Final exam

Physics 1B, Spring 2017

Name:

UCLA ID number:

Lecture: ☐ 4 ☐ 5

Section (number, meeting time, or TA name):

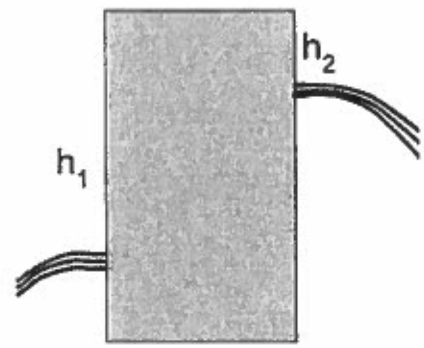
Please write solutions with some minimal derivation in the space provided below each problem; it is not sufficient to give just the final answer. The level of detail should be such that a grader, or your fellow classmate would understand how you solved the problem. You may use the back sides of each page as scrap paper.

Each problem is worth 30 points. If a problem has several parts, each part is worth the same number of points.

Time to complete the exam: 120 min

1	2	3	4	5	6	total

Problem 1.



Two small holes are cut on the opposite sides of the tank, at the heights h_1 and h_2 below the surface of the water. The water density is ρ. Each hole has a radius r, which is much smaller than the size of the tank or the heights h_1 and h_2.

(a) What is the difference in speeds $\Delta v = |v_1 - v_2|$ of the water coming out of the two holes?

(b) As the water flows out of the tank through the two holes, it produces a reaction force on the tank. Calculate the resultant force of reaction from the two jets flowing in the opposite directions. (One can measure this force by placing the tank on a frictionless surface and connecting it to a spring with a known spring constant.)

Solutions

Problem 1

a) Bernoulli Equation: $P + \frac{\rho v^2}{2} + \rho g h = constant$. To get the speed of the water coming out of the lower hole (with heights h_1 from the water surface): we choose one point at the water surface and one point near the hole. We have:

$$P_0 = P_0 + \frac{\rho v_1^2}{2} + \rho g(-h_1)$$

Note that we choose $h = 0$ for water surface and therefore $h = -h_1$ for the point near the hole. Then we have:

$$v_1 = \sqrt{2gh_1}$$

To get v_2, we make a substitution $1 \leftrightarrow 2$ for the equation above, we have:

$$v_2 = \sqrt{2gh_2}$$

Then $\Delta v = |v_1 - v_2| = \sqrt{2gh_1} - \sqrt{2gh_2}$.

b) The direction of the force due to the water coming out of the lower hole is in right direction. Now let's work out the magnitude. Consider dt time after you open the hole, we have in general

$$\frac{dp}{dt} = F$$

Now our work is to work out the change of momentum dp in dt time. Note that

$$dp = v_1 dm = \pi r^2 \rho v_1 dl = \pi r^2 \rho v_1^2 dt$$

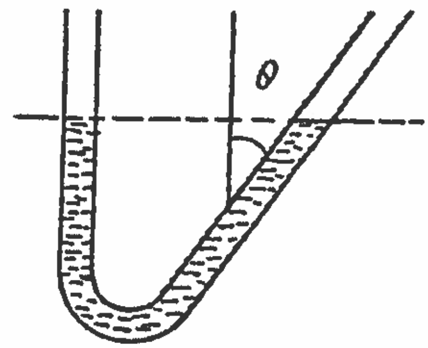
Then we have $F_1 = \pi r^2 \rho v_1^2$
To get F_2, we make a substitution $1 \leftrightarrow 2$ for the equation above, we have:$F_2 = \pi r^2 \rho v_2^2$
Note the direction of F_2 is in left direction, so finally we have:

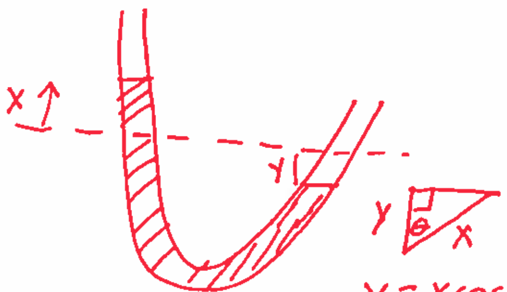
$$F_{total} = F_1 - F_2 = \pi r^2 \rho (v_1^2 - v_2^2) = 2\pi r^2 \rho g(h_1 - h_2)$$

The direction is in right direction.

