MIDTERM EXAM #2 (FORM A)

PHYSICS 1B LECTURE 4

INSTRUCTOR: HECTOR GARCIA VASQUEZ

Tuesday, May 21st, 2019

10:00 AM – 10:50 AM

Last Name: Wildenhain

First Name: Patrick

University ID: 105165778

DO NOT TURN PAGE UNTIL INSTRUCTED

You will have 50 minutes to complete this exam. One standard 3" x 5" index note card is permitted. Books and all other notes are not allowed. **Calculators are not allowed**. All other electronics are not allowed and must be put away. Both pen and pencil are allowed.

Please write your answer in the space below the problem. Scratch paper will be provided. **You must write legibly and demonstrate your reasoning to get full credit**. For clarity, please draw a box around your final answer.

Q1	38
Q2	38
Q3	24
TOTAL	100

Integrals

$$\int_{x_1}^{x_2} \frac{x}{(x^2+u^2)^{3/2}}\,dx = -\frac{1}{\sqrt{x^2+u^2}}\Bigg|_{x_1}^{x_2}$$

$$\int_{x_1}^{x_2} \frac{1}{(x^2+u^2)^{3/2}}\,dx = \frac{x}{u^2\sqrt{x^2+u^2}}\Bigg|_{x_1}^{x_2}$$

$$\int_{x_1}^{x_2} x^n\,dx = \frac{1}{n+1}x^{n+1}\Bigg|_{x_1}^{x_2} \qquad (n \neq 1)$$

$$\int_{x_1}^{x_2} \frac{1}{x}\,dx = ln(x)|_{x_1}^{x_2}$$

$$\int_{x_1}^{x_2} cos(x)\,dx = sin(x)|_{x_1}^{x_2}$$

$$\int_{x_1}^{x_2} sin(x)\,dx = -cos(x)|_{x_1}^{x_2}$$

Trig

$$\cos(0) = \cos(2\pi) = 1$$

$$\cos\left(\frac{\pi}{2}\right) = 0$$

$$\cos\left(\theta \pm \frac{\pi}{2}\right) = \cos\left(\frac{\pi}{2} \pm \theta\right) = \mp\sin(\theta)$$

$$\cos(\theta \pm \pi) = \cos(\pi \pm \theta) = -\cos(\theta)$$

$$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$$

$$\sin(0) = \sin(2\pi) = 0$$

$$\sin\left(\frac{\pi}{2}\right) = 1$$

$$\sin\left(\theta \pm \frac{\pi}{2}\right) = \pm\cos(\theta),\ \ \sin\left(\frac{\pi}{2} \pm \theta\right) = \cos(\theta)$$

$$\sin(\theta \pm \pi) = -\sin(\theta),\ \ \sin(\pi \pm \theta) = \mp\sin(\theta)$$

$$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$$

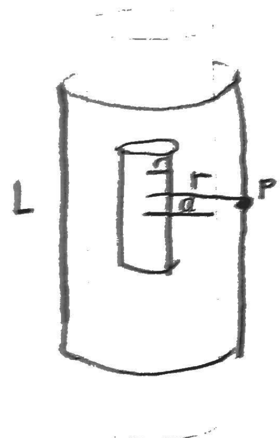
Gauss' Law, cylindrical shell:

$$\oint_S \vec{E}\cdot\hat{n}\,dA = 2\pi r L E = \frac{q_{enc}}{\varepsilon_0}$$

Question 1

An infinitely long cylinder of radius a has a uniform volume charge density ρ.

(a) Find the electric field $\vec{E}$ at a point outside the cylinder.
(b) Find the electric field $\vec{E}$ at a point inside the cylinder.
(c) Find the potential at the surface of the cylinder ($r = a$), with respect to a reference point at a radial distance $b > a$.



a. $r > a$ $\quad \vec{E}\cdot 2\pi r L = \frac{q_{enc}}{\varepsilon_0}\hat{r}$

$q_{enc} = \pi a^2 L \rho \qquad \vec{E} = \frac{\pi a^2 L \rho}{\varepsilon_0}\cdot\frac{1}{2\pi r L}\hat{r}$

$$\boxed{\vec{E}_{out} = \frac{a^2\rho}{2\varepsilon_0 r}\hat{r}}$$

b. $r < a$ $\quad \vec{E}\cdot 2\pi r L = \frac{q_{enc}}{\varepsilon_0}\hat{r}$

$q_{enc} = \pi r^2 L \rho \qquad \vec{E} = \frac{\pi r^2 L \rho}{\varepsilon_0}\cdot\frac{1}{2\pi r L}\hat{r}$

$$\vec{E}_{in} = \frac{r^2\rho}{2\varepsilon_0 r} = \boxed{\frac{r\rho}{2\varepsilon_0}\hat{r}}$$

c. $V = V_P - V_{ref} = V_P - V_R = -\int_R^P \vec{E}\cdot d\vec{l} = -\int_R^P \frac{a^2\rho}{2\varepsilon_0 r}\,dr$

