Problem 1.

A rectangular flat-bottom barge with a bottom area $A = 50\,\mathrm{m}^2$ is loaded so that the bottom is at $H = 1\mathrm{m}$ below the surface. The density of water is $\rho = 10^3 kg/m^3$, and the water surface is perfectly still.

(a) Calculate the mass of the barge.

$V = AH$

According to Archimedes' principle, $\rho g V = F_b = mg$

$m = \rho V = \rho A H = \boxed{50000 \text{ kg}}$

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(b) A round hole with radius $r = 1$ cm is made in the bottom of the barge, and the water starts leaking in. When the water level reaches $h = 5$ cm, a bilge alarm will alert the barge operator. How long will it take for the water to reach the level 5 cm? (Assume that the Bernoulli's equation is applicable.)

$P_1 + \rho g y_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g y_2 + \frac{1}{2}\rho v_2^2$

hole: P_1, v_1, y_1

below hole: P_2, v_2, y_2

$y_1 = y_2$

$h = 1m$

$v_2 = 0$

$P_1 = P_0$

$P_2 = P_0 + \rho g h$

$P_0 + \frac{1}{2}\rho v_1^2 = P_0 + \rho g h$

$\frac{1}{2}\rho v_1^2 = \rho g h$

$\frac{1}{2} v_1^2 = g h$

$v_1 = \sqrt{2gh}$

$A = \pi r^2$

Volume flow rate: $\frac{dV}{dt} = A v_1 = A\sqrt{2gh} = 0.00139\ m^3/s$

Volume when water level is 5cm $= 0.05m(50m^2) = 2.5\ m^3$

$\frac{2.5\,m^3}{0.00139\,m^3/s} = \boxed{1797.5\ s}$

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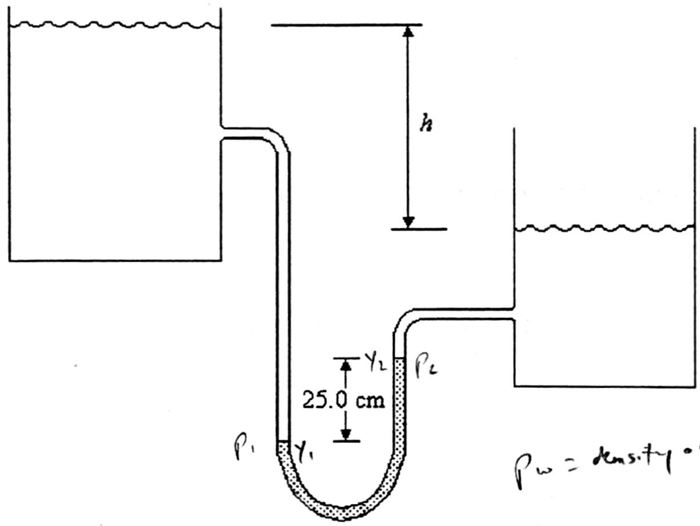
Problem 2

The two water reservoirs shown in the figure are open to the atmosphere, and the water has density 1000 kg/m3. The manometer contains incompressible mercury with a density of 13,600 kg/m3. What is the difference in elevation *h* if the manometer reading *m* is 25.0 cm?



ρ_w = density of water

ρ_m = density of mercury

$P_1 = P_2 + \rho_w g h$

$P_1 - P_2 = \rho_m g (y_2 - y_1)$

$P_2 + \rho_w g h - P_2 = \rho_m g (y_2 - y_1)$

$h = \frac{\rho_m}{\rho_w}(y_2 - y_1) = \boxed{3.4\ m}$

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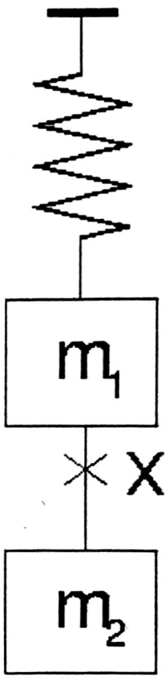
Problem 3

The two weights, $m_1 = 1$ kg and $m_2 = 2$ kg hang on the spring as shown in the figure. Initially the system is in equilibrium, and the weights are at rest. When the string connecting the two weights is cut at point X, the lower one falls, and the upper one begins to oscillate with an amplitude $A = 0.2$ m.



What is the period of these oscillations?

$-kx = -mg$

$kA = (m_1 + m_2)g$

$k = \dfrac{(m_1+m_2)g}{A}$

$T = 2\pi\sqrt{\dfrac{m_1}{k}} = 2\pi\sqrt{\dfrac{m_1 A}{(m_1+m_2)g}} = \boxed{0.52\ \text{s}}$

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Problem 4

A simple pendulum has a length of 220 cm.

(a) What is its period of oscillations?

$T = 2\pi\sqrt{\frac{L}{g}} = \boxed{2.98\ s}$

(b) What is the period of oscillations inside an elevator moving up with an acceleration 2.2 m/s^2

$T = 2\pi\sqrt{\frac{L}{g+2.2}} = \boxed{2.69\ s}$

(c) What is the period of the same pendulum on Mars, where the acceleration of gravity is about 0.37 that on Earth?

$T = 2\pi\sqrt{\frac{L}{0.37g}} = \boxed{4.89\ s}$

Problem 5

Two violinists are trying to tune their instruments in an orchestra. One is producing the desired frequency of 440.0 Hz. The other is producing a frequency of 448.4 Hz. By what percentage should the out-of-tune musician change the tension in his string to bring his instrument into tune at 440.0 Hz?

$f_1 = 440 = \frac{1}{2L}\sqrt{\frac{F_1}{\mu}}$

$f_2 = 448.4 = \frac{1}{2L}\sqrt{\frac{F_2}{\mu}}$

$\frac{f_1}{f_2} = \frac{\sqrt{F_1}}{\sqrt{F_2}}$

$0.9813 = \frac{\sqrt{F_1}}{\sqrt{F_2}}$

$0.9629 = \frac{F_1}{F_2}$

$0.9629 F_2 = F_1$

$1 - 0.9629 = 0.0371$

$\boxed{\text{lower by } 3.71\%}$ ✓

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Problem 6

Standing waves of frequency 60 Hz are produced on a string that has mass per unit length 0.015 kg/m. With what tension must the string be stretched between two supports if adjacent nodes in the standing wave are to be 0.7 m apart?

$d = L = \frac{\lambda}{2} = 0.7\ m$ $\frac{d}{2} = L = \lambda = 1.4\ m$

$60 = n \frac{1}{2L}\sqrt{\frac{F}{\mu}}$

$\sqrt{\frac{F}{\mu}} = \frac{120L}{n}$

$F = \frac{(120L)^2\mu}{n^2} = (120d)^2\mu = \boxed{105.84\ N}$

distance between adjacent nodes $= d = \frac{L}{n}$