20W-PHYSICS1B-2 Midterm 2

[redacted]

TOTAL POINTS

90 / 95

QUESTION 1

Problem 1 15 pts

1.1 (A) 5 / 5

✓ **+ 5 pts Correct**

+ 0 pts Incorrect

1.2 (B) 5 / 5

✓ **+ 5 pts Correct**

+ 0 pts Incorrect

1.3 (C) 5 / 5

✓ **+ 5 pts Correct**

+ 0 pts Incorrect

QUESTION 2

Problem 2 30 pts

2.1 A 6 / 10

+ 10 pts Correct

+ 7 pts integral for charge (but other issues with integral or otherwise)

✓ **+ 6 pts No integral but rho*V**

+ 1 pts r>R with mistakes

+ 3 pts attempt

+ 3 pts Correct flux answer

+ 3 pts Put flux in terms of E

- 2 pts bigger math error

- 1 pts Small error (ex. forgot to list flux for r>R or vice versa)

+ 0 pts Blank

2.2 B 9 / 10

✓ **+ 10 pts Correct**

+ 6.5 pts Incomplete but correct direction

✓ **- 1 pts math and or labeling**

+ 3.5 pts Attempt

- 2 pts Generic error or stating 0 field outside of cylinder

+ 0 pts Blank

2.3 C 10 / 10

✓ **+ 10 pts Correct**

+ 6 pts Calculation of Integral of E field plus errors

+ 4.5 pts Attempt

+ 2.5 pts Writing anything related to potential

- 2.5 pts Major Math or sign error

- 1 pts Minor math error

+ 0 pts Blank

QUESTION 3

Problem 3 20 pts

3.1 A 10 / 10

✓ **+ 10 pts Correct**

+ 6.5 pts General vector addition with errors in components or denominators

+ 4.5 pts Attempt with vector addition

+ 2 pts Other attempt

- 2 pts Math or geometry error

- 1 pts minor error

+ 0 pts Blank

3.2 B 10 / 10

✓ **+ 10 pts Correct**

+ 6 pts Correct relationship or voltage

+ 4 pts 0 between charges or partial credit with osc

+ 3 pts Other approach

- 2 pts No mention oscillations or says that there are osc,

- 1 pts math or labeling error

+ 0 pts Blank

QUESTION 4

Problem 4 30 pts

4.1 A 10 / 10

✓ **+ 10 pts Correct**

+ 6.5 pts E = -del V and attempt with major errors

+ 4 pts Attempt, or just writing E = -del V with no math

+ 2 pts Writing down anything related to E field

- 1 pts minor math error

- 2 pts major math error(s)

+ 0 pts Blank

4.2 B 10 / 10

✓ **+ 10 pts Correct**

+ 8 pts E*A but no complete numerical evaluation

+ 3.5 pts Attempt

+ 2 pts Correct numerical answer (7.82 (57.2 for wrong start) nC or 18.5 (150 for wrong start) nC)

- 2 pts math error or other labeling mistakes (this rubric option and -3 for some volume integration)

- 3 pts A is not surface area of sphere

+ 0 pts Blank

4.3 C 10 / 10

✓ **+ 10 pts Correct**

+ 5.5 pts Energy expression

+ 2.5 pts Other attempt

- 3 pts major errors (general)

- 1 pts math error or no explicit evaluation

+ 0 pts Blank

Write your name here:

[redacted]

Write your UCLA ID here

[redacted]

Midterm 2, Physics 1B, Version B

- Please write your name and UID in the boxes on the front page and your name in the boxes at the top of the odd numbered pages.
- Closed book, **one** 5x3in note card (both sides) allowed.
- Scientific Calculators allowed, no computers or smartphones, please put books and notebooks in your backpacks.
- If a problem is ambiguous, notify the instructor. Clarifications will be written on the blackboard. Check the board occasionally.
- Time for exam: 60 minutes
- There are 4 questions, check that your exam has all 12 pages.
- Useful quantities

