20S-PHYSICS1B-4 Midterm 1

ZACK HIRSCHHORN

TOTAL POINTS

25.25 / 29.5

QUESTION 1
14 pts

1.1 5 / 5

✓ **+ 1 pts** Used Newtons Second Law for static equilibrium of the buoy with buoyant force and weight opposing each other.

✓ **+ 1 pts** Noted that the weight of the buoy has magnitude $$\rho_w Vg/2$$

✓ **+ 2 pts** Noted that the buoyant force on the buoy is $$\rho_w V_\mathrm{submerged} g$$

✓ **+ 1 pts** Solved for $$V_\mathrm{submerged}$$ to obtain $$V/2$$

1.2 7 / 8

✓ **+ 1 pts** Wrote down Newton's Second Law $$F_B - Mg = M\ddot z$$ (in terms of magnitudes) or $$\vec F_B + M\vec g = M\vec a$$ (in terms of vectors)

✓ **+ 2 pts** Noticed that when weight is subtracted from total buoyant force, all that remains is the change in buoyant force due to the change in volume from the perturbation

✓ **+ 1 pts** Noticed that after a small perturbation, volume displaced and therefore buoyant force will change relative to what it was in static equilibrium.

✓ **+ 2 pts** Used geometry (similar triangles and difference of cone volumes) to determine the change in volume due to a perturbation is $$-(V/2)[1-(1-z/h)^3]$$ or something equivalent, OR correctly wrote down the small-$$z$$ approximation to this expression with a correct argument about how it was generated.

+ **1 pts** Obtained the small-$$z$$ equation of motion $$\ddot z = -(3g/h) z$$.

+ **0.5 pts** Obtained partially-correct (incorrect sign) small-$$z$$ equation of motion $$\ddot z = (3g/h) z$$.

✓ **+ 1 pts** Correctly deduced angular frequency based on equation of motion written. (Answer just needs to be consistent with equation of motion, but note that correct answer is $$\sqrt{3g/h}$$.

1.3 0.75 / 1

✓ **+ 0.25 pts** Equated $$\omega$$ expression from 1.2 to $$2\pi/T$$, then solved for $$h$$.

✓ **+ 0.5 pts** Used $$T = 1\,\mathrm s$$

+ **0.25 pts** Obtained $$h\approx 0.74\,\mathrm m$$

+ **0 pts** none of the above

QUESTION 2
8.5 pts

2.1 3 / 3

✓ **+ 3 pts** Argument is air-tight.

+ **2.5 pts** Argument is essentially correct with one or two errors.

+ **2 pts** Reasonable attempt but a few serious flaws in argument.

+ **1 pts** Something written but mostly incorrect.

+ **0 pts** Nothing correct.

2.2 4 / 4

✓ **+ 1 pts** Noted that for static equilibrium of the block, tension up the ramp and weight pulling down along the ramp need to sum to zero.

✓ **+ 1 pts** Found that component of weight pulling down the ramp has magnitude $$Mgh/\ell$$.

✓ **+ 0.5 pts** Used the relation between wave speed, tension, and mass density $$v = \sqrt{F_T/\mu}$$

✓ **+ 0.5 pts** Used (or derived) the expression for the frequency of the $$n^\mathrm{th}$$ harmonic $$f_n = nv/(2\ell)$$

✓ + 1 pts **Combined all of these steps together to obtain $$f_n = \frac{n}{2\ell}\sqrt{Mgh/m}$$**

- 0.5 pts No work shown for tension solution

- 0.5 pts Did not simplify linear mass density into givens

- 0.5 pts Incorrect trigonometry

2.3 1.5 / 1.5

✓ + 1 pts **Used answer from before and/or similar argumentation to find the following expression for the linear mass density: $$\mu = Mghn^2/(4\ell^3f_n^2)$$**

✓ + 0.5 pts **Plugged in values and correctly computed $$\mu \approx 7\times 10^{-3}\,\mathrm{kg}/\mathrm m = 7\,\mathrm g/\mathrm m$$**

+ 1.125 pts Incorrect answer, but used correct method based on solution from 2.2

- 0.1 pts Incorrect units

- 0.3 pts Math error

+ 0.75 pts Incorrect calculation for mu

QUESTION 3

7 pts

3.1 4 / 6

✓ + 1 pts **Correctly wrote down Bernoulli's Principle in symbolic form as applied comparing points 1 and 2 along a streamline.**

✓ + 0.5 pts **Converted flow rate into speed using $$Q = Av$$**

✓ + 0.5 pts **Noted that $$A_1 = \pi(d/2)^2$$ and $$A_2 = \pi(D/2)^2$$**

✓ + 1 pts **Noted that only difference in heights of points 1 and 2 matters (or set one of them to zero) and plugged in $$H$$ accordingly.**

+ 0.5 pts Made an argument as to why the pressure to the right of the stopper equals $$p_2$$ even after the screw is opened.

