Physics 1B **Name (Print)** [redacted]
Fall 2016
Midterm 2
11/08/16
Time Limit: 50 Minutes University ID [redacted]

This exam contains 5 pages (including this cover page) and 4 problems. Check to see if any pages are missing. Enter all requested information on the top of this page, and put your initials on the top of every page, in case the pages become separated.

You may *not* use your books, notes, but you can use your calculator on this exam.

The following rules apply:

- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- If you need more space, use the back of the pages; clearly indicate when you have done this.

Do not write in the table to the right.

Problem	Points	Score
1	15	15
2	25	16 +2
3	20	14+4
4	20	20
Total:	80	65+4 81.25%

69+2=71

[redacted]

1. (15 points) Horseshoe bats (genus Rhinolophus) emit sounds from their nostrils, then listen to the frequency of the sound reflected from their prey to determine the prey's speed. (The "horseshoe" that gives the bat its name is a depression around the nostrils that acts like a focusing mirror, so that the bat emits sound in a narrow beam like a flashlight.) A Rhinolophus flying at speed v_{bat} emits sound of frequency f_{bat}; the sound it hears reflected from an insect flying toward it has a higher frequency f_{refl}.

   If the bat emits a sound at a frequency of 80.1 kHz, the speed of the insect is $v_{insect} = 3.7$ m/s, the bat travels at a speed of $v_{bat} = 3.9$ m/s, use 344 m/s for the speed of sound in air. What is the frequency f_{refl} the bat hears from the insect?

bat → 3.9 m/s 3.7 m/s ← insect

80.1 kHz

$$f_{bug} = \frac{V + 3.7}{V - 3.9}(f_{bat}) \quad \frac{347.7}{340.1}(80100) = 81889.94413$$

$$f_{bat}' = \frac{344+3.9}{344-3.7}(81889.94413) = 83{,}718.81153 \text{ Hz}$$

$$\boxed{83718.811.53 \text{ Hz}}$$

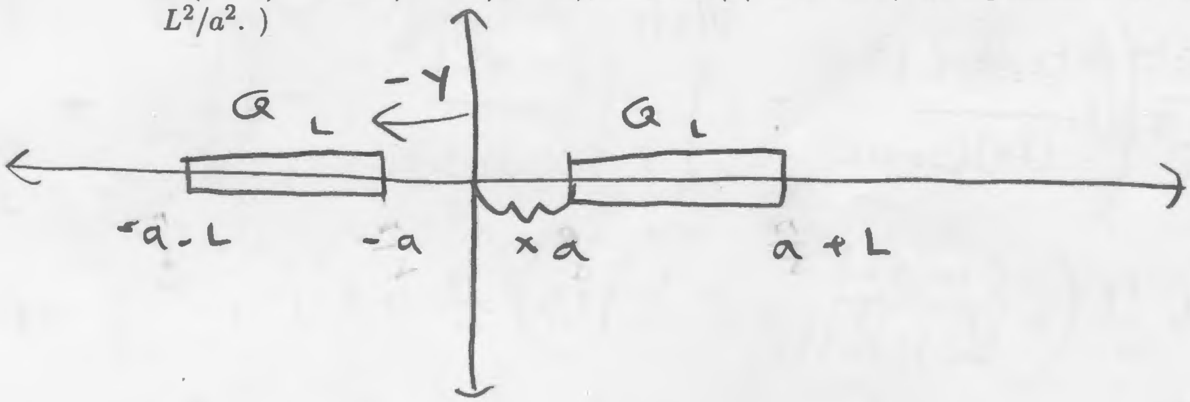
15

[redacted]

2. Two thin rods of length L lie along the x-axis, one between x=a and x=a+L and the other between x=$-a$ and x=$-a-L$. Each rod has positive charge Q distributed uniformly along its length.

   (a) (15 points) Calculate the electric field produced by the second rod at points along the positive x-axis?

   (b) (10 points) The magnitude of the force that one road exerts on the other is $F = \frac{kQ^2}{L^2} ln\left(\frac{(2a+L)^2)}{2a(2a+2L)}\right)$, assuming $a >> L$, find the magnitude of the force reduces to. (Hint: Use the expansion $ln(1+z) = z - z^2/2 + z^3/3 - ...$), valid for $|z| \ll 1$. Carry all expansion to at least order L^2/a^2.)



a) let "x" be the distance from origin to point on x axis

$\vec{E} = \int_{\text{over source}} k\frac{dq}{r^2}$ 15 $dq = \lambda dx$, $\lambda = \frac{Q}{L}$

$r = x + (-y) = x - y$ (y = distance origin to rod)

$\vec{E} = k\frac{Q}{L}\int_{-a}^{-a-L} \frac{1}{(x-y)^2}dx = -k\frac{Q}{L}\left[-(x-y)^{-1}\right]_{-a}^{-a-L} =$

$-k\frac{Q}{L}\left(\frac{1}{x+a+L} - \frac{1}{x+a}\right) = \boxed{k\frac{Q}{L}\left(\frac{1}{x+a} - \frac{1}{x+a+L}\right)}$

b) $F = \frac{kQ^2}{L^2}\left(2\ln(2a+L) - \ln(4a^2+4aL)\right)$

As a becomes very large, length L becomes more and more irrelevant (in comparison to a, so we can disregard L and treat the two like point charges.

$$F = \frac{kq_1q_2}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{Q^2}{(2a)^2} = \boxed{\frac{1}{4\pi\varepsilon_0}\frac{Q^2}{4a^2}}$$ 2

(attempt 1)

Using expansion: $z = \frac{(2a+L)^2}{(2a)(2a+2L)} - 1$, but as $a \gg L$

$$z - \frac{z^2}{2} + \frac{z^3}{3}$$

$$F = \frac{kQ^2}{L^2}\left(\left(\frac{(2a+L)^2}{(2a)(2a+2L)} - 1\right) - \left(\frac{(2a+L)^2}{(2a)(2a+2L)} - 1\right)^2 + \left(\frac{(2a+L)^2}{(2a)(2a+2L)} - 1\right)^3 \cdots\right)$$

↑ If you do all the algebra eventually it simplifies to the case of 2 pt. charges.

