20S-PHYSICS1B-4 Midterm 2

ZACK HIRSCHHORN

TOTAL POINTS

32.5 / 34.5

QUESTION 1

Treacherous Triangle Trickery 20 pts

1.1 3 / 3

- ✓ **+ 2 pts Essentially completely correct reasoning.**
- **+ 1 pts** A few errors made.
- **+ 0 pts** Essentially incorrect.
- ✓ **+ 1 pts Field at center is zero.**

1.2 3 / 3

- ✓ **+ 1 pts Potential due to first charge correct.**
- ✓ **+ 1 pts Potential due to second charge correct.**
- ✓ **+ 1 pts Potential due to third charge correct.**
- **+ 0 pts** None correct.
- **+ 1 pts** None correct but significant progress

1.3 3 / 3

- ✓ **+ 1 pts Gradient of potential is shown to be zero.**
- ✓ **+ 2 pts Computation of gradient is correct.**
- **+ 1 pts** Computation of gradient has some errors.
- **+ 0 pts** Computation of gradient is essentially incorrect.

1.4 2.5 / 2.5

- ✓ **+ 2.5 pts All correct**
- **+ 0.5 pts** Graph asymptotes to $$\pm\infty$$ (depending on the assumed sign of $$q$$) as $$x\to -d$$ from the left.
- **+ 0.5 pts** Graph asymptotes to $$\pm\infty$$ (depending on the assumed sign of $$q$$) as $$x\to -d$$ from the right.
- **+ 0.5 pts** Graph asymptotes to $$0$$ as $$x\to-\infty$$
- **+ 0.5 pts** Graph asymptotes to $$0$$ as $$x\to+\infty$$
- **+ 0.5 pts** From $$-d$$ to $$+\infty$$, graph basically depicted as decreasing (or increasing if $$q<0$$) with the exception of a feature near $$x = d/2$$ that looks either like a bump or a flat portion -- something that makes it clear that it's recognized what the two charges not on the $$x$$\-axis do to the potential there.
- **+ 0 pts** None correct

1.5 3 / 4

- ✓ **+ 0.5 pts Order $$x^0$$ term is nonzero.**
- ✓ **+ 1 pts Order $$x^0$$ term has value $$3kq/d$$**
- ✓ **+ 1 pts Order $$x$$ term is zero.**
- ✓ **+ 0.5 pts Order $$x^2$$ term is nonzero.**
- **+ 1 pts** Order $$x^2$$ term has value $$3kq/(4d^3) x^2$$
- **+ 0 pts** None correct.
- **+ 4 pts** All correct

1.6 2 / 2.5

- **+ 0.5 pts** Reasoning contains observation that charges $$q$$ and $$Q$$ need to be same sign for charge $$q$$ to remain in a vicinity of the origin. (or opposite if the expansion in 1.5 had an incorrect negative x^2 term)
- ✓ **+ 2 pts Reasoning is essentially correct. (or if V was incorrect in 1.5, leading to the wrong conclusion)**
- **+ 1 pts** Reasoning has some errors.
- **+ 0 pts** Reasoning is incorrect.

1.7 1.5 / 2

- ✓ **+ 0.5 pts Attempted to determine the equation of motion $$\ddot x = F(x)/m$$**
- ✓ **+ 0.5 pts Attempted to Taylor expand the equation of motion so as to obtain the small-$$x$$ equation of motion.**
- ✓ **+ 0.5 pts Found that the equation of motion for**

small $$x$$ is of the form $$\ddot x = -\omega^2 x$$

+ **0.5 pts** Found that $$T = 2\pi\sqrt{2md^3/(3kQq)}$$
+ **0.2 pts** Answer off by small coefficient
+ **0 pts** Incorrect approach

1 Test charge Q missing

QUESTION 2

2 Auditory Airplane Inference 5.5 / 5.5

✓ **+ 1 pts Noted that when the plane is coming toward the observer and is far from the observer, the observed and emitted frequencies are related by $$f_o = \frac{v}{v-v_s} f_s$$**

✓ **+ 1 pts Noted that when the plane is going away from the observer and is far from the observer, the observed and emitted frequencies are related by $$f_o = \frac{v}{v+v_s} f_s$$**

+ **1 pts** Found that the speed of the source is given in terms of the toward and away observed frequencies and the speed of sound as $$v_s = \frac{f_{o, \mathrm{toward}}-f_{o, \mathrm{away}}}{f_{o, \mathrm{toward}}+ f_{o, \mathrm{away}}}v$$

