20S-PHYSICS1B-4 Midterm 2

RYAN VUONG

TOTAL POINTS

34 / 34.5

QUESTION 1

Treacherous Triangle Trickery 20 pts

1.1 3 / 3

✓ **+ 2 pts** **Essentially completely correct reasoning.**

+ 1 pts A few errors made.

+ 0 pts Essentially incorrect.

✓ **+ 1 pts** **Field at center is zero.**

1.2 3 / 3

✓ **+ 1 pts** **Potential due to first charge correct.**

✓ **+ 1 pts** **Potential due to second charge correct.**

✓ **+ 1 pts** **Potential due to third charge correct.**

+ 0 pts None correct.

+ 1 pts None correct but significant progress

1.3 3 / 3

✓ **+ 1 pts** **Gradient of potential is shown to be zero.**

✓ **+ 2 pts** **Computation of gradient is correct.**

+ 1 pts Computation of gradient has some errors.

+ 0 pts Computation of gradient is essentially incorrect.

1.4 2.5 / 2.5

✓ **+ 2.5 pts** **All correct**

+ 0.5 pts Graph asymptotes to $$\pm\infty$$ (depending on the assumed sign of $$q$$) as $$x\to -d$$ from the left.

+ 0.5 pts Graph asymptotes to $$\pm\infty$$ (depending on the assumed sign of $$q$$) as $$x\to -d$$ from the right.

+ 0.5 pts Graph asymptotes to $$0$$ as $$x\to-\infty$$

+ 0.5 pts Graph asymptotes to $$0$$ as $$x\to+\infty$$

+ 0.5 pts From $$-d$$ to $$+\infty$$, graph basically depicted as decreasing (or increasing if $$q<0$$) with the exception of a feature near $$x = d/2$$ that looks either like a bump or a flat portion -- something that makes it clear that it's recognized what the two charges not on the $$x$$\-axis do to the potential there.

+ 0 pts None correct

1.5 4 / 4

+ 0.5 pts Order $$x^0$$ term is nonzero.

+ 1 pts Order $$x^0$$ term has value $$3kq/d$$

+ 1 pts Order $$x$$ term is zero.

+ 0.5 pts Order $$x^2$$ term is nonzero.

+ 1 pts Order $$x^2$$ term has value $$3kq/(4d^3) x^2$$

+ 0 pts None correct.

✓ **+ 4 pts** **All correct**

1.6 2 / 2.5

+ 0.5 pts Reasoning contains observation that charges $$q$$ and $$Q$$ need to be same sign for charge $$q$$ to remain in a vicinity of the origin. (or opposite if the expansion in 1.5 had an incorrect negative x^2 term)

✓ **+ 2 pts** **Reasoning is essentially correct. (or if V was incorrect in 1.5, leading to the wrong conclusion)**

+ 1 pts Reasoning has some errors.

+ 0 pts Reasoning is incorrect.

1.7 2 / 2

✓ **+ 0.5 pts** **Attempted to determine the equation of motion $$\ddot x = F(x)/m$$**

✓ **+ 0.5 pts** **Attempted to Taylor expand the equation of motion so as to obtain the small-$$x$$ equation of motion.**

✓ **+ 0.5 pts** **Found that the equation of motion for** small $$x$$ is of the form $$\ddot x = -\omega^2 x$$

✓ + **0.5 pts** **Found that $$T = 2\pi\sqrt{2md^3/(3kQq)}$$**

+ **0.2 pts** Answer off by small coefficient

+ **0 pts** Incorrect approach

QUESTION 2

2 Auditory Airplane Inference 5.5 / 5.5

✓ + **1 pts** **Noted that when the plane is coming toward the observer and is far from the observer, the observed and emitted frequencies are related by $$f_o = \frac{v}{v-v_s} f_s$$**

✓ + **1 pts** **Noted that when the plane is going away from the observer and is far from the observer, the observed and emitted frequencies are related by $$f_o = \frac{v}{v+v_s} f_s$$**

✓ + **1 pts** **Found that the speed of the source is given in terms of the toward and away observed frequencies and the speed of sound as $$v_s = \frac{f_{o, \mathrm{toward}}-f_{o, \mathrm{away}}}{f_{o, \mathrm{toward}}+ f_{o, \mathrm{away}}}v$$**