Problem 2.

Calculate the period of oscillations of mercury of mass m poured into a thin bent tube whose right arm forms an angle θ with the vertical. The tube has a radius r, which is much smaller than the height of the mercury or the length of the tube. The density of mercury is ρ. Neglect viscosity and friction.





$y = x\cos\theta$

$$F = -\Delta p \cdot A = -(\rho g x + \rho g y)\pi r^2$$

$$= -\rho g \pi r^2 (1+\cos\theta) x$$

$$m \frac{d^2x}{dt^2} = -\rho g \pi r^2 (1+\cos\theta) x$$

$$\frac{d^2x}{dt^2} = -\frac{\rho g \pi r^2 (1+\cos\theta)}{m} x = -\omega^2 x$$

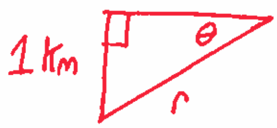
$$\omega^2 = \frac{\rho g \pi r^2}{m}(1+\cos\theta)$$

$$T = \frac{2\pi}{\omega}$$

Problem 3. On a remote planet, the atmosphere is different from Earth, but it allows life to exist. The speed of sound on that planet is $v_s = 100$ m/s.

(a) A spacecraft is flying horizontally at a height of 1 km with a speed of 200 m/s, exploring the terrain. How far will the spacecraft be from an animal on the surface, when the animal hears the sonic boom? (Calculate the distance along the inclined line connecting the animal to the spacecraft, not the distance projected on the ground.)

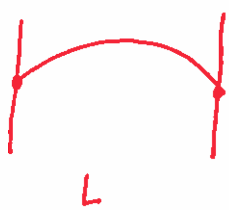
$$M = \frac{V}{V_S} = 2 \qquad \sin\theta = \frac{1}{M} = \frac{1}{2}$$



$$\sin\theta = \frac{1\ \text{km}}{r}$$

$$r = \frac{1}{\sin\theta}\ \text{km} = 2\ \text{km}$$

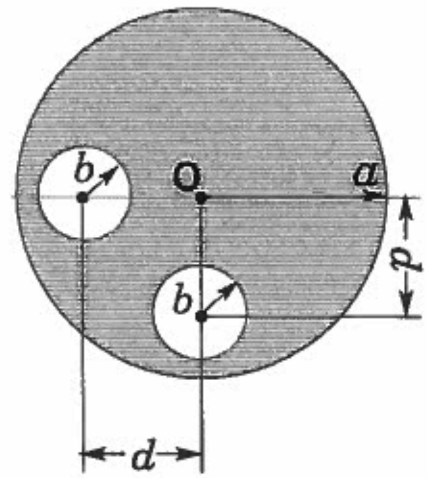
(b) Musical instruments sound differently on this planet compared to their sound on Earth. Calculate the fundamental frequency of a standing wave in a closed-end organ pipe that has a length L=2 m.



$$\lambda = 2L = 4\ \text{m}$$

$$f = \frac{V}{\lambda} = \frac{100}{4}\ \text{Hz} = 25\ \text{Hz}$$

Problem 4. A non-conducting sphere of radius a with a constant positive charge density ρ has two identical hollow spherical cavities (with zero charge density), as shown in the figure.



(a) What is the magnitude of the electric field E at point O in the center of the large sphere?

(b) What is the electric potential at point O, assuming it is zero at infinity?

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Final Solutions

1 Problem 4

a. By the principle of superposition, this problem can be treated as one completely solid sphere (radius a) of charge density ρ combined with two smaller spheres (radius b) of charge density $-\rho$ placed at the location of the holes. The electric field at the origin is then just the sum of the field contributions from these three spheres. By symmetry, the solid sphere produces no field at its own center and by Gauss' law the two small spheres act like point charges. As such, the electric field is given by:

$$\vec{E} = \frac{kq}{d^2}\hat{x} + \frac{kq}{d^2}\hat{y} \tag{1}$$

where q is the total charge of each small sphere. We can then express the total charge in terms of the charge density and the volume giving $q = -\frac{4\pi b^3 \rho}{3}$

