

20W-PHYSICS1B-2 Midterm 1

TOTAL POINTS

83 / 95

QUESTION 1

Concept questions 15 pts

1.1 5 / 5

1.2 5 / 5

1.3 5 / 5

QUESTION 2

Problem 2 30 pts

2.1 7 / 10

2.2 8 / 10

2.3 10 / 10

QUESTION 3

Problem 3 20 pts

3.1 Part A 10 / 10

3.2 Part B 10 / 10

QUESTION 4

Problem 4 20 pts

4.1 Part A 10 / 10

4.2 Part B 10 / 10

QUESTION 5

5 Problem 5 3 / 10

+ 1 Point adjustment

Write your name here:

Write your UCLA ID here

Midterm 1, Physics 1B, Version B

- Please write your name and UID in the boxes on the front page and your name in the boxes at the top of the odd numbered pages.
- Closed book, **one** 5x3in note card (both sides) allowed.
- Scientific Calculators allowed, no computers or smartphones, please put books and notebooks in your backpacks.
- If a problem is ambiguous, notify the instructor. Clarifications will be written on the blackboard. Check the board occasionally.
- Time for exam: 60 minutes
- There are 5 questions, check that your exam has all 12 pages.

Good Luck !!

-additional space for calculation-

A large, empty rectangular box with a thin black border, occupying most of the page. It is intended for students to perform calculations related to the problem above.

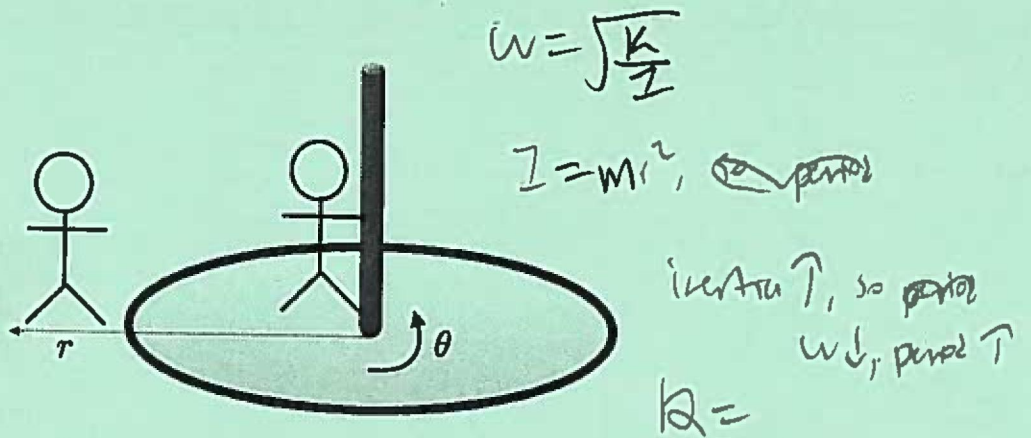
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Problem 1: [15pts] Concept questions

a) [5pts] An children's playground toy can be viewed as a torsion pendulum undergoing simple harmonic oscillator motion. The angular displacement can be described by

$$\theta(t) = A \cos\left(\frac{2\pi}{T}t\right) \quad (1)$$

At time $t = T/2$ another child quickly jumps onto the toy (moving radially), what happens to the amplitude and the period of the oscillation ?



Circle the correct answer

- A The period is unchanged and the the amplitude is unchanged
- B The period is increases and the amplitude is unchanged.
- C The period increases and the amplitude increases
- D The period is unchanged and the the amplitude decreases
- E The period decreases and the amplitude decreases
- F The period is decreases and the amplitude is unchanged.
- G The period decreases and the amplitude increases

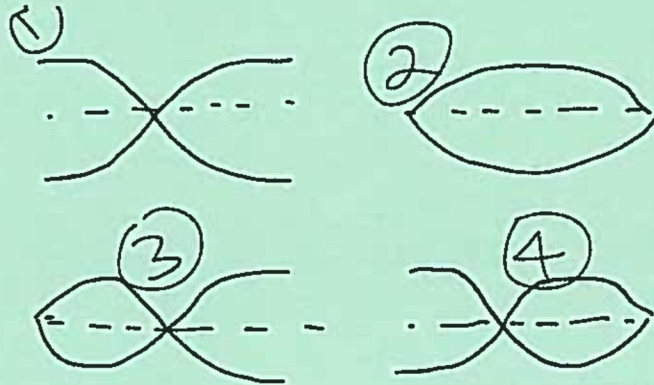
b) [5pts] Consider the statement that energy is transported in a standing wave

Circle the correct answer

- A is incorrect, since energy transported to the right and left cancel each other out
- B is correct, since the standing wave is a superposition of left and right moving waves
- C energy will be transported in a longitudinal standing wave, not a transverse one
- D the answer depends on whether we have fixed ends or free ends.
- E is correct, since there is work done when the string is displaced.
- F energy will be transported in a transverse standing wave, not a longitudinal one

Write your name here:

c) [5pts] Which of the following could describe the pressure amplitude in an organ pipe that is open on both ends ?



