Midterm exam 1

Physics 1B, Spring 2016

Name:
UCLA ID
Lecture

Please write solutions with some minimal derivation in the space provided below each problem; it is not sufficient to give just the final answer. The level of detail should be such that a grader, or your fellow classmate would understand how you solved the problem.

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Problem 1.

A rectangular flat-bottom barge with a bottom area $A = 100\,\text{m}^2$ is loaded so that the bottom is at $H = 1\text{m}$ below the surface. The density of water is $\rho = 10^3\text{kg/m}^3$, and the water surface is perfectly still.

(a) Calculate the mass of the barge.

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$$\Sigma F = 0 = F_B - F_g$$
$$0 = \rho_w g H A - mg$$
$$mg = \rho_w g H A$$
$$m = \rho_w H A$$
$$= (10^3)(1)(100)$$
$$= \boxed{10^5\ \text{kg}}$$

10 (b) A round hole with radius $r = 2$ cm is made in the bottom of the barge, and the water starts leaking in. When the water level reaches $h = 5$ cm, a bilge alarm will alert the barge operator. How long will it take for the water to reach the level 5 cm? (Assume that the Bernoulli's equation is applicable.)

water going in — water outside

$$\tfrac{1}{2}\rho v_1^2 + \rho g h_1 + P_a = P_a + \rho g h_2 + \tfrac{1}{2}\rho v_2^2$$
$$\tfrac{1}{2}\rho v_1^2 + 0 = \rho g h_2 + 0$$
$$v_1 = \sqrt{2 g h_2} = \sqrt{2 \cdot 9.8 \cdot 1} \approx 4.427\ \text{m/s}$$

$$V = AH = \pi r^2 h(t) \quad \text{where } h(t) = v \cdot t$$
$$AH = \pi r^2 v t$$
$$t = \frac{AH}{\pi r^2 v} = \frac{100 \cdot (0.05)}{\pi (0.02)^2 (4.427)} = \boxed{890}$$

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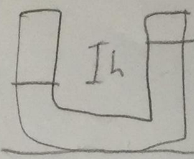

$\frac{N}{m^2} = \frac{kg \cdot m}{s^2} \cdot \frac{1}{m^2}$

$\frac{kg}{m^3} \cdot \frac{m}{s^2} \cdot m$

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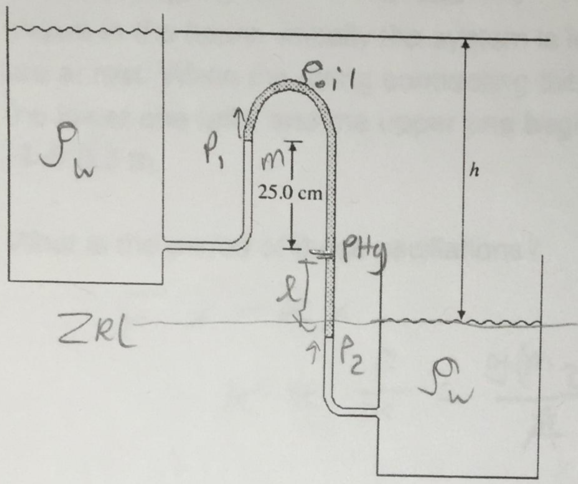
Problem 2

The two water reservoirs shown in the figure are open to the atmosphere, and the water has density 1000 kg/m³. The manometer contains incompressible oil with a density of 820 kg/m³. What is the difference in elevation h if the manometer reading m is 25.0 cm?

$\Delta y = h$



$\Delta P = P_2 - P_1 = \rho_{oil} g \Delta y$

$P_2 - P_1 = 820 \cdot 9.8 \cdot 0.25$

$P_2 - P_1 = 2009 \frac{N}{m^2}$

Bernoulli's eqn:

$P_1 + \rho_w g y_1 = P_2 + \rho_w g h + \rho_{Hg}$ $(+\rho_{air} \cdot g h)$

$P_1 + \rho_w g h = P_2 + \rho_w g(0)$

$P_2 - P_1 = \rho_w g h$

$h = \frac{P_2 - P_1}{\rho_w g} = \frac{2009}{9.8 \cdot 1000} = 0.205\,m$

$h = 0.205\,m$ ✗

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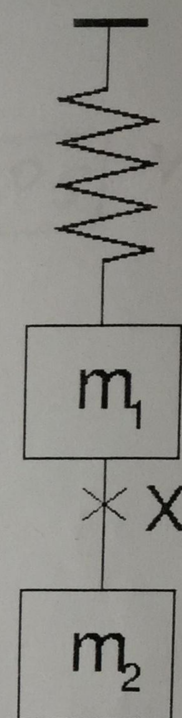
Problem 3

The two weights, $m_1 = 1$ kg and $m_2 = 2$ kg hang on the spring as shown in the figure. Initially the system is in equilibrium, and the weights are at rest. When the string connecting the two weights is cut at point X, the lower one falls, and the upper one begins to oscillate with an amplitude $A = 0.2$ m.



What is the period of these oscillations?