Finally, the magnitude of the field at the origin is:

$$|\vec{E}| = \sqrt{E_x^2 + E_y^2} = \sqrt{(-\frac{k\frac{4\pi b^3\rho}{3}}{d^2})^2 + (-\frac{k\frac{4\pi b^3\rho}{3}}{d^2})^2} = \sqrt{2}\frac{k4\pi b^3\rho}{3d^2} \tag{2}$$

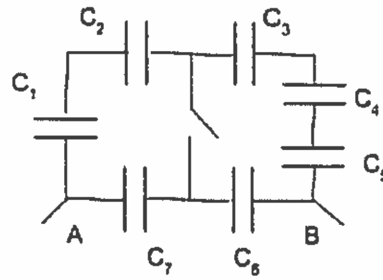
b. As in part (a) we can find the potential V at the origin by treating the system as the superposition of the three spheres. Since the observation point is outside of the small spheres, they again act like point charges and contribute $\frac{kq}{d}$ each to the potential. However, the observation point is inside the large sphere so we cannot use the point charge formula. Instead, as derived in class, the potential at the center of a solid sphere of charge with radius a and charge Q is $\frac{3kQ}{2a}$. Since the potential is a scalar quantity the total potential is just the scalar sum of all these contributions giving:

$$V = \frac{kq}{d} + \frac{kq}{d} + \frac{3kQ}{2a} \tag{3}$$

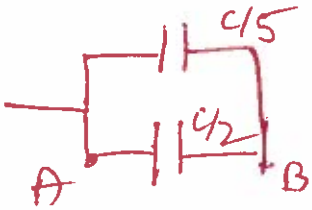
Finally, since the total charge of each small sphere is $q = -\frac{4\pi b^3\rho}{3}$ and the total charge of the large sphere is $Q = \frac{4\pi a^3\rho}{3}$:

$$V = -\frac{k4\pi b^3\rho}{3d} - \frac{k4\pi b^3\rho}{3d} + k2\pi a^2\rho \tag{4}$$

Problem 5. Every capacitor in the circuit has the same capacitance:
$C_1 = C_2 = C_3 = C_4 = C_5 = C_6 = C_7 = C$.
Express your answers in terms of C.

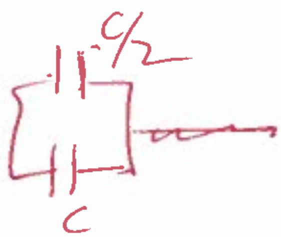


(a) Calculate the equivalent capacitance between points A and B when the switch is open.



$C_{eff} = \frac{7C}{10}$

(b) Calculate the equivalent capacitance between A and B when the switch is closed.

①


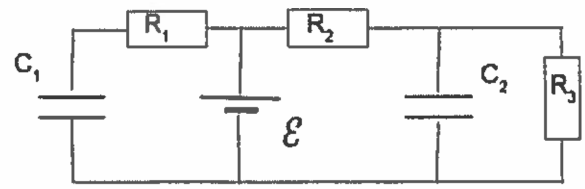
②


① & ② in series $\Rightarrow C_{eff} = \frac{12C}{17}$

(c) If the potential difference $V_{AB} = V$ is applied between points A and B when the switch is open, what is the charge Q_1 on capacitor C_1?

$\frac{CV}{5}$

Problem 6. Three resistors R_1, R_2, R_3 and two capacitors C_1, C_2 are connected to an ideal battery with emf $\mathcal{E}$ as shown in the figure. Express your answers in $R_1, R_2, R_3, C_1, C_2, C_3, \mathcal{E}$.



(a) Assuming that enough time has elapsed for the currents to be time-independent, calculate the currents I_1, I_2, I_3 in the resistors R_1, R_2, R_3, respectively.

at steady state, capacitor is open-loop

$i_1 = 0$

$i_2 = i_3 = \dfrac{\varepsilon}{R_2 + R_3}$

(b) Calculate the charge Q_1 on capacitor C_1.

$Q = CV \Rightarrow Q_1 = C_1 \varepsilon \ (\because i_1 = 0)$

(c) Calculate the charge Q_2 on capacitor C_2.

$Q_2 = C_2 \cdot V_2$

$V_2 = \varepsilon - i_2 \cdot R_2 = \varepsilon - \dfrac{\varepsilon}{R_2 + R_3} R_2 = \varepsilon \cdot \dfrac{R_3}{R_2 + R_3}$

$\Rightarrow Q_2 = C_2 \cdot \dfrac{\varepsilon R_3}{R_2 + R_3}$