[redacted]

3. (points) A student enters the room to find an insulating sphere of radius a has an embedded nonuniform charge density:
$\rho(r) = \frac{\rho_0}{2}(1 + r/a)$, $r \leq a$,
where ρ_0 is positive constant charge density inside $r \leq a$.

(a) (10 points) What is the total charge contained in the non-conducting sphere of radius a?

(b) (10 points) What is the electric field everywhere is space: considering the following regimes: $r < a$, $r > a$

a) Imagine integrating shells

$A = 4\pi r^2$

$dq = \frac{\rho_0}{2}\left(1 + \frac{r}{a}\right)$

$$Q_{enc} = \int_0^a 4\pi r^2 \cdot \frac{\rho_0}{2}\left(1 + \frac{r}{a}\right) dr =$$

$$2\pi\rho_0 \int_0^a r^2\left(1 + \frac{r}{a}\right) dr = 2\pi\rho_0 \int_0^a r^2 + \frac{r^3}{a}\, dr =$$

$$2\pi\rho_0 \left[\frac{r^3}{3} + \frac{r^4}{4a}\right]_0^a = 2\pi\rho_0\left(\frac{a^3}{3} + \frac{a^4}{4a}\right) = 2\pi\rho_0\left(\frac{a^3}{3} + \frac{a^3}{4}\right)$$

$$= \boxed{\pi\rho_0\left(\frac{7}{6}a^3\right)}$$

✓ 10

b) If $r > a$

Gauss' Law

$$\oint\!\!\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$$ ✓ 2

Sphere (uniform share), of radius a

$$4\pi \cancel{a^2}\, r^2\, \vec{E} = 2\pi\rho_0\left(\frac{7}{12}a^3\right)$$

$$\vec{E} = \frac{1}{2}\rho_0\left(\frac{7}{12}a\right)$$

$$\boxed{\vec{E} = \rho_0\left(\frac{7}{24}a\right)}$$

the given a

2 pts. for consistency.

If $r < a$

$$\oint\!\!\oint \vec{E} \cdot d\vec{a} = \frac{Q_{enc}}{\varepsilon_0}$$

sphere of radius $R (= r)$

+4

$$4\pi R^2 \vec{E} = \frac{\int_0^R 4\pi r^2 \cdot \frac{\rho_0}{2}\left(1+\frac{r}{a}\right) dr}{\varepsilon_0}$$ ← same as part A

Yes. but the integral is not the same.

$$4\pi R^2 \vec{E} = 2\pi \rho_0 \left(\frac{7}{12} R^3\right) \frac{1}{2}$$

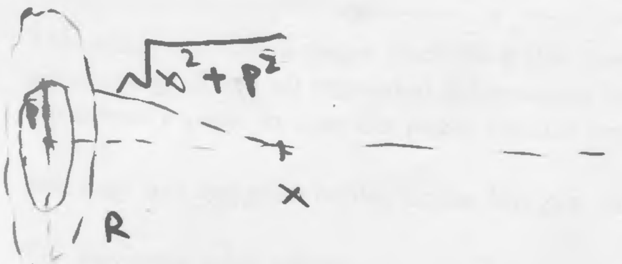
$$\boxed{\vec{E} = \rho_0 \left(\frac{7}{24} R\right)}$$ when R is radius of your shell, $R<a$, $R=r$

[redacted]

4. (points) A disk with radius R has uniform surface charge density σ.

   (a) (10 points) By regarding the disk as series of thin concentric rings, calculate the electric potential V at a point on the disk's axis a distance x from the center of the disk. Assume that the potential is zero at infinity.

   (b) (10 points) Calculate the electric field using $\vec{E} = -\nabla V$.



$$dq = 2\pi \rho \sigma \, d\rho$$

$$r = \sqrt{x^2+\rho^2}$$

$$V = k \int \frac{dq}{r} = k \int_0^R \frac{2\pi \rho \sigma \, d\rho}{\sqrt{x^2+\rho^2}} =$$

$$2\pi\sigma k \int_0^R \frac{\rho}{\sqrt{x^2+\rho^2}} d\rho = 2\pi\sigma k \left[\sqrt{x^2+\rho^2} \right]_0^R =$$

$$2\pi\sigma k \left(\sqrt{x^2+R^2} - \sqrt{x^2} \right) = \boxed{2\pi\sigma k \left(\sqrt{x^2+R^2} - x \right)}$$ (always positive)

b) $\vec{E} = -\nabla V \leftarrow$ V has ~~no~~ y or z component, only x

$$= -2\pi\sigma k \left(\frac{x}{\sqrt{x^2+R^2}} - 1 \right) \hat{x}$$

$$\boxed{= 2\pi\sigma k \left(1 - \frac{x}{\sqrt{x^2+R^2}} \right) \hat{x}}$$