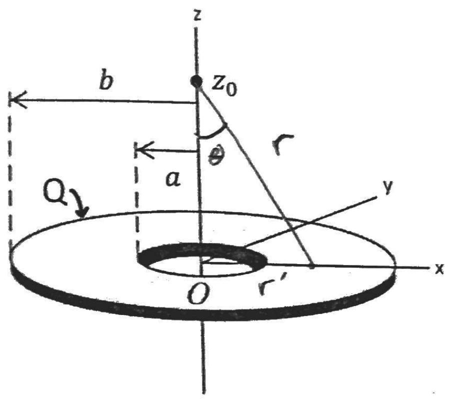
$= -\frac{a^2\rho}{2\varepsilon_0}\int_R^P \frac{1}{r}\,dr = -\frac{a^2\rho}{2\varepsilon_0}\Big[\ln|r|\Big]\Big|_R^P \qquad P = a, \quad R = b$

$$= -\frac{a^2\rho}{2\varepsilon_0}(\ln a - \ln b) = -\frac{a^2\rho}{2\varepsilon_0}\ln\left(\frac{a}{b}\right) = \boxed{\frac{a^2\rho}{2\varepsilon_0}\ln\left(\frac{b}{a}\right)}$$

Question 2

A thin disk with a circular hole at its center has inner radius a and outer radius b. The disk has positive charge Q distributed uniformly on its surface. The disk lies in the xy-plane, with its center at the origin.

(a) Find the surface charge density σ on the disk.
(b) Find the electric field $\vec{E}$ for a point z_0 above the center of the disk.



(a) $\sigma = \dfrac{Q}{\text{Area}}$ $\quad$ Area $= \pi b^2 - \pi a^2$

$$\boxed{\sigma = \frac{Q}{\pi(b^2 - a^2)}} \quad +1C$$

(b) $\vec{E}(z) = \dfrac{1}{4\pi\varepsilon_0} \displaystyle\int \frac{dq}{r^2}\,\hat{r}$ $\quad +3$

$dq = \sigma\, dA \qquad dA = 2\pi r'\, dr' \quad +2 \qquad z = z_0$

$r = \sqrt{r'^2 + z^2} \qquad \hat{r} = \cos\theta\, \hat{k} = \dfrac{z}{\sqrt{r'^2 + z^2}}\,\hat{k} \quad +1D$

$$\vec{E}(z) = \frac{1}{4\pi\varepsilon_0} \int_a^b \frac{\frac{Q}{\pi(b^2-a^2)} \cdot 2\pi r'\,dr'}{r'^2 + z^2} \cdot \frac{z}{\sqrt{r'^2 + z^2}}\,\hat{k} \quad +1C$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q \cdot 2\pi}{\pi(b^2-a^2)} \cdot z \int_a^b \frac{r'\,dr'}{(r'^2 + z^2)^{3/2}}\,\hat{k}$$

$$= \frac{Qz}{2\pi\varepsilon_0(b^2-a^2)} \left(-\frac{1}{\sqrt{r'^2 + z^2}}\right)\Bigg|_a^b \hat{k}$$

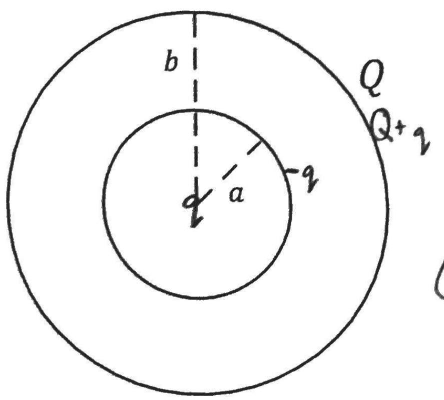
$$\boxed{\vec{E}(z) = \frac{Qz}{2\pi\varepsilon_0(b^2-a^2)} \left(\frac{1}{\sqrt{a^2+z^2}} - \frac{1}{\sqrt{b^2+z^2}}\right)\hat{k}}$$

x3

Question 3

A spherical conducting shell has inner radius a and outer radius b. The total charge on the conductor is Q. If a point charge q is placed inside the cavity at the origin:

(a) Find the charge on the inner surface ($r = a$).
(b) Find the charge on the outer surface ($r = b$).
(c) Find the electric field $\vec{E}$ inside the cavity, at a distance $r < a$ from the shell center.
(d) Find the electric field $\vec{E}$ outside the conductor, at a distance $r > b$ from the shell center.



(a) $\boxed{-q}$ on inner surface

(b) $\boxed{Q+q}$ on outer surface

(c) $r < a$ Use spherical Gaussian surface

$$\oint_S \vec{E} \cdot \hat{n}\, dA = E \cdot 4\pi r^2$$

$$\frac{q_{enc}}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{q}{\varepsilon_0}$$

$$\boxed{\vec{E}_{in} = \frac{q}{4\pi \varepsilon_0 r^2} \hat{r}}$$

(d) $r > b$ Use spherical Gaussian surface

$$\oint_S \vec{E} \cdot \hat{n}\, dA = E \cdot 4\pi r^2 \qquad \frac{q_{enc}}{\varepsilon_0} = \frac{Q+q}{\varepsilon_0}$$

$$E \cdot 4\pi r^2 = \frac{Q+q}{\varepsilon_0}$$

$$\boxed{\vec{E}_{out} = \frac{Q+q}{4\pi \varepsilon_0 r^2} \hat{r}}$$