

Name

StudentID:

Midterm 1, Winter 2019, Version C

- Please write your name and UID on the front page, if you separate pages or use extra paper please write your name and UID on each one of them.
- Closed book, **one** 5x3in note card allowed.
- Calculators allowed, no computers or smartphones.
- If a problem is ambiguous, notify the instructor. Clarifications will be written on the blackboard. Check the board occasionally.
- Time for exam: 60 minutes
- There are 4 questions, check that your exam has all 12 pages

Good Luck !!

Please do not write in this box

Problem	Points	of Total
1	5+5	15
2	27	30
3	30	20
4	16	30
Total	78	95

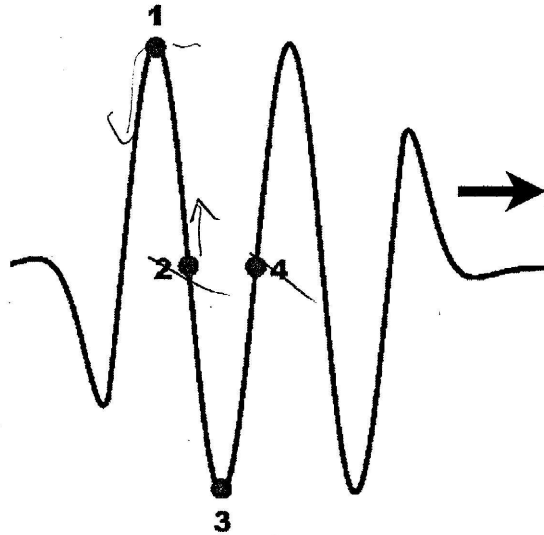
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Problem 1: [15pts] Concept questions

a) [5pts] A child swings on a swing (you can model the system as a simple pendulum undergoing small oscillations). The child holds a bag of sand. When the child goes through the bottom she drops the bag of sand. What will happen to the amplitude and the frequency of the oscillation after the bag has been dropped?

- A The frequency decreases and the amplitude increases
- B The frequency decreases and the amplitude decreases
- C The frequency is unchanged and the amplitude increases
- D The frequency is unchanged and the amplitude decreases
- E The frequency is unchanged and the amplitude is unchanged.

b) [5pts] A wave travels on a stretched cord; a snapshot of the displacement of the cord (at one instant in time) is shown below. If the wave is propagating to the right, at which point does the cord have a (non-zero) acceleration upwards?



- A 3.
- B 2.
- C 1.
- D 4.

c) [5pts] You play a guitar string and excite the fundamental standing mode on the string (frequency f , wavelength λ). A sound wave is generated which allows you to hear the guitar. What about the sound wave is different from the standing wave on the string?

- A wave speed only
- B wave speed and frequency only
- C wave speed, wavelength, and frequency
- D wave speed and wavelength only
- E frequency and wavelength only

v_{sound}

F they are all the same

Problem 2: [30pts]

A mass $m = 50g$ is attached to an ideal spring on a frictionless horizontal surface. The block is pulled to stretch the spring 10 cm , then released gently. After a little time the block passes through the equilibrium position with a velocity of 1 m/s

- a) [10pts] What is the block's period of oscillation?
 b) [10pts] What's the block speed at the point when the spring is compressed by 5 cm ?
 c) [10pts] You add a friction force with $F = -bv$ acting on the mass. You observe that after one oscillation the mechanical energy is $1/e$ of the value at $t = 0$. What is b ?

a) $x(t) = A \cos(\omega t + \phi)$
 $v(t) = -A\omega \sin(\omega t + \phi)$

At equilibrium, $x(t) = 0 \rightarrow \cos(\omega t + \phi) = 0 \rightarrow \sin(\omega t + \phi) = 1$.
 $= v(t) = -A\omega = 1\text{ m/s}$.
 (from $\cos^2\theta + \sin^2\theta = 1$)

The max amplitude is $10\text{ cm} = 10 \times 10^{-2}\text{ m}$

$\omega = \left| -\frac{v_{\max}}{A} \right| = \left| \frac{1\text{ m/s}}{.1\text{ m}} \right| = 10\text{ Hz}$

$T = \frac{2\pi}{\omega} = \frac{2\pi}{10\text{ Hz}} = .63\text{ sec}$ ✓

b) From $A = .1\text{ m}$ and $\omega = 10\text{ Hz}$,

$|v(t)| = |-(.1\text{ m})(10\text{ Hz})(\sin(\omega t + \phi))|$

When $x(t) = .05\text{ m}$, $\cos(\omega t + \phi) = \frac{.05\text{ m}}{.1\text{ m}} = \frac{1}{2}$.
 $\sin(\omega t + \phi) = \sqrt{1 - \cos^2(\omega t + \phi)} = \sqrt{1 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$