✓ **+ 0.5 pts Used the beginning part (small enough to make clear that student recognized middle transition section needed to be ignored) of the provided waveform to determine an estimate for $$f_{o, \mathrm{toward}}$$**

✓ **+ 0.5 pts Used the end part (small enough to make clear that student recognized middle transition section needed to be ignored) of the provided waveform to determine an estimate for $$f_{o, \mathrm{away}}$$**

+ **0.5 pts** Estimated $$f_{o, \mathrm{toward}} \approx 44\,\mathrm{Hz}$$ (can be off by around 25%).

✓ **+ 0.5 pts Estimated $$f_{o, \mathrm{away}} \approx 12\,\mathrm{Hz}$$ (can be off by around 25%).**

+ **0.5 pts** Estimated $$v_s \approx 200\,\mathrm m/\mathrm s$$ (can be off by around 25%)
+ **0 pts** Incorrect approach

+ 2 Point adjustment

QUESTION 3

True or False 9 pts

3.1 3 / 3

+ **1 pts** True.
+ **2 pts** Essentially correct, complete reasoning with maybe a minor error.

✓ **+ 3 pts Fully correct reasoning.**

+ **1 pts** Reasoning has some errors or is incomplete, but some stuff correct.
+ **0 pts** Essentially incorrect reasoning.

3.2 3 / 3

✓ **+ 3 pts Fully correct reasoning.**

+ **2 pts** Essentially correct, complete reasoning with maybe a minor error.
+ **1 pts** Reasoning has some errors or is incomplete, but some stuff correct.
+ **0 pts** Essentially incorrect reasoning.

3.3 3 / 3

✓ **+ 3 pts Fully correct reasoning.**

+ **2 pts** Essentially correct, complete reasoning with maybe a minor error.
+ **1 pts** Reasoning has some errors or is incomplete, but some stuff correct.
+ **0 pts** Essentially incorrect reasoning.

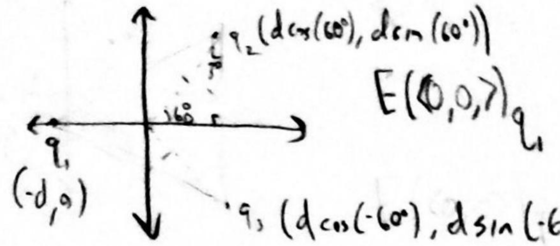
Physics 1B, Spring 2020, Midterm 2

Instructions.

1. Complete this exam packet, and submit it to Gradescope before the end of the exam period.
2. Make sure all final work is in only the spaces provided, and make sure to upload a document of exactly the length of this packet.
3. If you are using a smartphone to generate a scan, please use a scanning app such as Adobe Scan to quickly generate an optimized PDF document.
4. There is extra space at the end of the packet in case the space below each problem isn't sufficient, but if you use that extra space, make sure to indicate that extra work is contained there when you show your work for a problem.
5. You are allowed to use the CCLE course page, the course textbook (OpenStax), and your notes, but you are not permitted to use any other internet resources.
6. A calculator (whatever type desired) is allowed.
7. You may not communicate about the contents of this exam with anyone during the exam period.
8. You may not logon to Campuswire during the exam period.
9. **Josh and the TAs will not be free to answer clarifying questions about the exam during the exam period.** If you believe there is an ambiguity, feel free to make a note of it to yourself and we will attempt to address it post-exam, but otherwise do your best to answer the questions based on what they say.
10. Violations of instructions pertaining to what you are not allowed to do during the exam will be considered cheating and will be reported to the Dean of Students.

1. Treacherous Triangle Trickery. Consider a charge distribution consisting of an equilateral triangle with a point charge q fixed at each of its vertices. Let d be the distance between the center of the triangle and each vertex, let the triangle's center be at the origin, and let one of its vertices lie on the x-axis at the point $x = -d$.

1.1. Compute the electric field at the center of the triangle by explicitly computing the sum of the electric fields due the charges at each vertex.