✓ + **0.5 pts** **Used the beginning part (small enough to make clear that student recognized middle transition section needed to be ignored) of the provided waveform to determine an estimate for $$f_{o, \mathrm{toward}}$$**

✓ + **0.5 pts** **Used the end part (small enough to make clear that student recognized middle transition section needed to be ignored) of the provided waveform to determine an estimate for $$f_{o, \mathrm{away}}$$**

✓ + **0.5 pts** **Estimated $$f_{o, \mathrm{toward}} \approx 44\,\mathrm{Hz}$$ (can be off by around 25%).**

✓ + **0.5 pts** **Estimated $$f_{o, \mathrm{away}} \approx 12\,\mathrm{Hz}$$ (can be off by around 25%).**

✓ + **0.5 pts** **Estimated $$v_s \approx 200\,\mathrm m/\mathrm s$$ (can be off by around 25%)**

+ **0 pts** Incorrect approach

QUESTION 3

True or False 9 pts

3.1 3 / 3

+ **1 pts** True.

+ **2 pts** Essentially correct, complete reasoning with maybe a minor error.

✓ + **3 pts** **Fully correct reasoning.**

+ **1 pts** Reasoning has some errors or is incomplete, but some stuff correct.

+ **0 pts** Essentially incorrect reasoning.

3.2 3 / 3

✓ + **3 pts** **Fully correct reasoning.**

+ **2 pts** Essentially correct, complete reasoning with maybe a minor error.

+ **1 pts** Reasoning has some errors or is incomplete, but some stuff correct.

+ **0 pts** Essentially incorrect reasoning.

3.3 3 / 3

✓ + **3 pts** **Fully correct reasoning.**

+ **2 pts** Essentially correct, complete reasoning with maybe a minor error.

+ **1 pts** Reasoning has some errors or is incomplete, but some stuff correct.

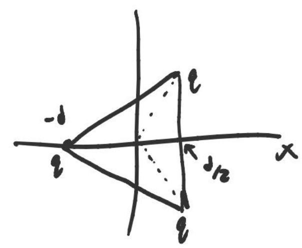
+ **0 pts** Essentially incorrect reasoning.

Instructions.

1. Complete this exam packet, and submit it to Gradescope before the end of the exam period.
2. Make sure all final work is in only the spaces provided, and make sure to upload a document of exactly the length of this packet.
3. If you are using a smartphone to generate a scan, please use a scanning app such as Adobe Scan to quickly generate an optimized PDF document.
4. There is extra space at the end of the packet in case the space below each problem isn't sufficient, but if you use that extra space, make sure to indicate that extra work is contained there when you show your work for a problem.
5. You are allowed to use the CCLE course page, the course textbook (OpenStax), and your notes, but you are not permitted to use any other internet resources.
6. A calculator (whatever type desired) is allowed.
7. You may not communicate about the contents of this exam with anyone during the exam period.
8. You may not logon to Campuswire during the exam period.
9. **Josh and the TAs will not be free to answer clarifying questions about the exam during the exam period**. If you believe there is an ambiguity, feel free to make a note of it to yourself and we will attempt to address it post-exam, but otherwise do your best to answer the questions based on what they say.
10. Violations of instructions pertaining to what you are not allowed to do during the exam will be considered cheating and will be reported to the Dean of Students.

1. Treacherous Triangle Trickery. Consider a charge distribution consisting of an equilateral triangle with a point charge q fixed at each of its vertices. Let d be the distance between the center of the triangle and each vertex, let the triangle's center be at the origin, and let one of its vertices lie on the x-axis at the point $x = -d$.

1.1. Compute the electric field at the center of the triangle by explicitly computing the sum of the electric fields due the charges at each vertex.



Side length $= \sqrt{3}\,d$

$$E_{\text{left charge}} = \frac{kq}{|d\hat{x}|^3}(d\hat{x}) = \frac{kq}{d^2}\hat{x}$$

$$E_{\text{top right}} = \frac{kq}{\left|-\frac{d}{2}\hat{x} - \frac{\sqrt{3}}{2}d\hat{y}\right|^3}\left(-\frac{d}{2}\hat{x} - \frac{\sqrt{3}}{2}\hat{y}\right)$$

$$= \frac{kq}{\left(\left(\frac{d}{2}\right)^2 + \frac{3d^2}{4}\right)^{3/2}}\left(-\frac{d}{2}\hat{x} - \frac{\sqrt{3}}{2}d\hat{y}\right)$$