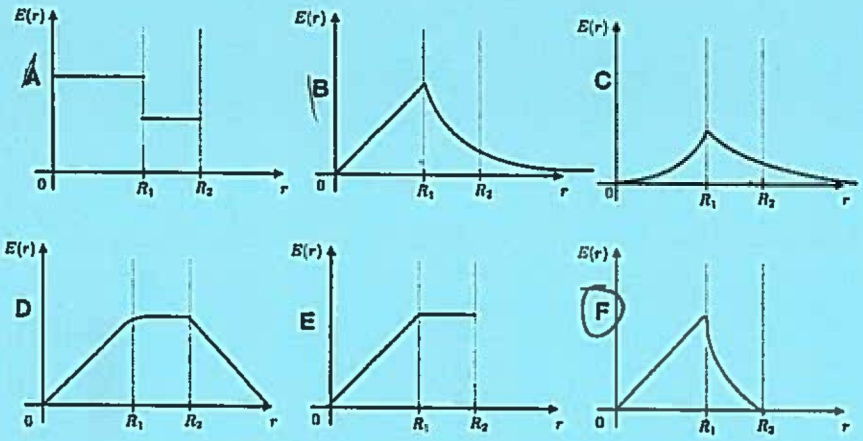
$$\begin{aligned} \epsilon_0 &= 8.85 \times 10^{-12} C^2 m^{-2} N^{-1} \\ g &= 9.81 m/s^2 \\ m_{electron} &= 9.11 \times 10^{-31} kg \\ m_{proton} &= 1.67 \times 10^{-27} kg \\ q_e &= -1.602 \times 10^{-19} C \end{aligned}$$

-additional space for calculation-

Write your name here: [redacted]

Problem 1: [15pts] Concept questions

a) [5pts] Charge Q is spread uniformly throughout a sphere of radius R_1. Surrounding that sphere is a thick shell of inner radius R_1 outer radius R_2. The shell carries charge $-Q$, uniformly spread. Which of the following plots best represents the radial component of the electric field as a function of radius ?



A

B

C

D

E

(F)

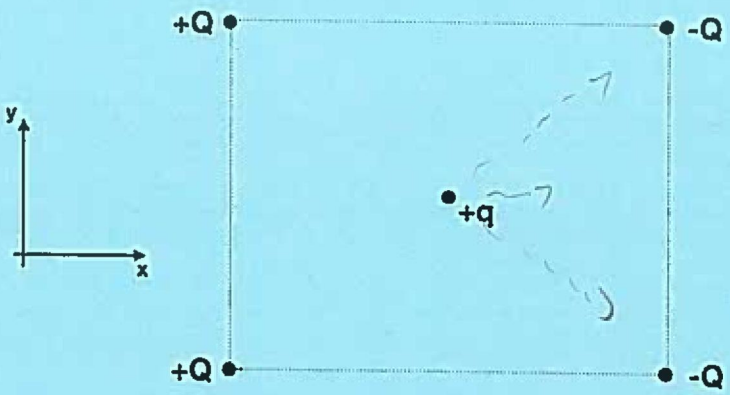
$E \cdot 4\pi R_1^2 = \frac{+Q}{\epsilon_0}$

$\frac{Q}{4\pi\epsilon_0}$

$E \cdot 4\pi r^2 = \frac{\frac{Q}{\frac{4}{3}\pi R_1^3} \cdot \frac{4}{3}\pi r^3}{\epsilon_0}$

$E = \frac{Qr}{4\pi\epsilon_0 R_1^3}$

b) [5pts] A charge with value $+q$ is placed at the center of a square which has four charges on its corners as shown. In what direction is the force on this charge?