$|v(t)| = |(.1\text{ m})(10\text{ Hz})\left(\frac{\sqrt{3}}{2}\right)| = .87\text{ m/s}$ ✓

c. We can express energy in terms of the amplitudes:

$\left(\frac{1}{2}\right)\left(\frac{1}{2}kA_0^2\right) = \left(\frac{1}{2}\right)kA_f^2$

$\frac{A_0}{\sqrt{e}} = A_f$

After one oscillation:
 $x(t) = A_f \cos(\omega t + \phi)$
 $= A_0 e^{-\frac{b}{2m}t} \cos(\omega t + \phi)$

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-additional space for calculation-

$$\text{Thus, } A_f = A_0 e^{-\frac{b}{2m}T} \quad (T = \text{period} = \frac{2\pi}{\omega} = .6 \text{ sec}).$$

$$\frac{A_2}{\sqrt{e}} = A_0 e^{-\frac{b}{2m}T}$$

↓

$$e^{-\frac{1}{2}} = e^{-\frac{b}{2m}T}$$

↓

$$\frac{1}{2} = \frac{b}{2m}T \rightarrow b = \frac{m}{T} = \frac{50 \times 10^{-3} \text{ kg}}{.6 \text{ sec}} \rightarrow \text{wrong period.}$$
$$= \boxed{.008 \text{ kg/s}} \quad -3$$

Problem 3: [30pts]

A tube open on both ends is filled with an unknown gas. The tube is 190cm in length and 3cm in diameter. By using different tuning forks, it is found that the resonant standing waves can be excited at frequencies of 315Hz, 420Hz and 525 Hz and there are no other resonant standing frequencies between 315 Hz and 525 Hz.

- a) [10pts] What is the speed of sound in this gas ?
- b) [10pts] Draw a picture of the 315Hz and 525Hz standing wave. Please draw the wave form for both pressure and displacement amplitudes.
- c) [10pts] If one end of the tube is now closed, what would be the frequency of the tuning fork needed to be to excite the second harmonic standing wave ? [If you could not do a) assume that $v = 370\text{m/s}$].

a) Let:

$$f_n = 315\text{ Hz}$$

$$f_{n+1} = 420\text{ Hz}$$

$$f_{n+2} = 525\text{ Hz}$$

Then:

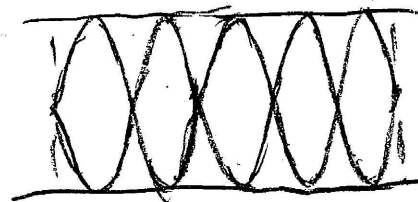
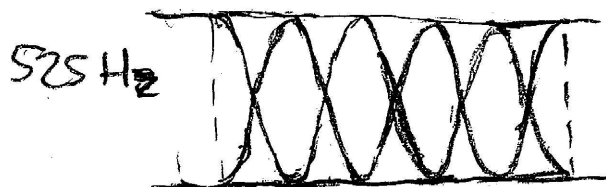
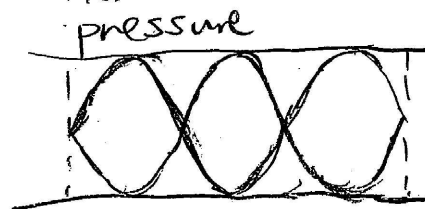
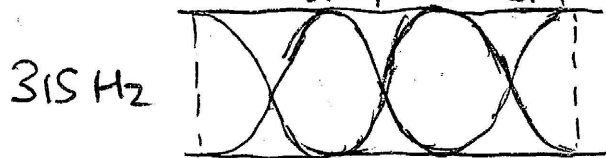
$$\frac{f_n}{f_{n+1}} = \frac{315\text{ Hz}}{420\text{ Hz}} = \frac{\frac{v}{2L}n}{\frac{v}{2L}(n+1)} = \frac{n}{n+1} = \frac{3}{4}$$

\downarrow
 $n=3$

$$v = \frac{f_n \cdot 2L}{n} = \frac{315\text{ Hz} \cdot (2)(190 \times 10^{-2}\text{ m})}{3}$$

$$= \boxed{399\text{ m/s}}$$

b) As stated before, $n=3$, for $f_3 = 315\text{ Hz}$.