$q_1 = q_2 = q_3 = q$

$$E(\langle 0,0\rangle)_{q_1} = \frac{kq_1}{d^2}\hat{x} \qquad E(\langle 0,0\rangle)_{q_2} = \frac{kq_2}{d^2}\langle -\cos(60°), -\sin(60°)\rangle$$

$$E(\langle 0,0\rangle)_{q_3} = \frac{kq_3}{d^2}\langle -\cos(-60°), -\sin(-60°)\rangle$$

$$\Sigma E(\langle 0,0\rangle) = E_{q_1}(\langle 0,0\rangle) + E_{q_2}(\langle 0,0\rangle) + E_{q_3}(\langle 0,0\rangle)$$

$$= \left\langle \frac{kq}{d^2}, 0\right\rangle + \left\langle \frac{-kq}{d^2}\cos(60°), \frac{-kq}{d^2}\sin(60°)\right\rangle + \left\langle \frac{-kq}{d^2}\cos(60°), \frac{kq}{d^2}\sin(60°)\right\rangle$$

$$= \left\langle \frac{kq}{d^2} - \frac{1}{2}\left(\frac{kq}{d^2}\right) - \frac{1}{2}\left(\frac{kq}{d^2}\right), \frac{-kq}{d^2}\sin(60°) + \frac{kq}{d^2}\sin(60°)\right\rangle$$

$$\boxed{\Sigma E(\langle 0,0\rangle) = \langle 0, 0\rangle = \vec{0}}$$

1.2. Let $V(x, y)$ be the electric potential as a function of position. Compute an expression for $V(x, y)$, and try to simplify it if possible.

$$V(x,y) = V_{q_1} + V_{q_2} + V_{q_3}$$

$$V_{q_1}(x,y) = \frac{kq}{\sqrt{(x-(-d))^2 + y^2}} \quad V_{q_2}(x,y) = \frac{kq}{\sqrt{(x - d\cos(60°))^2 + (y - d\sin(60°))^2}} \quad V_{q_3}(x,y) = \frac{kq}{\sqrt{(x - d\cos(-60°))^2 + (y - d\sin(-60°))^2}}$$

$$= \frac{kq}{\sqrt{(x+d)^2 + y^2}} \qquad = \frac{kq}{\sqrt{(x-\frac{d}{2})^2 + (y - \frac{\sqrt{3}d}{2})^2}} \qquad = \frac{kq}{\sqrt{(x-\frac{d}{2})^2 + (y + \frac{\sqrt{3}d}{2})^2}}$$

$$\boxed{V(x,y) = \frac{kq}{\sqrt{(x+d)^2+y^2}} + \frac{kq}{\sqrt{(x-\frac{d}{2})^2 + (y-\frac{\sqrt{3}d}{2})^2}} + \frac{kq}{\sqrt{(x-\frac{d}{2})^2 + (y+\frac{\sqrt{3}d}{2})^2}}}$$

$$V(x,y) = kq\left(((x+d)^2+y^2)^{-\frac{1}{2}} + \left((x-\tfrac{d}{2})^2 + (y - \tfrac{\sqrt{3}d}{2})^2\right)^{-\frac{1}{2}} + \left((x-\tfrac{d}{2})^2 + (y+\tfrac{\sqrt{3}}{2})^2\right)^{-\frac{1}{2}}\right)$$

I don't think simplifying anymore would be helpful

1.3. If a point charge Q is placed at rest at the origin, will it remain at rest? Justify using electric potential and symbolic computation.

We want to find $\nabla V(0,0)$. If $\nabla V = 0$, Q will remain at rest.

$$\frac{dx}{dV} = kq\left(-\frac{1}{2}\left((x+d)^2+y^2\right)^{-\frac{3}{2}}(2x+2d) - \frac{1}{2}\left(\left(x-\frac{d}{2}\right)^2+\left(y-\frac{\sqrt{3}d}{2}\right)^2\right)^{-\frac{3}{2}}(2x-d)\ldots\right.$$

$$\left.\ldots - \frac{1}{2}\left(\left(x-\frac{d}{2}\right)^2+\left(y+\frac{\sqrt{3}d}{2}\right)^2\right)^{-\frac{3}{2}}(2x-d)\right)$$

$$\frac{dx}{dV} = kq\left(-\frac{x+d}{\left((x+d)^2+y^2\right)^{\frac{3}{2}}} - \frac{2x-d}{2\left(\left(x-\frac{d}{2}\right)^2+\left(y-\frac{\sqrt{3}d}{2}\right)^2\right)^{\frac{3}{2}}} - \frac{2x-d}{2\left(\left(x-\frac{d}{2}\right)^2+\left(y+\frac{\sqrt{3}d}{2}\right)^2\right)^{\frac{3}{2}}}\right)$$