A +x direction

B The force is zero

C -y direction

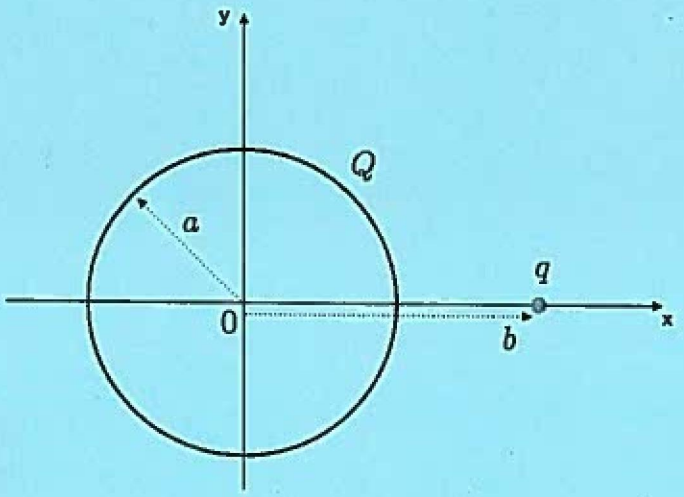
D -x direction

E +y direction

Write your name here:

[redacted]

c) [5pts] A thin insulating shell with radius a is centered at the origin and has charge Q uniformly distributed on the surface. A point charge q is localized at $x = b$ on the x-axis (The figure shows a cross section in the x-y plane). What is the electric field at the origin ?



Q charge on surface does nothing, no net electric field

Electric field at point charge $= \frac{1}{4\pi\epsilon_0}\frac{q}{b^2}$

A $\vec{E} = \left(\frac{1}{4\pi\epsilon_0}\frac{Q}{a^2} - \frac{1}{4\pi\epsilon_0}\frac{q}{b^2}\right)\hat{e}_x$

B $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q+q}{|a-b|^2}\hat{r}$

C $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{b^2}\hat{e}_x$

D $\vec{E} = -\frac{1}{4\pi\epsilon_0}\frac{q}{b^2}\hat{e}_x$

E $\vec{E} = 0$

F $\vec{E} = \left(\frac{1}{4\pi\epsilon_0}\frac{Q}{a^2} + \frac{1}{4\pi\epsilon_0}\frac{q}{b^2}\right)\hat{e}_x$

where $\hat{e}_x$ is the unit vector in the x-direction and $\hat{r}$ is the unit vector in the radial direction.

Problem 2: [30pts] An infinitely long, solid, non-conducting cylinder has radius R and is centered along the z-axis. The volume charge density depends on the distance r from the central axis as

$$\rho(r) = \begin{cases} \frac{1}{3}\rho_0\left(\frac{R}{r}\right) & r \le R \\ 0 & r > R \end{cases} \tag{1}$$

a) [10pts] Calculate the electric flux through a closed cylindrical surface of radius r and length $2L$ centered around the z-axis, for all $0 \le r \le \infty$.

$\Phi_E = \frac{q_{en}}{\epsilon_0}$ $\quad q_{en} = \rho V \quad V = \pi r^2 \cdot 2L = 2\pi r^2 L$

$r \le R$: $q_{en} = \frac{1}{3}\rho_0\left(\frac{R}{r}\right) \cdot 2\pi r^2 L$, $\Phi_E = \boxed{\frac{2\rho_0 R \pi r L}{3\epsilon_0}}$

$r > R$: $q_{en} = \frac{1}{3}\rho_0\left(\frac{R}{r}\right) \cdot \pi R^2 \cdot 2L$, $\Phi_E = \boxed{\frac{2\rho_0 \pi R^3 L}{3\epsilon_0}}$

b) [10pts] Calculate the electric field (magnitude and direction) as a function of the radius r, for all $0 \le r \le \infty$.