$$E_{\text{bottom left}} = \frac{kq}{\left(\left(\frac{d}{2}\right)^2 + \frac{3d^2}{4}\right)^{3/2}}\left(-\frac{d}{2}\hat{x} + \frac{\sqrt{3}}{2}d\hat{y}\right)$$

$$\text{Sum: } \frac{kq}{d^3}\left(-\frac{d}{2}\hat{x} + \cancel{\frac{\sqrt{3}}{2}d\hat{y}}\right) + \frac{kq}{d^3}\left(-\frac{d}{2}\hat{x} - \cancel{\frac{\sqrt{3}}{2}d\hat{y}}\right) + \frac{kq}{d^2}\hat{x} = \boxed{0}$$

1.2. Let $V(x, y)$ be the electric potential as a function of position. Compute an expression for $V(x, y)$, and try to simplify it if possible.

$$V(x,y) = kq\left(\frac{1}{\sqrt{(x+d)^2+y^2}} + \frac{1}{\sqrt{\left(x-\frac{d}{2}\right)^2+\left(y-\frac{\sqrt{3}}{2}d\right)^2}} + \frac{1}{\sqrt{\left(x-\frac{d}{2}\right)^2+\left(y+\frac{\sqrt{3}}{2}d\right)^2}}\right)$$

1.3. If a point charge Q is placed at rest at the origin, will it remain at rest? Justify using electric potential and symbolic computation.

Q will remain at rest.

$$\nabla V(x,y) = -kq\left(\frac{(x+d)\hat{x} + y\hat{y}}{[(x+d)^2+y^2]^{3/2}} + \frac{(x-\frac{d}{2})\hat{x} + (y-\frac{\sqrt{3}}{2}d)\hat{y}}{[(x-\frac{d}{2})^2+(y-\frac{\sqrt{3}}{2}d)^2]^{3/2}} + \frac{(x-\frac{d}{2})\hat{x} + (y+\frac{\sqrt{3}}{2}d)\hat{y}}{[(x-\frac{d}{2})^2+(y+\frac{\sqrt{3}}{2}d)^2]^{3/2}} \right)$$

$$\nabla V(0,0) = -kq\left(\frac{\hat{x}}{d^2} + \frac{-\frac{1}{2}\hat{x} - \frac{\sqrt{3}}{2}\hat{y}}{d^2} + \frac{-\frac{1}{2}\hat{x} + \frac{\sqrt{3}}{2}\hat{y}}{d^2} \right)$$

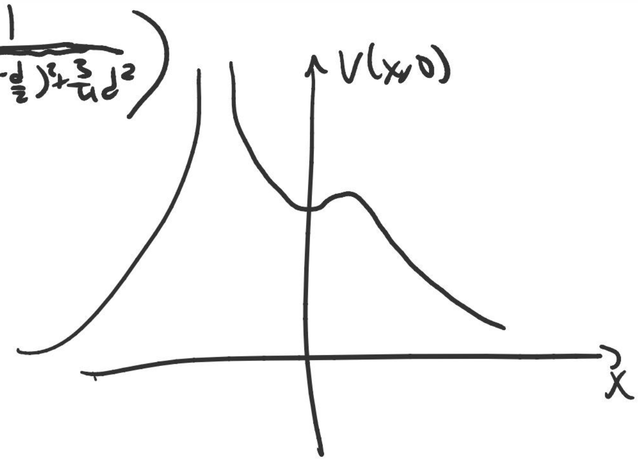
$$= 0$$

Since $\nabla V(0,0)=0$, electric field at $(0,0)$ is 0, so the charge will remain at rest.

1.4. Sketch the graph of $V(x, 0)$ versus x.

$$V(x,0) = kq\left(\frac{1}{x+d} + \frac{1}{\sqrt{(x-\frac{d}{2})^2 + \frac{3}{4}d^2}} + \frac{1}{\sqrt{(x-\frac{d}{2})^2 + \frac{3}{4}d^2}}\right)$$

