20W-PHYSICS1B-2 Midterm 2

PREET MODI

TOTAL POINTS

83.5 / 95

QUESTION 1

Problem 1 15 pts

1.1 (A) **5 / 5**

✓ + **5 pts** Correct

+ **0 pts** Incorrect

1.2 (B) **5 / 5**

✓ + **5 pts** Correct

+ **0 pts** Incorrect

1.3 (C) **5 / 5**

✓ + **5 pts** Correct

+ **0 pts** Incorrect

QUESTION 2

Problem 2 30 pts

2.1 A **9 / 10**

✓ + **10 pts** Correct

+ **7 pts** integral for charge (but other issues with integral or otherwise)

+ **6 pts** No integral but rho*V

+ **1 pts** r>R with mistakes

+ **3 pts** attempt

+ **3 pts** Correct flux answer

+ **3 pts** Put flux in terms of E

- **2 pts** bigger math error

✓ - **1 pts** Small error (ex. forgot to list flux for r>R or vice versa)

+ **0 pts** Blank

2.2 B **6.5 / 10**

+ **10 pts** Correct

✓ + **6.5 pts** Incomplete but correct direction

- **1 pts** math and or labeling

+ **3.5 pts** Attempt

- **2 pts** Generic error or stating 0 field outside of cylinder

+ **0 pts** Blank

2.3 C **10 / 10**

✓ + **10 pts** Correct

+ **6 pts** Calculation of Integral of E field plus errors

+ **4.5 pts** Attempt

+ **2.5 pts** Writing anything related to potential

- **2.5 pts** Major Math or sign error

- **1 pts** Minor math error

+ **0 pts** Blank

QUESTION 3

Problem 3 20 pts

3.1 A **9 / 10**

✓ + **10 pts** Correct

+ **6.5 pts** General vector addition with errors in components or denominators

+ **4.5 pts** Attempt with vector addition

+ **2 pts** Other attempt

- **2 pts** Math or geometry error

✓ - **1 pts** minor error

+ **0 pts** Blank

3.2 B **7 / 10**

✓ + **10 pts** Correct

+ **6 pts** Correct relationship or voltage

+ **4 pts** 0 between charges or partial credit with osc

+ **3 pts** Other approach

✓ - **2 pts** No mention oscillations or says that there are osc,

✓ - **1 pts** math or labeling error

+ **0 pts** Blank

QUESTION 4

Problem 4 30 pts

4.1 A 10 / 10

✓ + **10 pts** **Correct**

+ **6.5 pts** E = -del V and attempt with major errors

+ **4 pts** Attempt, or just writing E = -del V with no math

+ **2 pts** Writing down anything related to E field

- **1 pts** minor math error

- **2 pts** major math error(s)

+ **0 pts** Blank

4.2 B 10 / 10

✓ + **10 pts** **Correct**

+ **8 pts** E*A but no complete numerical evaluation

+ **3.5 pts** Attempt

+ **2 pts** Correct numerical answer (7.82 (57.2 for wrong start) nC or 18.5 (150 for wrong start) nC)

- **2 pts** math error or other labeling mistakes (this rubric option and -3 for some volume integration)

- **3 pts** A is not surface area of sphere

+ **0 pts** Blank

4.3 C 7 / 10

✓ + **10 pts** **Correct**

+ **5.5 pts** Energy expression

+ **2.5 pts** Other attempt

✓ - **3 pts** **major errors (general)**

- **1 pts** math error or no explicit evaluation

+ **0 pts** Blank

Write your name here:

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Write your UCLA ID here

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Midterm 2, Physics 1B, Version A

- Please write your name and UID in the boxes on the front page and your name in the boxes at the top of the odd numbered pages.
- Closed book, **one** 5x3in note card (both sides) allowed.
- Scientific Calculators allowed, no computers or smartphones, please put books and notebooks in your backpacks.
- If a problem is ambiguous, notify the instructor. Clarifications will be written on the blackboard. Check the board occasionally.
- Time for exam: 60 minutes
- There are 4 questions, check that your exam has all 12 pages.
  - Useful quantities

$$
\begin{aligned}
\epsilon_0 &= 8.85 \times 10^{-12} C^2 m^{-2} N^{-1} \\
g &= 9.81 m/s^2 \\
m_{electron} &= 9.11 \times 10^{-31} kg \\
m_{proton} &= 1.67 \times 10^{-27} kg \\
q_e &= -1.602 \times 10^{-19} C
\end{aligned}
$$