$r \le R$:

$E \cdot 2\pi r \ell = \frac{\rho(r) \cdot \pi r^2 \ell}{\epsilon_0}$ $\quad E = \frac{\rho r}{2\epsilon_0} = \frac{1}{3}\rho_0\left(\frac{R}{r}\right) \cdot \frac{r}{2\epsilon_0}$

$= \boxed{\frac{\rho_0 R}{6\epsilon_0}}$

$r > R$:

$E \cdot 2\pi r \ell = \frac{\frac{1}{3}\rho_0\left(\frac{R}{r}\right) \cdot \pi R^2 \ell}{\epsilon_0}$ $\quad \boxed{E = \frac{\rho_0 R^3}{6r^2\epsilon_0}}$

Write your name here: [redacted]

c) [10pts] Calculate the potential difference $V(r=R)-V(r=0)$.

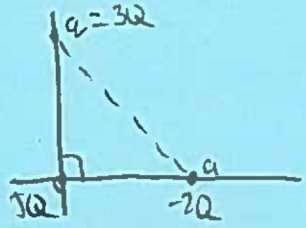
$$V(R)-V(0) = -\int_0^R \vec{E}\cdot d\vec{r}$$

$$= -\int_0^R \frac{\rho_0 r}{6\epsilon_0}\, dr$$

$$= \boxed{-\frac{\rho_0 R^2}{6\epsilon_0}}$$

Problem 3: [20pts] A charge $5Q$ is located at the origin, $x = 0, y = 0, z = 0$, and a charge $-2Q$ is located at $x = a, y = 0, z = 0$ (both charges remain fixed throughout this problem and $Q > 0$, assume $a > 0$).

a) [10pts] Find the force $\vec{F}$ acting on a test charge $q = 3Q$ located along the y-axis (i.e. $x = 0$, y arbitrary)



$$\vec{F}_{3Q\text{-}5Q} = \frac{1}{4\pi\epsilon_0}\frac{(3Q)(5Q)}{y^2} = \frac{1}{4\pi\epsilon_0}\frac{15Q^2}{y^2}$$

$$\vec{F}_{3Q\text{-}2Q,y} = \frac{1}{4\pi\epsilon_0}\frac{(-2Q)(3Q)}{a^2+y^2}\cdot\sin\theta = \frac{-3Q^2}{2\pi\epsilon_0}\frac{y}{(a^2+y^2)^{3/2}}$$

$$\vec{F}_{3Q\text{-}2Q,x} = \frac{1}{4\pi\epsilon_0}\frac{(-2Q)(3Q)}{a^2+y^2}\cdot\cos\theta = \frac{-3Q^2}{2\pi\epsilon_0}\frac{a}{(a^2+y^2)^{3/2}}$$

$$\vec{F} = \left(\frac{1}{4\pi\epsilon_0}\frac{15Q^2}{y^2} - \frac{3Q^2}{2\pi\epsilon_0}\frac{y}{(a^2+y^2)^{3/2}}\right)\hat{j} + \left(\frac{3Q^2}{2\pi\epsilon_0}\frac{a}{(a^2+y^2)^{3/2}}\right)\hat{i}$$

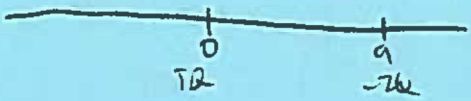
@ $2a$: $\frac{k(\sqrt{2}Q)}{2a^2} - \frac{k(2Q)}{a^2}$ $\frac{k(\sqrt{2}Q)}{(\sqrt{2}a)^2}$ $\frac{k\sqrt{2}Q}{\frac{1}{2}a^2} = \frac{kQ\sqrt{2}}{\frac{1}{4}a^2}$

@ $3a$: $\frac{k(\sqrt{2}Q)}{9a^2} - \frac{k(2Q)}{4a^2}$ $= \frac{2kQ}{a^2}$

Write your name here:

[redacted]

b) [10pts] Find the location(s) on the x-axis (i.e. $y = 0$) where the electric field is zero. Placed at this location, would a test charge $q = \frac{1}{5}Q$ which is constrained to move along the x-axis (i.e. $y = 0$ at all times for this test charge) undergo small oscillations ?