$$= kq\left(\frac{1}{x+d} + \frac{2}{\sqrt{x^2 - dx + d^2}}\right)$$



1.5. Compute the Taylor expansion of $V(x, 0)$ about $x = 0$ up to the term of order x^2.

$$V(0,0) = kq\left(\frac{1}{d} + \frac{2}{d}\right) = \frac{3kq}{d}$$

$$V' = kq\left(-\frac{1}{(x+d)^2} - \frac{(2x-d)}{(x^2-dx+d^2)^{3/2}}\right)$$

$$V'(0,0) = kq\left(-\frac{1}{d^2} + \frac{1}{d^2}\right) = 0$$

$$V'' = kq\left(\frac{2}{(x+d)^3} - \left(\frac{2}{(x^2-dx+d^2)^{3/2}} - \frac{\frac{3}{2}(2x-d)(2x-d)}{(x^2-dx+d^2)^{5/2}}\right)\right)$$

$$V''(0,0) = kq\left(\frac{2}{d^3} - \left(\frac{2}{d^3} - \frac{3d^2}{2d^5}\right)\right) = \frac{3kq}{2d^3}$$

$$\boxed{V(x,0) \approx \frac{3kq}{d} + \frac{3kq}{4d^3}x^2}$$

1.6. If a point charge Q is placed at the origin and then given a sufficiently small kick in the x-direction, will it remain in the vicinity fo the origin forever? Does it depend on the sign of Q? Does it matter if the kick is to the left or right? Justify all answers carefully.

$$V(x,0) \approx \frac{3kq}{d} + \frac{3kq}{4d^3} x^2$$

$$V'(x,0) = \frac{3kq}{2d^3} x$$

$$V''(x,0) = \frac{3kq}{2d^3}$$

Since $V''(x,0)$ is positive, there is a potential valley @ the origin.

Since there is a potential valley and negative charges climb out of valleys, all negative Q values won't remain in the vicinity of the origin, regardless of direction.

If Q is positive and the kick is "sufficiently" small enough then it will oscillate about the origin because once it moves up one part of the valley, it won't be able to go over it and it'll go back down, and this repeats creating oscillation.

* rest of answer on p.6.

1.7. If there is a case where the charge Q will oscillates under a small push in the x-direction, determine the period of small oscillations if the charge in the center has mass m. If there is not such a case of oscillatory motion, explain how you know this.

$F_{net} = m\ddot{x}$

$\ddot{x} = -\omega^2 x$

$V(x,0) \approx \frac{3kq}{d} + \frac{3kq}{4d^3}x^2$

$\nabla V(x,0) = \frac{3kq}{2d^3}x$

$m\ddot{x} = -\nabla V(x,0)Q = -\frac{3kqQ}{2d^3}x = m\ddot{x}$

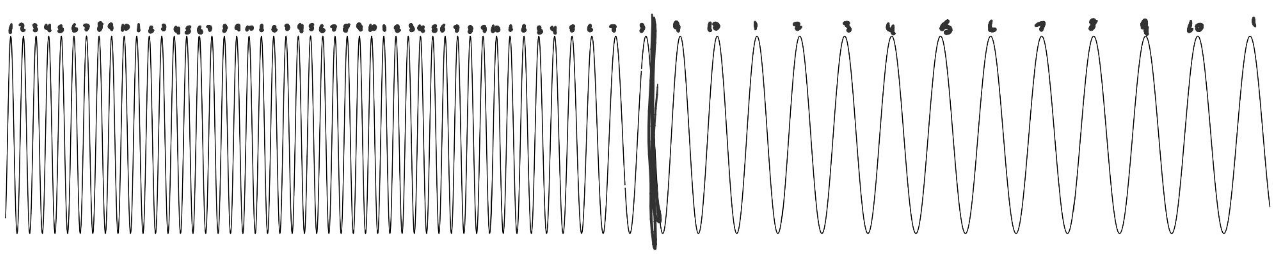
$\ddot{x} = +\frac{3kqQ}{2md^3}\cancel{x} = +\omega^2 \cancel{x}$

$\omega = \sqrt{\frac{3kqQ}{2md^3}}$

$T = \frac{2\pi}{\omega}$

$$\boxed{T = \frac{2\pi}{\sqrt{\frac{3kqQ}{2md^3}}}}$$

2. Auditory Airplane Inference. An airplane is flying past you some distance away at a constant speed in a straight line, and you use an app on your phone to record the sound it generates which has a constant emission frequency. The app outputs the following graph representing the pressure as a function of time in the air surrounding the phone. The graph displays 2 seconds of sensing data with the middle of the graph representing the moment of closest approach of the plane. Compute how fast the plane is going as accurately as you can, and clearly explain and show the logic and any algebra behind your computation.