-additional space for calculation-

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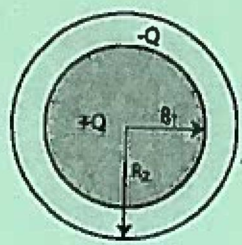
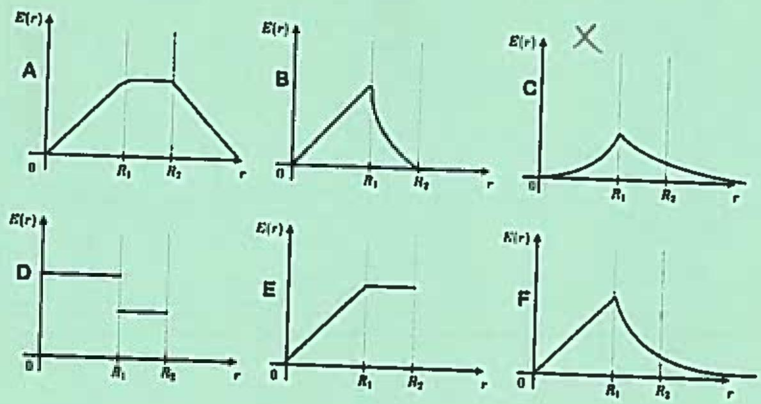
Problem 1: [15pts] Concept questions

a) [5pts] Charge Q is spread uniformly throughout a sphere of radius R_1. Surrounding that sphere is a thick shell of inner radius R_1 outer radius R_2. The shell carries charge $-Q$, uniformly spread. Which of the following plots best represents the radial component of the electric field as a function of radius ?





A

(B)

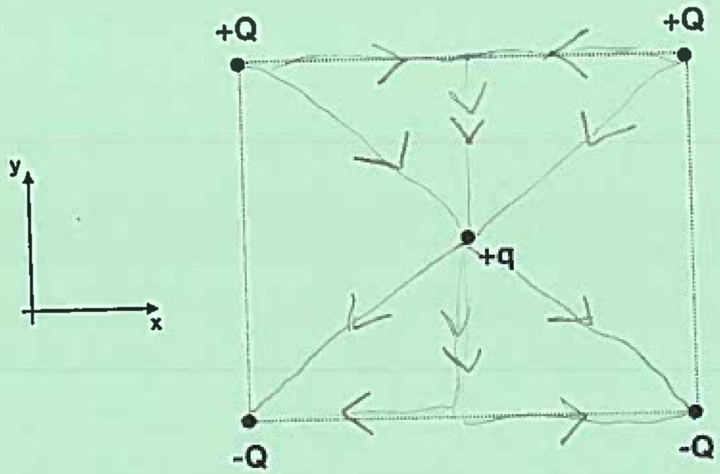
C

D

E

F

b) [5pts] A charge with value $+q$ is placed at the center of a square which has four charges on its corners as shown. In what direction is the force on this charge?



A +x direction

B The force is zero

C -y direction

D -x direction

E +y direction

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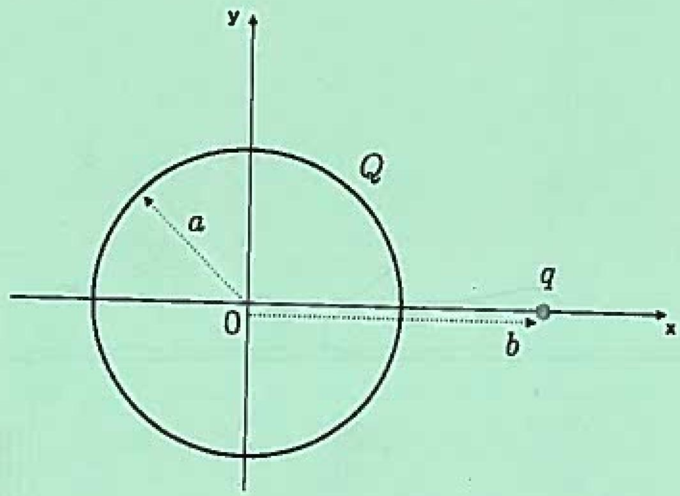
c) **[5pts]** A thin insulating shell with radius a is centered at the origin and has charge Q uniformly distributed on the surface. A point charge q is localized at $x = b$ on the x-axis (The figure shows a cross section in the x-y plane). What is the electric field at the origin ?