+ 0.5 pts Asserted that the pressure to the right of the stopper equals $$p_2$$ even after the screw is opened.

+ 0.5 pts Made an argument as to why the pressure at the ram equals $$p_2$$ even after the screw is opened (Pascal's Principle).

+ 0.5 pts Asserted that the pressure at the ram equals $$p_2$$ even after the screw is opened.

✓ + 1 pts **Computed the force on the ram by the fluid (and thus the force it exerts on the object it's crushing) by multiplying the pressure in the fluid in contact with the ram by the ram's area.**

3.2 0 / 1

+ 1 pts Plugged in all values and got approximately right answer $$F_\mathrm{ram} \approx 412\,\mathrm N$$.

✓ + 0 pts **Plugged in values did not show approximately right answer $$F_\mathrm{ram} \approx 412\,\mathrm N$$.**

+ 0.25 pts Found correct solution from incorrect 3.1 formula

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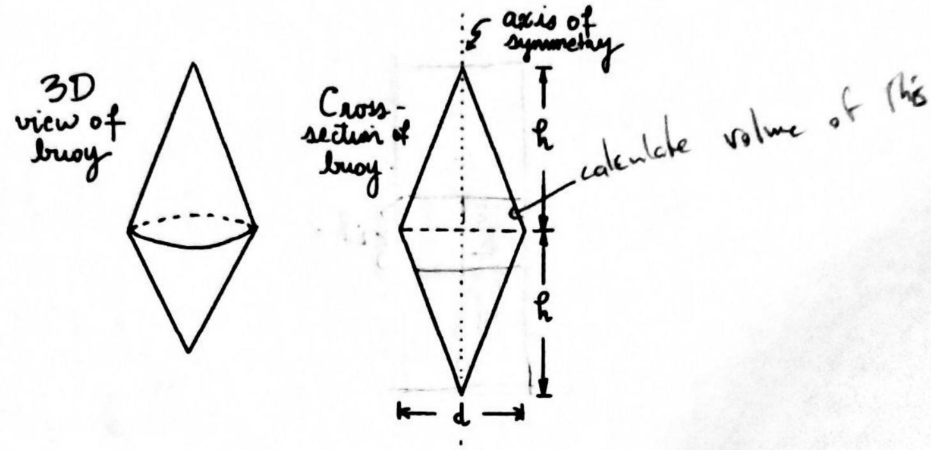
Instructions.

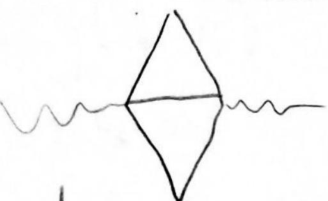
1. Complete this exam packet, and submit it to Gradescope before the end of the exam period.
2. Make sure all final work is in only the spaces provided, and make sure to upload a document of exactly the length of this packet.
3. If you are using a smartphone to generate a scan, please use a scanning app such as Adobe Scan to quickly generate an optimized PDF document.
4. There is extra space at the end of the packet in case the space below each problem isn't sufficient, but if you use that extra space, make sure to indicate that extra work is contained there when you show your work for a problem.
5. You are allowed to use the CCLE course page, the course textbook (OpenStax), and your notes, but you are not permitted to use any other internet resources.
6. A calculator (whatever type desired) is allowed.
7. You may not communicate about the contents of this exam with anyone during the exam period.
8. You may not logon to Campuswire during the exam period.
9. **Josh and the TAs will not be free to answer clarifying questions about the exam during the exam period.** If you believe there is an ambiguity, feel free to make a note of it to yourself and we will attempt to address it post-exam, but otherwise do your best to answer the questions based on what they say.
10. Violations of instructions pertaining to what you are not allowed to do during the exam will be considered cheating and will be reported to the Dean of Students.