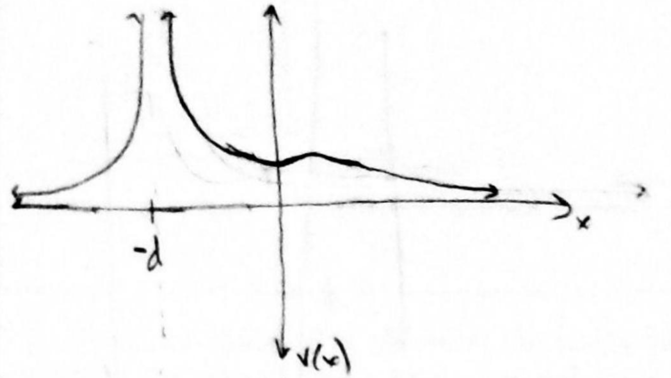
$$\frac{dy}{dV} = kq\left(\frac{-y}{\left((x+d)^2+y^2\right)^{\frac{3}{2}}} - \frac{2y-\sqrt{3}d}{2\left(\left(x-\frac{d}{2}\right)^2+\left(y-\frac{\sqrt{3}d}{2}\right)^2\right)^{\frac{3}{2}}} - \frac{2y+\sqrt{3}d}{2\left(\left(x-\frac{d}{2}\right)^2+\left(y+\frac{\sqrt{3}}{2}\right)^2\right)^{\frac{3}{2}}}\right)$$

$$\nabla V = kq\left\langle \frac{-(x+d)}{\left((x+d)^2+y^2\right)^{\frac{3}{2}}} - \frac{2x-d}{2\left(\left(x-\frac{d}{2}\right)^2+\left(y-\frac{\sqrt{3}d}{2}\right)^2\right)^{\frac{3}{2}}} - \frac{2x-d}{2\left(\left(x-\frac{d}{2}\right)^2+\left(y+\frac{\sqrt{3}d}{2}\right)^2\right)^{\frac{3}{2}}},\ \frac{-y}{\left((x+d)^2+y^2\right)^{\frac{3}{2}}}\ldots\right.$$

$$\left.\ldots - \frac{2y-\sqrt{3}d}{2\left(\left(x-\frac{d}{2}\right)^2+\left(y-\frac{\sqrt{3}d}{2}\right)^2\right)^{\frac{3}{2}}} - \frac{2y+\sqrt{3}d}{2\left(\left(x-\frac{d}{2}\right)^2+\left(y+\frac{\sqrt{3}}{2}\right)^2\right)^{\frac{3}{2}}}\right\rangle$$

$$\nabla V(0,0) = kq\left\langle \frac{-d}{d^3} - \frac{-d}{2d^3} - \frac{-d}{2d^3},\ 0 - \frac{-\sqrt{3}d}{2d^3} - \frac{\sqrt{3}d}{2d^3}\right\rangle$$ Extra Work

1.4. Sketch the graph of $V(x,0)$ versus x.



$$V(x,0) = \frac{kq}{|x+d|} + \frac{kq}{\sqrt{x^2-xd+d^2}} + \frac{kq}{\sqrt{x^2-xd+d^2}}$$

$$V(x,0) = kq\left(\frac{1}{|x+d|} + \frac{2}{\sqrt{x^2-xd+d^2}}\right)$$

1.5. Compute the Taylor expansion of $V(x,0)$ about $x = 0$ up to the term of order x^2.

Need to find $V'(x,0)$ and $V''(x,0)$

$$V'(x,0) = kq\left(\left((x^2+2xd+d^2)^{-\frac{1}{2}}\right)' + \left(2(x^2-xd+d^2)^{-\frac{1}{2}}\right)'\right)$$

$$= kq\left(-\tfrac{1}{2}(x^2+2xd+d^2)^{-\frac{3}{2}}(x^2+2xd+d^2)' + \left(-(x^2-xd+d^2)^{-\frac{3}{2}}(x^2-xd+d^2)'\right)\right)$$

$$= kq\left(-\tfrac{1}{2}(2x+2d)(x^2+2xd+d^2)^{-\frac{3}{2}} - (2x-d)(x^2-xd+d^2)^{-\frac{3}{2}}\right)$$