$F = -kx$

$k = \frac{F}{x} = \frac{g(m_2)}{A} = \frac{9.8 \cdot 2}{0.2} = 98 \frac{N}{m}$

$T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{1}{98}} = \boxed{0.635\,s}$ ✓

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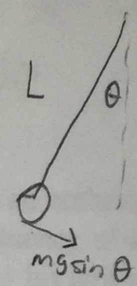
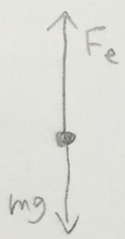


Problem 4

A simple pendulum has a length of 120 cm.

(a) What is its period of oscillations?

$$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1.2}{9.8}} = \boxed{2.20\,s}$$ ✓



(b) What is the period of oscillations inside an elevator moving up with an acceleration 1.2 m/s²

$$\Sigma F = ma = -mg + F_{elevator}$$

$$mg + ma = F_{elevator}$$

$$m(g+a) = F_{elevator}$$

(g+a): net acceleration

$$T = 2\pi\sqrt{\frac{L}{g+a}} = 2\pi\sqrt{\frac{1.2}{9.8+1.2}} = \boxed{2.08\,s}$$ ✓

(c) What is the period of the same pendulum on Mars, where the acceleration of gravity is about 0.37 that on Earth?

$$g_{mars} = 0.37 \cdot g_{earth} = 0.37 \cdot 9.8 = 3.626\ m/s^2$$

$$T = 2\pi\sqrt{\frac{L}{g_{mars}}} = 2\pi\sqrt{\frac{1.2}{3.626}} = \boxed{3.61\,s}$$ ✓

$\frac{m}{s} \cdot \frac{m}{s} \quad \frac{m}{s^2}$

$\mu = 2, v = 8, \lambda = 5,$

$F_2 = 157080.125N \quad F_1 = \sout{2420000} \; 151250\,N$

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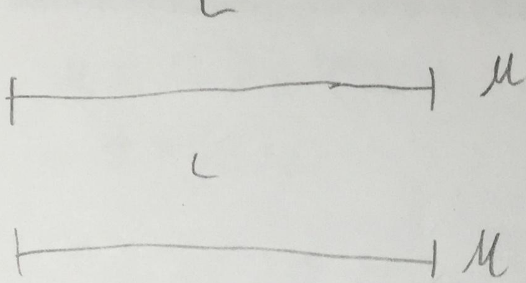
Problem 5

Two violinists are trying to tune their instruments in an orchestra. One is producing the desired frequency of 440.0 Hz. The other is producing a frequency of 448.4 Hz. By what percentage should the out-of-tune musician change the tension in his string to bring his instrument into tune at 440.0 Hz?



$f_1 = 440$ Hz ← what we want

$f_2 = 448.4$ Hz

$v = \sqrt{\frac{F}{\mu}} = v = f\lambda \rightarrow f = \frac{v}{\lambda}$

$f = \frac{v}{\lambda}\sqrt{\frac{F}{\mu}}$ Let: $F_1 = \alpha F_2$

$f_1 = 440 = \frac{v}{\lambda}\sqrt{\frac{F_1}{\mu}}$

$f_2 = 448.4 = \frac{v}{\lambda}\sqrt{\frac{F_2}{\mu}} = \frac{v}{\lambda}\sqrt{\frac{F_1}{\alpha\mu}}$

$448.4 = \frac{1}{\sqrt{\alpha}}\left(\frac{v}{\lambda}\sqrt{\frac{F_1}{\mu}}\right) = \frac{1}{\sqrt{\alpha}} f_1$

$448.4 = \frac{1}{\sqrt{\alpha}} 440$

$\frac{1}{448.4} = \sqrt{\alpha} \cdot \frac{1}{440}$

$\alpha = \left(\frac{440}{448.4}\right)^2$

$= 0.963$

$1 - 0.963 = \boxed{3.71\%}$ ✓

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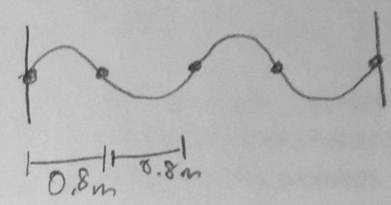
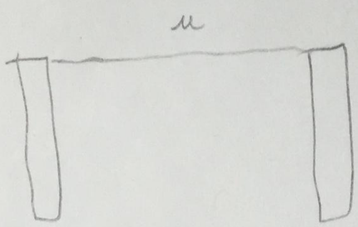
Problem 6

Standing waves of frequency 50 Hz are produced on a string that has mass per unit length 0.025 kg/m. With what tension must the string be stretched between two supports if adjacent nodes in the standing wave are to be 0.8 m apart?

$\mu = 0.025 \, kg/m$

$f = 50 \, Hz$





$\lambda = 0.8 \cdot 2 = 1.6 \, m$

$v = f\lambda = 1.6 \cdot 50 = 80 \, m/s$

$v = \sqrt{\frac{F}{\mu}} \rightarrow v^2 \mu = F$

$\frac{m^2}{s^2} \cdot \frac{kg}{m} = \frac{kg\,m}{s^2}$

$F = (80)^2 \cdot (0.025)$

$= \boxed{160 \, N}$ ✓

1	20
2	12
3	20
4	20
5	20
6	20
TOT	112