1. Bobbing Buoy. A buoy is a floating device that can have many purposes, but often as a locator for ships. Collin constructs a hollow metal buoy by welding together two identical cones of height h and diameter d. The resulting double-cone buoy has average density equal to half the density of seawater meaning that its total mass divided by total volume is half the density of seawater. Throughout this problem, you can ignore the air above the sea's surface.



1.1. When the buoy is at rest in a calm ocean, with its axis of symmetry perpendicular to the water's surface, what fraction of it will be submerged?

$\rho_b = \frac{1}{2}\rho_w$



$V = \frac{2}{3}\pi\left(\frac{d}{2}\right)^2 h$

b = % below water

FBD

$F_B = bV\rho_w g$

$mg = V\rho_b g$

When the buoy is floating $F_B = mg$.

$b\cancel{V}\rho_w \cancel{g} = \cancel{V}\rho_b \cancel{g}$

$b = \frac{\rho_b}{\rho_w}$

$b = \frac{\frac{1}{2}\rho_w}{\rho_w} = \frac{1}{2}$

percent submerged is 50%

1.2. If the buoy is at rest at time $t = 0$ and is then pushed down slightly into the water and let go, what will be its angular frequency ω of small oscillations in terms of the given variables? Note: the volume of a cone of height h and base area A is $hA/3$.

$z = 0$ will be equilibrium point, where half the buoy is above water and half in below.

$z+$ is pointing up ↑

$z-$ is pointing down ↓.

$$\Sigma F(z) = mg - \rho_w g\,(V_{below})$$

$$\Sigma F(z) = \tfrac{1}{2}\rho_w (V_a + V_b) g - \rho_w g V_b$$

$$\Sigma F(z) = \tfrac{1}{2}\rho_w V_a g + \tfrac{1}{2}\rho_w V_b g - \rho_w V_b g$$

$$\Sigma F(z) = \tfrac{1}{2}\rho_w V_a g - \tfrac{1}{2}\rho_w V_b g$$

$$\Sigma F(z) = \tfrac{1}{2}\rho_w g (V_a - V_b)$$

$$ma = \tfrac{1}{2}\rho_w g (V_a - V_b)$$

$$\tfrac{1}{2}\rho_w (V_a + V_b)\, a = \tfrac{1}{2}\rho_w g (V_a - V_b)$$

$$a = \frac{g(V_a - V_b)}{(V_a + V_b)}$$

$= -V_{that}$

$$a = \frac{12 g\,(V_a - V_b)}{\pi d^2 h}$$

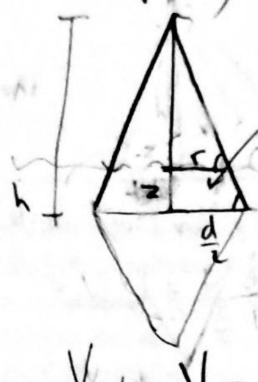
cuz z is small $z \gg z^2$, can disregard z^2

$$\ddot{z} = -\frac{11}{12} g - \frac{gz}{6h} - \frac{z^2 g}{12h}$$

$$\ddot{z} = -\frac{11}{12} g - \frac{gz}{6h}$$

$$\ddot{z} = -\frac{g}{6h}\left(z + \frac{11}{2} h\right)$$

$$\boxed{w = \sqrt{\frac{g}{6h}}}$$



want volume of this whether is above waterline or below

$$\frac{h - z}{h} = \frac{2r}{d}$$

$$r = \frac{d(h-z)}{2h}$$

$$V_{that} = V - \frac{h\pi}{3}\left(\frac{d(h-z)}{2h}\right)^2$$

$$V_{that} = V - \frac{h\pi}{3}\left(\frac{d^2 (h-z)^2}{4h^2}\right)$$

$$V_{that} = V - \frac{\pi d^2 (h-z)^2}{12h}$$

$$V_{that} = \frac{\pi h d^2}{12} - \frac{\pi d^2 (h-z)^2}{12h}$$

$$V_{that} = \frac{\pi d^2 h}{12}\left(1 - \frac{(h-z)^2}{12h^2}\right)$$

$$V_{that} = \frac{\pi d^2 h}{12}\left(1 - \frac{h^2}{12h^2} + \frac{2hz}{12h^2} + \frac{z^2}{12h^2}\right)$$

$$V_{that} = \frac{\pi d^2 h}{12}\left(\frac{11}{12} + \frac{z}{6h} + \frac{z^2}{12h}\right)$$