$$= kq\left(\frac{-(2x+2d)}{2(x^2+2xd+d^2)^{\frac{3}{2}}} - \frac{2x-d}{(x^2-xd+d^2)^{\frac{3}{2}}}\right)$$

$$V''(x,0) = kq\left(\left(-\tfrac{1}{2}(2x+2d)(x^2+2xd+d^2)^{-\frac{3}{2}}\right)' - \left((2x-d)(x^2-xd+d^2)^{-\frac{3}{2}}\right)'\right)$$

$$V''(x,0) = kq\left(-\tfrac{1}{2}\left[2(x^2+2xd+d^2)^{-\frac{3}{2}} + (2x+2d)\left(-\tfrac{3}{2}\right)(x^2+2xd+d^2)^{-\frac{5}{2}}(2x+2d)\right] - \dots\right.$$

$$\dots \left.\left[2(x^2-xd+d^2)^{-\frac{3}{2}} + (2x-d)\left(-\tfrac{3}{2}\right)(x^2-xd+d^2)^{-\frac{5}{2}}(2x-d)\right]\right)$$

$$V''(x,0) = kq\left(-(x^2+2xd+d^2)^{-\frac{3}{2}} + \tfrac{3}{4}(2x+2d)^2(x^2+2xd+d^2)^{-\frac{5}{2}} - 2(x^2-xd+d^2)^{-\frac{3}{2}} + \dots\right.$$

$$\dots \left.\tfrac{3}{2}(2x-d)^2(x^2-xd+d^2)^{-\frac{5}{2}}\right)$$

$$V''(x,0) = kq\left(\frac{-1}{(x^2+2xd+d^2)^{\frac{3}{2}}} + \frac{3(2x+2d)^2}{4(x^2+2xd+d^2)^{\frac{5}{2}}} - \frac{2}{(x^2-xd+d^2)^{\frac{3}{2}}} + \frac{3(2x-d)^2}{2(x^2-xd+d^2)^{\frac{5}{2}}}\right)$$

Extra Work

1.6. If a point charge Q is placed at the origin and then given a sufficiently small kick in the x-direction, will it remain in the vicinity fo the origin forever? Does it depend on the sign of Q? Does it matter if the kick is to the left or right? Justify all answers carefully.

We can use the derivative and double derivative of the taylor expansion of $V(x,0)$ around $x=0$ to find the shape of $V(x,0)$ around 0 and see how the charge Q will behave.

$V(x,0) = \frac{3kq}{d} + \frac{3kq}{2d^3}x^2$ ← taylor expansion

$V'(x,0) = \frac{3kq}{d^3}x$, $V'(0,0)=0 \Rightarrow$ So $(0,0)$ is a C.P.

$V''(x,0) = \frac{3kq}{d^3}$. we know q, k, and $d>0$, so $V''(0,0)>0$, so $V(x,0)$ is a local min at $x=0$.

So if $Q>0$, Q will oscillate around the origin forever once kicked smally because positive charges "roll down" potential hills, and V at $x=0$ is a "valley" that will trap Q. The direction doesn't matter because its a local min (hills in both directions).

But if $Q<0$, then the charge will "roll up" potential hills, Therefore because $(0,0)$ is a local min for $V(x,0)$, a kick in either direction no matter how small, will cause Q to "roll up" the hill and leave the vicinity forever.

Just visually looking at the graph from 1.4 confirms this :).

1.7. If there is a case where the charge Q will oscillates under a small push in the x-direction, determine the period of small oscillations if the charge in the center has mass m. If there is not such a case of oscillatory motion, explain how you know this.

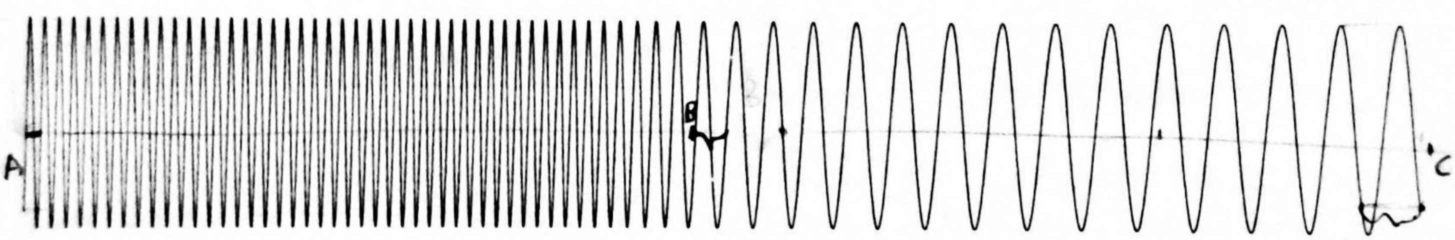
If $Q>0$ it will oscillate. To find the period we need to find an equation $\ddot{x} = -w^2 x$. We have the potential around $x=0$, if we find the Electric field from that we can get an equation we want. Normally we find E from $-\nabla V$, but because we are only working in one dimension, we will use the one dimensional equivalent: the derivative. (We will use the taylor series of $V(x,\rho)$ around 0 to approximate V around 0).

