Physics 1B

MIDTERM 2 - July 18, 2016

The exam lasts 60 minutes. You may consult both sides of a single 3″ × 5″ notecard, otherwise the exam is closed book and closed notes. A graphing calculator is allowed.

Show all your work in order to receive credit for your answer! Include any supporting diagrams and calculations. Give units and appropriate significant figures for numerical answers, show the magnitude and direction of vectors, and clearly indicate your final answers.

Do not begin the exam until everyone is instructed to do so. Your signature below indicates your adherence to the University's policies of academic integrity.

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Signature: YT

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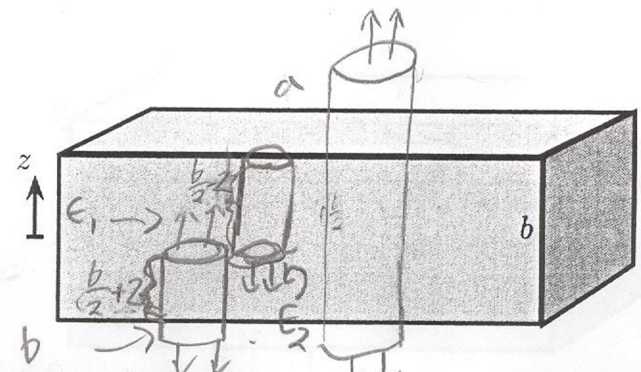
Question Number	Maximum Points	Points Earned
1	30	28
2	30	~~23~~ 24
3	30	29
Total:	90	81

from lab



28

1. A very wide slab of charge has thickness b and is filled with charge of uniform density ρ. The plane $z = 0$ is at the center of the slab, as in the figure.

   Find the magnitude and direction of the electric field vector $\boldsymbol{E}$ at points located:

   (a.) (10) outside the slab ($|z| > b/2$);

$\sigma = \rho \cdot b$

Gauss's law:

$$\iint \vec{E} \cdot d\vec{A} = \frac{q_{enclosed}}{\epsilon_0}$$

$q_{enclosed} = \rho \cdot b \cdot area$

$$2E\,\cancel{A} = \frac{\rho b\,\cancel{Area}}{\epsilon_0}$$

$$E = \frac{\rho b}{2\epsilon_0}$$ direction away from center of slab

+10

   (b.) (10) inside the slab ($|z| < b/2$).

bottom cylinder: $q_{enclosed} = \rho \cdot (z + \frac{b}{2}) \cdot area$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$$

$$2 \cdot \epsilon \cdot area = \frac{\rho \cdot (z + \frac{b}{2}) \cdot \cancel{area}}{\epsilon_0}$$

$$\epsilon_1 = \frac{\rho \cdot (z + \frac{b}{2})}{2\epsilon_0}$$ ← chg from below (direction = up)

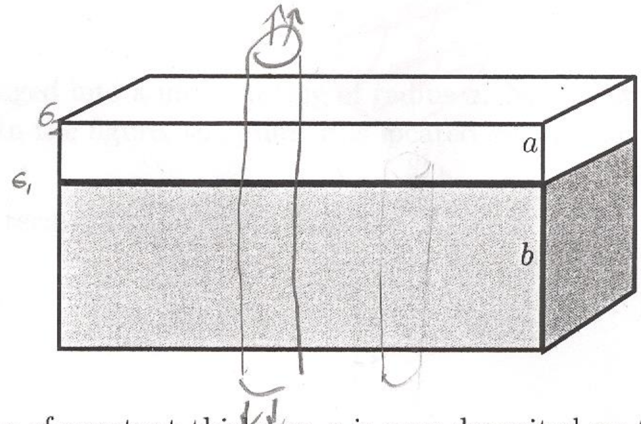
top cylinder: $2 \cdot \epsilon \cdot \cancel{area} = \frac{\rho \cdot (\frac{b}{2} - z) \cdot \cancel{area}}{\epsilon_0}$

+10

$$\epsilon_2 = \frac{\rho \cdot (\frac{b}{2} - z)}{2\epsilon_0}$$ ← chg from above (direction = down)

$$\epsilon = \epsilon_1 - \epsilon_2 = \frac{\rho(z + \frac{b}{2} - (\frac{b}{2} - z))}{2\epsilon_0} = \frac{\rho\, 2z}{2\epsilon_0} = \boxed{\frac{\rho z}{\epsilon_0}}$$

direction: away from center of slab



A conducting layer of constant thickness a is now deposited on top of the slab. If the conductor is neutral, find

(c.) (10) the charge densities on the surfaces (top and bottom planes) of the conductor.