Circle the correct answer

- A 2 only
- B 3 only
- C 4 only
- D 1 only
- E None of them since they are not the fundamental standing wave
- F Both 3 and 4

Problem 2: [30pts] A string is made out of steel with density $\rho = 7.85 \text{ g/cm}^3$ and has a diameter of $d = 0.4 \text{ mm}$. One end is located at $x = 0$ and the other end is located at $x = L$, with $L = 6.00 \text{ m}$. At the end of the string located at $x = 0$, an external apparatus acts on the end, starting at time $t = 0$. The transverse displacement at $x = 0$ as a function of time is given by

$$y(t) = A \sin(\omega t) \quad (2)$$

Where $A = 3.00 \text{ mm}$ and $\omega = 10^3 \frac{\text{rad}}{\text{s}}$. For times $t > 0$, a traveling wave is produced (neglect any effect of reflected waves for this problem)

a) [10pts] If the string is held under tension $T = 60.00 \text{ N}$ what is the wave speed and the wavelength of the traveling wave?

$$\mu = \frac{m}{L} \quad \rho = \frac{7.85 \text{ g}}{\text{cm}^3} \quad \mu = \frac{7.85 \text{ g}}{\text{cm}^3} \cdot \left(\frac{1000 \text{ cm}}{1 \text{ m}}\right)^3 \cdot \frac{1 \text{ kg}}{1000 \text{ g}} \cdot 7.54 \cdot 10^{-7} \text{ m}^2 = 0.00592 \text{ kg/m}$$

$$\mu_{\text{area}} = \left(0.2 \text{ mm} \times \frac{1 \text{ m}}{1000 \text{ mm}}\right)^2 \pi = 1.26 \times 10^{-7} \text{ m}^2 \cdot 6.00 \text{ m} = 7.54 \times 10^{-7} \text{ m}^3$$

$$v = \sqrt{\frac{60.00 \text{ N}}{0.00592 \text{ kg/m}}} = \boxed{101 \text{ m/s}} \quad v = f\lambda \quad \lambda = \frac{v}{f} \quad f = \frac{\omega}{2\pi}$$

$$\lambda = \frac{2\pi v}{\omega} = \frac{2\pi (101 \text{ m/s})}{10^3 \text{ rad/s}} = \boxed{0.633 \text{ m}}$$

b) [10pts] Write down an expression $y(x, t)$ for the displacement of traveling wave

$$y = f(x - vt) \quad @ x = 0; y = A \cos(-\omega t + \phi_0)$$

$$y(x, t) = f(x - 101 \text{ m/s} t) \quad \cos(-30^\circ) = \sin(30^\circ)$$

$$y(x, t) = A \cos\left(\frac{2\pi}{0.633 \text{ m}} (x - 101 \text{ m/s} t) + \phi_0\right) \quad \sin \theta = \cos(90 - \theta)$$

$$\sin(\omega t) = \cos(-\omega t + \phi_0) \quad \sin(\omega t) = \cos(90 - \omega t)$$

$$\cos(90 + \omega t)$$

$$\boxed{y(x, t) = 3.00 \text{ mm} \sin\left(\frac{2\pi}{0.633 \text{ m}} (x - 101 \text{ m/s} t)\right)}$$

$$\sin(-30^\circ) = -\sin(30^\circ)$$

Write your name here:

c) [10pts] Find the time t at which the displacement of the string at $x = L/2$ is $-A$ for the first time [If you could not do a) assume $v = 175\text{m/s}$].

$$y = 101 \text{ m/s} \quad y(x,t) = 3.00 \text{ mm} \cos\left(\frac{2\pi}{0.613 \text{ m}}(x + 101 \text{ m/s} \cdot t) + \frac{\pi}{2}\right)$$

$$y(x,t) = 3.00 \cos\left(29.78 + 1002.53t + \frac{\pi}{2}\right)$$

$$y(x,t) = -3.00 \text{ mm} \sin\left(\frac{2\pi}{0.613 \text{ m}}(x - 101 \text{ m/s} \cdot t)\right)$$

$$-3.00 \text{ mm} \sin(29.78 - 1002.53t)$$

$$\sin(29.78 - 1002.53t) = 1$$

$$t = 0.0297 \text{ s}$$

Problem 3: [20pts] You place a speaker connected to a sine wave generator close to one end of an organ pipe of unknown length L . You slowly increase the frequency of the sine wave generator from zero and for the frequencies $f_a = 85\text{Hz}$ and $f_b = 255\text{Hz}$ you find a resonant standing wave (but at no other frequency up to f_b). Assume that the speed of sound in air is $v = 340\frac{\text{m}}{\text{s}}$.

a) [10pts] Does the pipe have two open ends or one open and closed end (Justify your reasoning)? Find the length of the pipe.

$$\frac{f_b}{f_a} = \frac{255\text{Hz}}{85\text{Hz}} = 3 \quad f_a = f_1, \quad f_b = f_3$$

the pipe has one open and one closed end, since it jumps f_1 to f_3 , skipping f_2 = one open and one closed pipes have n values of $1, 3, 5, \dots$.