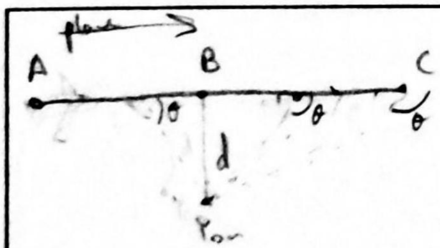
$E(x,\rho) = -(V(x,\rho))'$

$E(x,\rho) = -\frac{3kq}{d^3}x$ $\quad \ddot{x} = -\frac{3kq}{d^3 m}x$ $\quad T = \frac{2\pi}{w}$

$F=ma$ so $-\frac{3kq}{d^3}x = m\ddot{x}$ $\quad -w^2 = -\frac{3kq}{d^3 m}$ $\quad w = \sqrt{\frac{3kq}{d^3 m}}$ $\quad \boxed{T = 2\pi\sqrt{\frac{d^3 m}{3kq}}}$

2. Auditory Airplane Inference. An airplane is flying past you some distance away at a constant speed in a straight line, and you use an app on your phone to record the sound it generates which has a constant emission frequency. The app outputs the following graph representing the pressure as a function of time in the air surrounding the phone. The graph displays 2 seconds of sensing data with the middle of the graph representing the moment of closest approach of the plane. Compute how fast the plane is going as accurately as you can, and clearly explain and show the logic and any algebra behind your computation.





Ok so were gonna have to use the doppler affect to calculate the speed of the plane. The doppler affect, when the source and the object are not on the same line is: note in Extra work

$$f_{observed} = f_{source}\left(\frac{V}{V - V_{source}\cos(\theta)}\right)$$

First we can find f_{source} when $\theta = 90$ (when the plane is at its closest point) because then $\cos(\theta)=0$ and $f_o = f_s\left(\frac{V}{V}\right)$, $f_o = f_s$. That would be at point B.

So to find f_s, we just have to measure f_o on the given graph.

f is waves per second. So at second B, I've measured of one full wave. Using a ruler, that wave is approximately $\frac{1}{40}$th of the entire graph. Meaning one full wave hit our phones in $\frac{1}{20}$th of a second (cuz the entire graph is two seconds). So the f is 20 waves/s. $\boxed{f_s = 20}$.

Now according to the textbook, the speed of sound in air is 331 m/s. so: $\boxed{v = 331 \text{ m/s}}$

So solving for Vsource, which is what we want:

$$\frac{f_o}{f_s} = \frac{v}{v - v_s \cos\theta} \Rightarrow \frac{f_o}{f_s} v - \frac{f_o}{f_s} v_s \cos\theta = v \Rightarrow -\frac{f_o}{f_s} v_s \cos\theta = v - \frac{f_o}{f_s} v$$

$$\Rightarrow \boxed{v_s = \frac{\frac{f_o}{f_s} v - v}{\frac{f_i}{f_s} \cos\theta}}$$

. Now we need to choose an appropriate θ for point C, measure the frequency at point C, and then we can calculate v_s. I'll choose θ to be 135° at point C, (45° less than the horizontal) but you can choose any value.

Near point C, I measure a full wave to be $\frac{1}{24}$th of the entire graph. This means f_o at pt. C = 12. So $\theta = 45$, $f_o = 12$. Let calculate v_s!!

$$\boxed{v_s = \frac{\frac{12}{20}(331) - 331}{\frac{12}{20}\cos(135^\circ)} = 312 \text{ m/s}}$$

Now we can check our answer by calculating the speed at point A (and cuz v_s is constant we know $\theta = 45$). Here a wave measures $\frac{1}{120}$ of the entire length, so f_o at point A = 60.

$$\boxed{v_s = \frac{\frac{60}{20}(331) - 331}{\frac{60}{20}\cos(45^\circ)} = 312 \text{ m/s}}$$

yay!!