E_{inside} is 0

field produced by surface charges:

$E = \frac{\sigma}{2\epsilon_0}$

E above the conductor is the same as below the slab, GL

$2E \, area = \frac{\rho \, b \, area}{\epsilon_0}$

$E = \frac{\rho b}{2\epsilon_0}$

$$\frac{\rho b}{2\epsilon_0} + \frac{\sigma_1}{2\epsilon_0} = 0$$

$$\sigma_1 = -\frac{\rho b}{2}$$

+ 8

$$\frac{\rho b}{2\epsilon_0} + \frac{\sigma_1}{2\epsilon_0} + \frac{\sigma_2}{2\epsilon_0} = \frac{\rho b}{2\epsilon_0}$$

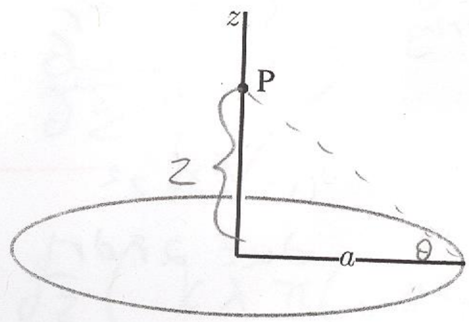
$$\frac{\sigma_1}{2\epsilon_0} + \frac{\sigma_2}{2\epsilon_0} = 0$$

$$\sigma_2 = \frac{\rho b}{2}$$

~~23~~ 24'

2. Charge Q is arranged into a uniform ring of radius a. Note that the shape is a circle or ring, not a disc. In the figure, the point P is located on the axis of the ring, the z axis, with $z > 0$.

Answer below in terms of the given quantities Q and a.



(a.) (10) Find an expression for the electric potential (voltage) at point P, where the potential is chosen to be zero at infinity.

6'

First find $\vec{E}$ at P: $Q = \lambda s = 2\pi\lambda a$

$$E = E_z = \int k\frac{dq}{r^2} = k\int_0^{2\pi} \frac{2\pi\lambda a}{\sqrt{a^2+z^2}}\,dz = ?$$

symmetry

$$k\frac{Qy}{(a^2+z^2)^{3/2}}$$

antiderivative

$$\Delta V = -\int_I^F \vec{E}\cdot d\vec{s} = -\int_\infty^z \frac{kQh}{(a^2+h^2)^{3/2}}\,dh = \frac{kQ}{\sqrt{r^2+h^2}}\Bigg|_\infty^z =$$

$$\boxed{\frac{kQ}{\sqrt{r^2+z^2}}} - 0$$

b) Disc has charge Q_{disk} =

8'

$$Q_{ring} = \lambda 2\pi r$$

$$\Delta V_{disk} = \int_0^r \Delta V_{ring} = \int_0^R \frac{k\lambda\pi r}{\sqrt{r^2+z^2}} dr = \frac{k\lambda\pi}{2}\int \frac{1}{\sqrt{u}} du =$$

$u = r^2 + z^2$

$du = 2r\,dr$

$$\frac{k\lambda\pi}{2}\left(2\sqrt{u}\right| =$$

$$k\lambda\pi\left(\sqrt{r^2+z^2}\right|_0^R =$$

$$k\lambda\pi\left(\sqrt{R^2+z^2} - z\right) =$$

(c.) (10) Find the electric field vector $\boldsymbol{E}$ of the disc at point P. In other words, give the electric field's x, y, and z components at P. Explain your answers.

10

E_x and E_y are 0 by symmetry

$$E_z = E_z = -\frac{\partial V}{\partial z}$$

$$= -\frac{d}{dz}\left(k\lambda\pi\left(\sqrt{R^2+z^2}-z\right)\right) =$$

$$k\lambda\pi - k\lambda\pi\left(\frac{z}{\sqrt{R^2-z^2}}\right)$$

?

3. An isolated conducting sphere of radius 1.50 m is charged to a voltage of 1.00×10^4 V (the reference point is at infinity).

(a.) (10) What is the total charge on the sphere?

$\vec{E} = k\frac{Q}{r^2}$

9

$V = -k\int_{\infty}^{1.5} \frac{Q}{r^2}\,dr = -k\frac{Q}{r}\Big|_{\infty}^{1.5} = k\frac{Q}{1.5\,m} = 10000$

$Q = \frac{1500}{k} =$

$1.67 \cdot 10^{-7}$ C

A second conducting sphere of radius 0.125 m and zero net charge is placed very far from the first. The spheres are now connected to each other with a fine conducting wire.

(b.) (10) How much charge flows through the wire when the spheres are connected?

10

old sphere   new sphere's charge

$V_1 = V_2$, $Q_{initial} = Q_1 + Q_2$

$\frac{kQ_1}{r_1} = \frac{kQ_2}{r_2}$ $1.67 \cdot 10^{-7} = Q_1 + Q_2$

$= 13\,Q_2$

OK

$Q_1 = \frac{r_1}{r_2} Q_2 = \frac{1.5}{0.125} Q_2 = 12\,Q_2$ $Q_2 = 1.28 \cdot 10^{-8}$ C

amount that moves into new sphere

$1.67 \cdot 10^{-7} = \frac{13}{12} Q_1$

$Q_1 = 1.54 \cdot 10^{-7}$

(c.) (10) After the spheres are connected, what is the electric field strength just outside the surface of each sphere?

$\vec{E} = k\frac{Q}{r^2}$

since they're far from each other we can ignore the effect of the other splere

$E_{sphere_1} = k\frac{q_1}{r_1^2} = k\frac{1.54\cdot 10^{-7}}{1.5^2} = \frac{8.99\cdot 1.84\cdot 10^{2}}{2.25} =$

10

615.3 OK

$E_{sphere_2} = k\frac{Q_2}{r_2^2} = k\frac{1.28\cdot 10^{-8}}{0.125^2} = \frac{8.99\cdot 1.28\cdot 10^{-1}}{0.015625} =$

$\frac{1.151}{0.015625} = 73.65$

10m

1 center of slab