$$f_1 = \frac{v}{4L} \quad 85\text{Hz} = \frac{340\text{m/s}}{4(L)}$$

$$L = \frac{85\text{Hz} \cdot 4}{340\text{m/s}} = \boxed{1\text{m}}$$

Write your name here:

b) [10pts] You close all ends of the pipe and fill it with an unknown gas. You scan through the frequencies of the sine wave generator and there is a resonance at $f_a = 450\text{Hz}$ and $f_b = 525\text{Hz}$ without any other resonances in between f_a and f_b . What is the speed of sound in the gas? [If you could not do a) assume $L = 1.20\text{m}$].

$$f_b = 525\text{ Hz} \quad f_a = 450\text{ Hz}$$

$$\frac{f_b}{f_a} = \frac{525\text{ Hz}}{450\text{ Hz}} = \frac{7}{6} \quad f_b = \frac{6 \cdot v_{\text{sound}}}{2 \cdot L}$$

$$450\text{ Hz} = \frac{6 \cdot v_{\text{sound}}}{2 \cdot 1\text{ m}}$$

$$v_{\text{sound}} = \frac{450\text{ Hz} \cdot 2\text{ m}}{6} = \boxed{150\text{ m/s}}$$

Problem 4: [20pts] A mass $m = 3.00\text{kg}$ is connected to an ideal spring with spring constant $k = 11.00\text{N/m}$. Neglect any friction in this problem.

The equilibrium of the spring is at $x = 0$. At $t = 0$ you compress the spring so that $x = -0.10\text{m}$ and give it a push so that the initial velocity is $v = -1.70\text{m/s}$. We describe the ensuing simple harmonic oscillations by

$$x(t) = A \cos(\omega t + \phi_0), \quad -\pi < \phi_0 \leq +\pi \quad (3)$$

a) [10pts] Find ω , A and ϕ_0 .

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{11.00\text{N/m}}{3.00\text{kg}}} = 1.91 \text{ rad/s} \leftarrow \omega$$

$$x = A \cos(\omega t + \phi_0) \quad x(0) = A \cos(\phi_0) = -0.10$$

$$v = -A\omega \sin(\omega t + \phi_0) \quad v(0) = -A(1.91 \text{ rad/s}) \sin(\phi_0) = -1.70 \text{ m/s}$$

$$x(0) = -0.10 = A \cos(1.68 \text{ rad}) \quad A \sin \phi_0 = 0.89 \text{ m rad}$$

$$A = \frac{-0.10}{\cos(1.68 \text{ rad})} \quad A \cos \phi_0 = -0.10 \text{ m rad}$$

$$\tan \phi_0 = -8.88$$

$$\phi_0 = -1.46 \text{ rad} + \pi = 1.68 \text{ rad} \leftarrow \phi_0$$

$$A = 0.89 \text{ m}$$

b) [10pts] At what time $t > 0$ does the mass reach the largest positive displacement for the first time? [If you could not do a) assume $\phi_0 = 1.80 \text{ rad}$]

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$$x = 0.89 \text{ m} \cos(1.91 \text{ rad/s} \cdot t + 1.68 \text{ rad})$$

$$\cos(1.91 \text{ rad/s} \cdot t + 1.68 \text{ rad}) = 1$$

$$1.91 \text{ rad/s} \cdot t + 1.68 \text{ rad} = 2\pi$$

$$t = 2.41 \text{ s}$$

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Problem 5) [10pts] Consider a mass M connected to a spring which has the following potential energy function (no other forces are present)

$$U(x) = \frac{1}{3}\alpha\left(\sqrt{1 + \frac{x^2}{x_0^2}} - 1\right) \quad (4)$$

Find the location of stable equilibrium and calculate the period of small oscillations of a mass M around the equilibrium in terms of M , α and x_0 .

$$U(x) = \frac{1}{3}\alpha\left(\sqrt{1 + \frac{x^2}{x_0^2}} - 1\right)$$

$$\frac{1}{2}kx^2 = \frac{1}{3}\alpha\left(\sqrt{1 + \frac{x^2}{x_0^2}} - 1\right)$$

$$U'(x) = 0; \quad U(x) = 0; \quad \frac{1}{3}\alpha\sqrt{1 + \frac{x^2}{x_0^2}} - \frac{1}{3}\alpha = 0$$

$x = 0$: stable equilibrium

small oscillation period:

$$\omega = \sqrt{\frac{k}{m}} \quad k = \frac{2\alpha}{3x^2}\left(\sqrt{1 + \frac{x^2}{x_0^2}} - 1\right)$$

$$2\pi f = \omega \quad \Rightarrow \quad T = \frac{2\pi}{\omega}$$

$$T = \frac{2\pi}{\omega}$$

$$T = 2\pi \sqrt{\frac{M}{\frac{2\alpha}{3x^2}\left(\sqrt{1 + \frac{x^2}{x_0^2}} - 1\right)}}$$

-additional space for calculation-