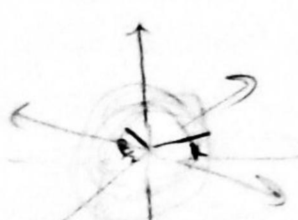
Note #2 in extra work

3. True or False questions. Determine whether or not each of the following statements is true. If a statement is true, prove it. If the statement is false, provide a counterexample and explain how it constitutes a counterexample. Diagrams can be useful in explaining such things.

3.1. If the electric potential in a certain region of space is constant, then the charge enclosed by any closed surface completely contained within that region is zero.

True. $E(\vec{r}) = -\nabla V(\vec{r})$, so if $V(\vec{r})$ is constant, it means $\nabla V = 0$, so that means $E(\vec{r})$ would be zero over that regoin. Now that we have proven the Electric field is zero in that region, we can use Gauses law to prove the charge enclosed by any closed surface is zero. Gauss' law states: $\int_S \vec{E} \cdot \hat{n} dA = \frac{q_{enc}}{\varepsilon_0}$, so $q_{enc} = \varepsilon_0 \left(\int_S \vec{E} \cdot \hat{n} dA \right)$. Sinces we've proven E is zero everywhere in the region $\int_S \vec{E} \cdot \hat{r} dA$ also has to be zero. Therefore $q_{enc} = 0$.

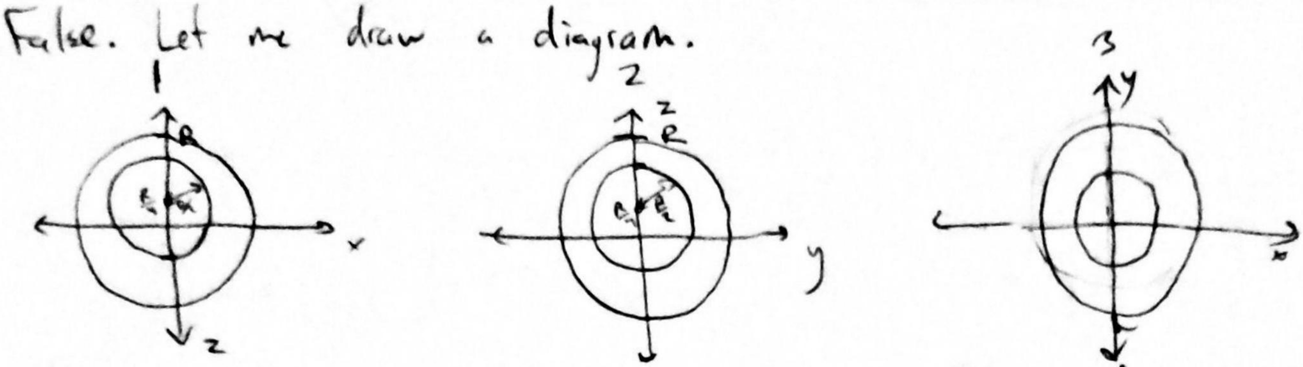
3.2. A sphere of radius R is centered at the origin. A total charge Q is uniformly distributed throughout its interior. A ball of radius $R/2$ centered at the the origin is carved out and discarded, leaving behind an empty cavity. The electric potential has the same value at every point inside the cavity.



True. First we need to find the Electric field inside the cavity. We can use Gauses law to do that. Because the cavity has rotational symmetry, we will use a Gaussian sphere that encloses just the cavity. We know that: $\int_S \vec{E} \cdot \hat{n} dA = \frac{q_{enc}}{\varepsilon_0}$. We know from symmetry that $\vec{E}(\vec{r}) = E(r)\hat{n}$ (where $E(r)$ is the mag. of the field based on distance from origin, and $\hat{n}$ is the outwards pointing normal vector in the $\hat{r}$ direction). So $\int_S \vec{E}(\vec{r}) \cdot \hat{n} dA = \int_S E(r)\hat{n} \cdot \hat{n} dA = \int_S E(r) dA = E(r) \int_S dA = E(r) \frac{4\pi R^2}{4}$. So $E(r)\pi R^2 = \frac{q_{in}}{\varepsilon_0}$. since $q_{enc} = 0$ inside the cavity that means $E(r) = 0$ inside the cavity. (So $\vec{E}(\vec{r}) = \vec{0}$ inside the cavity).
To find V we use $V(\vec{r}) = -\int_{r_0}^{r} \vec{E} \cdot d\vec{l}$. I will choose r_0 to be $(0,0)$, and $V(r_0)$ to be equal 0. Because $\vec{E}(\vec{r}) = \vec{0}$ for all $\vec{r}$ inside the cavity, $V(\vec{r})$ will equal 0 for all $\vec{r}$ inside the cavity. (The fact that we got 0 was because I chose r_0 to be inside the cavity for ease of computation. Had r_0 been outside the cavity it would have been a different constant).