A $\vec{E} = \left(\frac{1}{4\pi\epsilon_0}\frac{Q}{a^2} + \frac{1}{4\pi\epsilon_0}\frac{q}{b^2}\right)\hat{e}_x$

B $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{b^2}\hat{e}_x$

(C) $\vec{E} = -\frac{1}{4\pi\epsilon_0}\frac{q}{b^2}\hat{e}_x$

D $\vec{E} = 0$

E $\vec{E} = \left(\frac{1}{4\pi\epsilon_0}\frac{Q}{a^2} - \frac{1}{4\pi\epsilon_0}\frac{q}{b^2}\right)\hat{e}_x$

F $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{Q+q}{|a-b|^2}\hat{r}$

where $\hat{e}_x$ is the unit vector in the x-direction and $\hat{r}$ is the unit vector in the radial direction.

$\pi r^2 l$

$2\pi r l \cdot dr$

Problem 2: [30pts] An infinitely long, solid, non-conducting cylinder has radius R and is centered along the z-axis. The volume charge density depends on the distance r from the central axis as

$$\rho(r) = \begin{cases} \frac{1}{4}\rho_0\left(\frac{R}{r}\right) & r \leq R \\ 0 & r > R \end{cases} \tag{1}$$

a) [10pts] Calculate the electric flux through a closed cylindrical surface of radius r and length $3L$ centered around the z-axis, for all $0 \leq r \leq \infty$.

$\Phi = \frac{q_{encl.}}{\varepsilon_0}$ ⇐ by Gauss's Law

$q_{encl.} = \int_0^{\infty} 2\pi r l \cdot dr \cdot \rho(r) = \int_0^{R} 3L \cdot 2\pi r \cdot \frac{1}{4} \rho_0 \left(\frac{R}{r}\right) \cdot dr$ ⇐ since $\rho(r) = 0$ for $r > R$

$= \frac{3L\pi \rho_0 R}{2} \int_0^R dr = \frac{3L\pi\rho_0 R^2}{2}$

$\rightarrow \Phi = \frac{3L\pi\rho_0 R^2}{2\varepsilon_0}$

b) [10pts] Calculate the electric field (magnitude and direction) as a function of the radius r, for all $0 \leq r \leq \infty$.

$\int E \cdot dA = \Phi$

$\rightarrow E\left[2\pi r 3L\right]_0^R = \frac{3L\pi\rho_0 R^2}{2\varepsilon_0}$

$\rightarrow E\, 2\pi R 3L = \frac{3L\pi\rho_0 R^2}{2\varepsilon_0}$

$\rightarrow E = \frac{\rho_0 R}{4\varepsilon_0}$

- direction is radially outwards.

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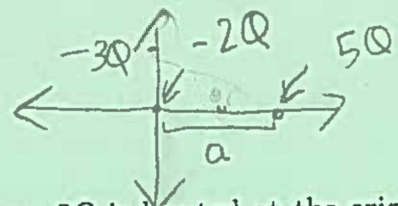
c) [10pts] Calculate the potential difference $V(r = R) - V(r = 0)$.

$$\Delta V = -\int_0^R E \cdot dr$$

$$= -\int_0^R \frac{\rho_0 R}{4\varepsilon_0} \cdot dr$$

$$= -\left[\frac{\rho_0 R r}{4\varepsilon_0}\right]_0^R$$

$$= \frac{-\rho_0 R^2}{4\varepsilon_0}$$



Problem 3: [20pts] A charge $-2Q$ is located at the origin, $x = 0, y = 0, z = 0$, and a charge $5Q$ is located at $x = a, y = 0, z = 0$ (both charges remain fixed throughout this problem and $Q > 0$, assume $a > 0$).

a) [10pts] Find the force $\vec{F}$ acting on a test charge $q = -3Q$ located along the y-axis (i.e. $x = 0$, y arbitrary)

$$F_{1y} = \frac{k(-2Q)(-3Q)}{y^2} = \frac{6kQ^2}{y^2}$$

$$F_{1x} = 0$$

$$F_{2y} = \frac{k(-3Q)(5Q)}{(y^2+a^2)} \cdot \frac{y}{\sqrt{y^2+a^2}} = \frac{-15kQ^2 y}{(y^2+a^2)^{3/2}}$$

$$F_{2x} = \frac{k(-3Q)(5Q)}{(y^2+a^2)} \cdot \frac{a}{\sqrt{y^2+a^2}} = \frac{-15kQ^2 a}{(y^2+a^2)^{3/2}}$$

