

$$n=1 \quad L = \frac{n}{2f_n} \sqrt{\frac{F}{\mu}} = \cancel{0.434 \text{ m}}$$

- (d) (5 points) What would be the length of a simple pendulum (point mass on the end of a mass-less string) that would oscillate at the same frequency as the open B string?

$$\omega = 2\pi f$$

$$\omega = 2\pi \cdot 247 \text{ Hz} = 494\pi$$

$$\omega = \sqrt{\frac{g}{l}}$$

$$494\pi \omega^2 = \frac{g}{l}$$

$$l = \frac{g}{\omega^2} = \frac{9.8}{(494\pi)^2} = 4.073 \times 10^{-6} \text{ m}$$