3.3. A sphere of radius R is centered at the origin. A total charge Q uniformly distributed throughout its interior. A ball of radius $R/2$ centered at the point $(x, y, z) = (0, 0, R/4)$ is carved out and discarded, leaving behind an empty cavity. The electric potential has the same value at every point inside the cavity.

False. Let me draw a diagram.



If the potential is constant, that means the Electric field inside the cavity would be zero everywhere, because $\vec{E} = -\nabla V$ and the gradient of a constant function is 0. I will show that the $\vec{E}$ is not zero at a point inside the cavity, which will therefore prove the electric potential is not constant inside the cavity.

From the 3rd graph (the birds-eye view) we can tell that the distribution is symmetric around the z-axis. Therefore a point on the z-axis (which would be at (0,0) on the third graph) would have an electric force of zero in the x+y directions (they all cancel out).

But lets take the origin for example. In the z-direction it will not be cancelled out. The force is related to total charge and distance. In the case of the origin, there is more total charge below AND it is closer than the charge up top. This means a positive charge at (0,0,0) will be pushed upwards. This shows the Electric field inside the cavity is not 0 everywhere, so the potential is not constant.

Space for extra work.

1.5) So Taylor expansion around 0 is:

$$V(x,0) = \frac{V(0,0)}{0!} + \frac{V'(0,0)}{1!}x + \frac{V''(0,0)}{2!}x^2$$

$$= \frac{3kq}{d} + \left(kq\left(\frac{-2d}{2(d^2)^{\frac{3}{2}}} - \frac{-d}{(d^2)^{\frac{3}{2}}}\right)\right)x + \left(kq\left(\frac{-1}{(d^2)^{\frac{3}{2}}} - \frac{2}{(d^2)^{\frac{3}{2}}} + \frac{3(2d)^2}{4(d^2)^{\frac{5}{2}}} + \frac{3(-d)^2}{2(d^2)^{\frac{5}{2}}}\right)\right)x^2$$

$$= \frac{3kq}{d} + \left(kq\left(\frac{-d}{d^3} + \frac{d}{d^3}\right)\right)x + \left(kq\left(\frac{-1}{d^3} - \frac{2}{d^3} + \frac{3d^2}{d^5} + \frac{3d^2}{2d^5}\right)\right)x^2$$

$$= \frac{3kq}{d} + 0 + kq\left(\frac{-3}{d^3} + \frac{3}{d^3} + \frac{3}{2d^3}\right)x^2$$

$$\boxed{= \frac{3kq}{d} + \frac{3kq}{2d^3}x^2}$$

1.3) $\nabla V(0,0) = \langle 0,0 \rangle = \vec{0}$

So because $\nabla V = \vec{0}$, we know that V is exactly flat at (0,0). And because (positive) charges "roll down" potential hills, we know Q will stay at rest - because it won't "roll" on a "flat surface".

2). Note on the formula:

-I found this formula when doing pset 4. Basically you need to multiply by $\cos(\theta)$ because the speed added to the sound that the observer hears is in the direction of the person. $V_s\cos\theta$ finds the component heading towards the person. I chose θ on the diagram so the speed would subtract when the source was heading towards the person ($\theta < 90°$) and add when heading away ($\theta > 90°$) as it should. Note continued on next page

Space for extra work.

Beyond the formula, we can conceptually understand why $f_o = f_s$ when $\theta = 90°$. Basically at that point the speed of the source is not affecting the speed of the sound that reaches the person because they are $\perp$.

Note #2 on 2) The speed that you get is dependent on θ, or basically the distance from the aircraft. So my speed is based on $\theta = 45°/135°$ and I had. I chose another θ I would have gotten a differen answer. However my method would have worked based on any θ.