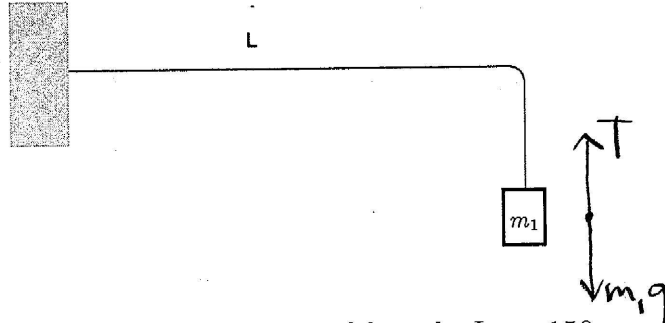


c) The frequency needed to excite the 2nd harmonic wave is:

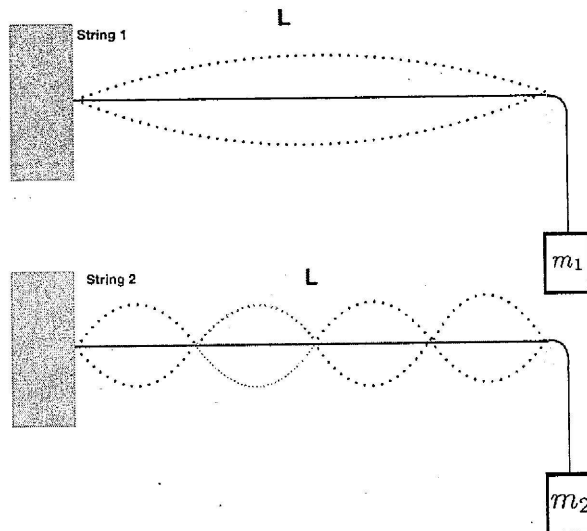
$$f_2 = \frac{2n-1}{4L} (v) = \frac{2(2)-1}{4(1.9\text{ m})} (399\text{ m/s}) = \boxed{157.5\text{ Hz}}$$

Problem 4: [20pts] [For this problem assume $g = 10\text{m/s}^2$]

a) [10pts] A string of length $L = 150\text{cm}$ and linear density $\mu = 5\text{gram/m}$ is held under tension by a pulley with $m_1 = 20\text{kg}$, find the wave speed v on the string and the wavelength λ for a sinusoidal wave of frequency $f = 1000\text{Hz}$.



b) [10pts] Now consider two identical strings of length $L = 150\text{cm}$ and linear density $\mu = 5\text{g/m}$. String 1 is held under tension with a pulley of $m_1 = 20\text{kg}$ and string 2 by a pulley with mass m_2 . Standing waves are excited on the two horizontal segments of the strings where string 1 is excited at the fundamental frequency and string 2 at its fourth harmonic frequency. You hear beats at a frequency of 2Hz . Find one possible value of m_2 .



$$a) v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{m_1 g}{\mu}} = \sqrt{\frac{(20 \text{ kg})(10 \text{ m/s}^2)}{(5 \times 10^{-3} \text{ kg/m})}} = 200 \text{ m/s}$$



$$\lambda f = v \rightarrow \lambda = \frac{v}{f} = \frac{200 \text{ m/s}}{1000 \text{ Hz}} = 0.20 \text{ m}$$

-additional space for calculation-

$$b) f_{\text{beat}} = |f_2 - f_1| \\ = 2 \text{ Hz.}$$

$$v = \sqrt{\frac{m_1 g}{M}}$$

$$f_1 = \frac{v}{2L} \overset{\text{fundamental}}{\downarrow} (1) = \frac{1}{2L} \sqrt{\frac{m_1 g}{M}}$$

$$= \frac{1}{2(150 \times 10^{-2} \text{ m})} \sqrt{\frac{(20 \text{ kg})(10 \text{ m/s}^2)}{5 \times 10^3 \text{ kg/m}}}$$

$$= 66.67 \text{ Hz}$$

$$f_2 = 68.67 \text{ Hz}, 64.67 \text{ Hz}$$

$$f_2 = \frac{v}{4L} (2(4) - 1) = \frac{7}{4L} \sqrt{\frac{m_2 g}{M}} \quad (4)$$

$$\left(\frac{4f_2 L}{7} \sqrt{\frac{M}{g}} \right)^2 = m_2$$

$$m_2 = \left(\frac{4(64.67 \text{ Hz})(150 \times 10^{-2} \text{ m})}{7} \right)^2 \frac{5 \times 10^3 \text{ kg/m}}{10 \text{ m/s}^2}$$

$$= \boxed{1.54 \text{ kg}} +$$