From 0s – 1s, the plane is moving towards me and from 1s–2s it's moving away from me.

0s–1s:

48 humps $\rightarrow \frac{1}{\frac{1s}{48}} = 48\text{Hz} \rightarrow f_o = \frac{V}{V - V_s} f_s$

1s–2s:

13 humps $\rightarrow \frac{1}{\frac{1s}{13}} = 13\text{Hz} \rightarrow f_o = \frac{V}{V + V_s} f_s$

doppler effect

$0_s - 1_s$:

$$48 = \frac{343}{343 - V_s} f_s \qquad \frac{48(343 - v_s)}{343} = f_s$$

$1_s - 2_s$:

$$13 = \frac{343}{343 + V_s} f_s \qquad \frac{13(343 + v_s)}{343} = f_s$$

$$\frac{48(343 - v_s)}{\cancel{343}} = \frac{13(343 + v_s)}{\cancel{343}}$$

$$16464 - 48v_s = 4459 + 13v_s$$

$$61v_s = 12005$$

$$\boxed{V_s \approx 197 \text{ m/s}}$$

3. True or False questions. Determine whether or not each of the following statements is true. If a statement is true, prove it. If the statement is false, provide a counterexample and explain how it constitutes a counterexample. Diagrams can be useful in explaining such things.

3.1. If the electric potential in a certain region of space is constant, then the charge enclosed by any closed surface completely contained within that region is zero.

True | If potential is constant, then electric field is zero everywhere inside the region since $-\nabla V = \vec{E}$ and $\nabla(\text{constant}) = 0$.

$\Phi_{\substack{\text{close}\\\text{surface}}} = \frac{q_{enc}}{\varepsilon_0} = \int \vec{E}(r)\cdot\hat{n}\,dA = |E(r)||\hat{n}|\int dA$

when $E(r)=0$ then $\Phi = 0$

When $\Phi = 0$, $\underline{q_{enc} = 0}$

3.2. A sphere of radius R is centered at the origin. A total charge Q is uniformly distributed throughout its interior. A ball of radius $R/2$ centered at the the origin is carved out and discarded, leaving behind an empty cavity. The electric potential has the same value at every point inside the cavity.

True | At every point inside the cavity, there is no electric field. This is because there is no charge inside of the cavity since the cavity is hollow and because the cavity is centered at the origin. Since electric field is zero and $V = \int \vec{E}\cdot d\vec{l}$, V has to be constant. Also, since the cavity is centered at the origin, there is rotational symmetry of the electric field and charge distribution, so we know electric field is zero because all field lines cancel out to zero.

r = location in cavity

$\Phi(r) = \int E\cdot\hat{n}\,dA = E(r)(4\pi r^2)$

$\Phi(r) = \frac{Q_{enc}}{\varepsilon_0}$ $\quad Q_{enc} = 0 \rightarrow E(r) = 0$

3.3. A sphere of radius R is centered at the origin. A total charge Q uniformly distributed throughout its interior. A ball of radius $R/2$ centered at the point $(x, y, z) = (0, 0, R/4)$ is carved out and discarded, leaving behind an empty cavity. The electric potential has the same value at every point inside the cavity.

False If the electric potential is constant throughout the cavity, it means that the electric field everywhere inside of the cavity would have to be zero. However this isn't true. This is because the cavity-sphere system isn't rotationally nor reflectionally symmetric everywhere. This means that there is going to be an unequal electric field distribution throughout the cavity due to positional imbalances.

Looking at the cavity-sphere system in detail, we can see that the part of the hollow sphere that's above the cavity has a smaller volume than the part of the hollow sphere that's below the cavity. This means that there is more charge below the cavity than above the cavity. If we compare the top of the cavity to the bottom of the cavity (both points being inside the cavity) we can see that the bottom of the cavity will have a greater electric field than the top of the cavity because the immediate charge surrounding the bottom of the cavity has a greater magnitude than the charge immediately surrounding the top of the cavity. Since the |electric field| at the top of the cavity is smaller than at the bottom, and $V = \int \vec{E} \cdot d\vec{l}$, $V_{top} < V_{bottom}$ and V isn't constant.

Space for extra work.

The direction of the kick doesn't matter because the valley exists on both sides of the origin.

Space for extra work.