$$F_x = \frac{6kQ^2}{y^2} - \frac{15kQ^2 y}{(y^2+a^2)^{3/2}}$$

$$F_y = \frac{-15kQ^2 a}{(y^2+a^2)^{3/2}}$$

$$F = \sqrt{F_x^2 + F_y^2}$$

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b) [10pts] Find the location(s) on the x-axis (i.e. $y = 0$) where the electric field is zero. Placed at this location, would a test charge $q = \frac{1}{10}Q$ which is constrained to move along the x-axis (i.e. $y = 0$ at all times for this test charge) undergo small oscillations ?

$$E_1 = \frac{k(-2Q)}{d^2} \; ; \; E_2 = \frac{k(5Q)}{(a-d)^2}$$

$$E_1 + E_2 = 0 \rightarrow \frac{2kQ}{d^2} = \frac{5kQ}{(a-d)^2} \rightarrow \frac{(a-d)^2}{d^2} = \frac{5}{2}$$

$$\rightarrow \frac{a-d}{d} = \sqrt{\frac{5}{2}} \rightarrow a - d = \sqrt{\frac{5}{2}}\, d \rightarrow a = \left(\sqrt{\frac{5}{2}} + 1\right) d$$

$$\rightarrow d = \frac{a}{\left(1 + \sqrt{\frac{5}{2}}\right)}$$ is the location on the x-axis where net $E = 0$.

The test charge would undergo small oscillation about the position $x = d$ since $x = d$, is a stable equillibrium point. This is because d lies between $x = 0$ and $x = a$. When a slight displacement is made to either side, the forces readjust and oppose the motion until equillibrium is reached again.

$(8.33r+1)^{-1}$

Problem 4: [20pts] An unknown charge distribution produces the following electrostatic potential

$$V(r) = \frac{V_0}{1 + \frac{r}{r_0}} \tag{2}$$

Where r is the radial distance from the origin and $V_0 = 1500$Volts and $r_0 = 12cm$.

a) [10pts] Calculate the electric field $\vec{E}(\vec{r})$ derived from this electrostatic potential

$$V(r) = \frac{1500}{1+\frac{r}{0.12}} = \frac{1500}{8.33r + 1}$$

$$E(\vec{r}) = -\frac{dV}{dr} = -\left(\frac{-1500}{(8.33r+1)^2} \cdot (8.33)\right)$$

$$E(r) = \frac{8.33\,V_0}{(8.33r+1)^2}$$

b) [10pts] How much charge is in a sphere of radius $R = 20cm$ centered around the origin ? [If you could not do a) assume $\vec{E} = \frac{2V_0}{r_0}\frac{1}{\sqrt{1+\frac{r^2}{r_0^2}}}\hat{r}$]

$$E(0.2) = \frac{8.33(1500)}{(8.33(0.2)+1)^2} = \frac{(8.33)(1500)}{(2.666)^2}$$

$$E = \frac{kQ}{r^2} \rightarrow Q = \frac{Er^2}{k} \rightarrow Q = \frac{(8.33)(1500)}{(2.666)^2} \cdot \frac{(0.2)^2}{(9\times10^9)}$$

$$\rightarrow Q = 7.81\times10^{-9}\,C$$

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c) [10pts] Consider a point mass with mass $m_1 = 2.30 \times 10^{-4} kg$ and charge $q_1 = -7.20 \times 10^{-6} C$, you release the point mass at the origin with velocity v_1 moving radially outward. What is the smallest value of v_1 so that the point mass makes it all the way to infinity ? (Neglect gravity in this problem)

$m = 2.30 \times 10^{-4}$ kg

$q_1 = -7.20 \times 10^{-6}$ C

KE = U at initial position

$U = qV$

$\rightarrow U = (-7.20 \times 10^{-6}) \left(\frac{1500}{1 + \frac{0.20}{0.12}} \right)$

$U = -4.05 \times 10^{-3}$

$|U| = 4.05 \times 10^{-3}$

$\rightarrow \frac{1}{2} m v^2 \geq 4.05 \times 10^{-3}$

$\rightarrow v \geq \sqrt{\frac{4.05 \times 10^{-3} \times 2}{2.30 \times 10^{-4}}}$

$\rightarrow v \geq 5.934$

Thus, the smallest value of v is 5.93 m/s for which the point mass makes it to infinity.

-additional space for calculation-