1.3. What should be the height h of each cone in the buoy so that the buoy will execute one oscillation period every second and therefore be usable as a clock with one-second accuracy?

$$f = \frac{\omega}{2\pi} \qquad h = \frac{g}{24\pi^2 f^2}$$

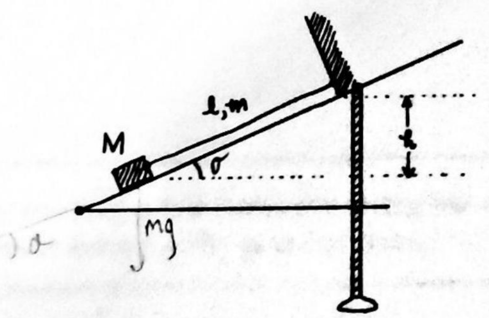
$$\omega = 2\pi f \qquad h = \frac{g}{24\pi^2 (1)^2}$$

$$\sqrt{\frac{g}{6h}} = 2\pi f \qquad \boxed{h = .041\,m}$$

$$\frac{g}{6h} = 4\pi^2 f^2$$

$$6h = \frac{g}{4\pi^2 f^2}$$

2. Mills' Musical Machine. A Mills banjo is a musical instrument which relies on being placed near the surface of the Earth to operate. It consists of a block of mass M that is being prevented from sliding down a frictionless incline by a thin string of length ℓ and mass m attached to a stationary wall. The steepness of the incline can be adjusted by adjusting the height h with a screw as indicated on the diagram. The string can be plucked like a guitar string. Assume that the block is sufficiently massive that the string can be treated as though it's fixed at both endpoints.



$\theta = \sin^{-1}\left(\frac{h}{\ell}\right)$

2.1. If all other variables besides h are held fixed, do you expect the frequencies of the harmonics on the string to increase, stay the same, or decrease if h is increased? Make as compelling a physical argument as you can without writing down any equations.

I think as h increases this means the angle of the block will increase, which will mean the force of tension on the rope will increase (cuz the blocks mass will be "held up less" by the incline) and held up more by the rope itself). As the rope becomes more tense/taught, it will vibrate easier, so the frequencies of the harmonics will increase.

2.2. What is the frequency of the n^{th} harmonic of the string in terms of the variables given in the problem statement?

$f_n = n\frac{v}{2L}$ $\quad L = \ell$ $\quad v = \sqrt{\frac{F_T}{\mu}}$ $\quad \mu = \frac{m}{\ell}$ $\quad F_T = Mg\sin\theta$

$F_T = Mg\sin\left(\sin^{-1}\left(\frac{h}{\ell}\right)\right)$

$F_T = \frac{Mgh}{\ell}$

$v = \sqrt{\frac{\frac{Mgh}{\cancel{\ell}}}{\frac{m}{\cancel{\ell}}}}$

$v = \sqrt{\frac{Mgh}{m}}$

$f_n = n\frac{\sqrt{\frac{Mgh}{m}}}{2\ell}$

$$\boxed{f_n = n\frac{\sqrt{Mgh}}{2\ell\sqrt{m}}}$$

2.3. If the block has a mass of 20 kg, the height h is 0.5 m, **the string has a length of 0.77 m, and the 5th harmonic** has frequency 445 Hz, what is the linear mass density μ **of the string?**

$f_n = n\frac{\sqrt{Mgh}}{2\ell\sqrt{m}}$

$\frac{1}{\sqrt{m}} = \frac{f_n 2\ell}{n\sqrt{Mgh}}$

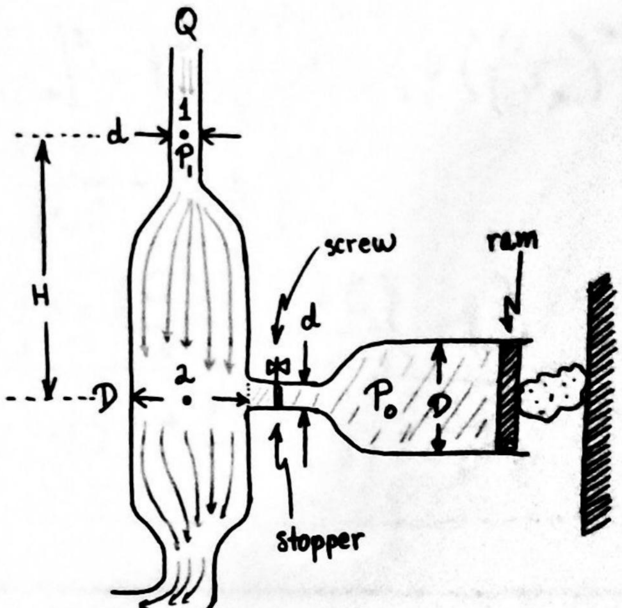
$\sqrt{m} = \frac{n\sqrt{Mgh}}{f_n 2\ell}$