E-field $= \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}$

$\frac{1}{4\pi\epsilon_0}\frac{\sqrt{2}Q}{r_1^2} - \frac{1}{4\pi\epsilon_0}\frac{2Q}{r_2^2} = 0$

$\frac{\sqrt{2}Q}{r_1^2} = \frac{2Q}{r_2^2}$ $\frac{1}{2} = \left(\frac{r_1}{r_2}\right)^2$ $\sqrt{\frac{1}{2}} = \frac{r_1}{r_2}$

$r_2\sqrt{\frac{1}{2}} = r_1$ $r_2 = x - a$

$(x-a)\sqrt{\frac{1}{2}} = x$ $\sqrt{\frac{1}{2}}x - \sqrt{\frac{1}{2}}a = x$

$\sqrt{\frac{1}{2}}x - x = \sqrt{\frac{1}{2}}a$ $x\left(\sqrt{\frac{1}{2}} - 1\right) = \sqrt{\frac{1}{2}}a$

$x = \frac{\sqrt{\frac{1}{2}}a}{\sqrt{\frac{1}{2}} - 1}$

check: if $a = 1$: $\frac{k\sqrt{2}Q}{k2Q}$ $-(\sqrt{10} - 3)$ $-2 \cdot \sqrt{10} - 7$

small oscillations?: if there is no E-field, then there are no oscillations here since the force is 0.

Problem 4: [20pts] An unknown charge distribution produces the following electrostatic potential

$$V(r) = \frac{V_0}{1 + \frac{r}{r_0}} \tag{2}$$

Where r is the radial distance from the origin and $V_0 = 2500$Volts and $r_0 = 15cm$.

a) [10pts] Calculate the electric field $\vec{E}(\vec{r})$ derived from this electrostatic potential

$E = -\frac{dV}{dr}$ $\quad V(r) = V_0 (1 + \frac{r}{r_0})^{-1}$ $\quad \frac{dV}{dr} = -V_0 (1 + \frac{r}{r_0})^{-2} \cdot \frac{1}{r_0}$

$\frac{dV}{dr} = \frac{-V_0}{(1 + \frac{r}{r_0})^2 r_0}$ $\quad E = \frac{V_0}{r_0 (1 + \frac{r}{r_0})^2}$

$\vec{E}(\vec{r}) = \frac{2500\ V}{0.15m\,(1 + \frac{r}{0.15m})^2}$

b) [10pts] How much charge is in a sphere of radius $R = 30cm$ centered around the origin ? [If you could not do a) assume $\vec{E} = \frac{2V_0}{r_0} \frac{1}{\sqrt{1 + \frac{r^2}{r_0^2}}} \hat{r}$]

$E(0.30m) = \frac{2500\ V}{0.15m\,(1+2)^2} = 1850\ N/C$

$1850\ N/C \cdot 4\pi (0.30m)^2 = \frac{q_{en}}{\epsilon_0}$

$q_{en} = 1.85 \times 10^{-8}\ C$

Write your name here: [redacted]

c) [10pts] Consider a point mass with mass $m_1 = 3.30 \times 10^{-4} kg$ and charge $q_1 = -6.20 \times 10^{-6} C$, you release the point mass at the origin with velocity v_1 moving radially outward. What is the smallest value of v_1 so that the point mass makes it all the way to infinity ? (Neglect gravity in this problem)

$m_1 = 3.30 \times 10^{-4}$ kg $q_1 = -6.20 \times 10^{-6}$ C

$qV + \frac{1}{2}mv_1^2 = qV + \frac{1}{2}mv^2$

@ infinity, this is 0

$V(r) = \frac{V_0}{1 + \frac{r}{r_0}}$ $(-6.20 \times 10^{-6} C)(2500 V) + \frac{1}{2}(3.30 \times 10^{-4} kg) v_1^2$

$-0.0155 J + 1.65 \times 10^{-4} kg \, v_1^2 = 0$

$1.65 \times 10^{-4} kg \, v_1^2 = 0.0155 J$

$v_1 = 9.68 \, m/s$

-additional space for calculation-