$m = \frac{n^2(Mgh)}{4f_n^2\ell^2}$

$m = 0.00522\text{ kg} = 5.22\text{ g}$

$$\boxed{\mu = 6.77\ \frac{g}{m}}$$

3. In the apparatus below, the Dynamic Aqua Crusher, water is injected into a vertical channel at a volume flow rate Q. The diameter of the channel at point 1 is d, and its diameter at point 2 is D. To the right of point 2 is a horizontal channel blocked by a stopper. The small diameter of this horizontal channel is d, and the large diameter is D. To the right of the stopper is a chamber with static water. When the stopper is clamped in place by a screw, the pressure in the chamber is p_0. When the clamp is unscrewed, the stopper is free to move. At the right-hand end of the chamber is a ram that can be used to crush things against a wall. Let ρ be the density of water, p_1 be the pressure at point 1 in the vertical channel, and H be the vertical distance between points 1 and 2. You may assume that $p_1 > p_0$.



3.1. Suppose that the clamp is unscrewed and the stopper is free to move. Find an expression in terms of the given variables for the force exerted by the ram on whatever it's crushing in this circumstance.

Pascal's principle states:

$\Delta P_{Left} = \Delta P_{right}$

$(P_2 - P_0) - P_0 = P_r - P_0$

$P_r = P_2 - P_0$

$\frac{F_r}{A_r} = P_2 - P_0$

$F_r = A_r (P_2 - P_0)$

$F_r = \frac{D^2 \pi}{4}(P_2 - P_0)$ ← now we need to find the pressure on the stopper, which $= P_2$.

F_r = force of ram, A_r = Area of ram

P_2 = Pressure at point 2, $P_r = F_r / A_r$ pressure of ram

To find the pressure on the stopper, we need to use Bernoulli's principle.

$$P_1 + \rho g z_1 + \frac{1}{2}\rho v_1^2 = P_2 + \rho g z_2 + \frac{1}{2}\rho v_2^2$$

The left side will be point 1, right side point 2

solving for this

$$P_1 + \rho g H + \frac{1}{2}\rho\left(\frac{Q}{(\frac{d}{2})^2\pi}\right)^2 = P_2 + \cancel{\rho g 0} + \frac{1}{2}\rho\left(\frac{Q}{(\frac{D}{2})^2\pi}\right)^2$$

$$P_2 = P_1 + \rho g H + \frac{1}{2}\rho\frac{16Q^2}{d^4\pi^2} - \frac{1}{2}\rho\frac{16Q^2}{D^4\pi^2}$$

$$F_r = \frac{D^2\pi}{4}\left(P_1 + \rho g H + \frac{8Q^2\rho}{d^4\pi^2} - \frac{8Q^2\rho}{D^4\pi^2} - P_0\right)$$

$$\boxed{F_r = \frac{D^2\pi}{4}\left(P_1 - P_0 + \rho g H + \frac{8Q^2\rho}{\pi^2}\left(\frac{1}{d^4} - \frac{1}{D^4}\right)\right)}$$

3.2. If $d = 1\,\text{cm}$ (.01 m), $D = 5\,\text{cm}$ (.05 m), $H = 1\,\text{m}$, $p_1 = 2\times10^5$ Pa, $p_0 = 10^5$ Pa, and $Q = 100\,\text{cm}^3/\text{s}$ (.0001), what is the magnitude of the force that the ram exerts on the object it's crushing?

$$F_r = .00196\left(10^5 + 10.62\rho\right)$$

$$\rho_{water} = 1000\ \text{kg/m}^3 \text{ (from TB)}$$

$$F_r = 196 + .021\rho$$

$$\boxed{F_r = 217\,\text{N}}$$

Space for extra work.

Space for extra work.