Write your name here:

Nicolas Trammer

Write your UCLA ID here

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Midterm #1, Physics 1B, Winter 2020

Section 1 – Thomas Dumitrescu

- Please write your name and UID in the boxes on the front page and your name in the boxes at the top of the odd numbered pages.

- Please write your answers within the margins outlined by the boxes on each page.

- Closed book, **one** 5x3in note card (both sides) allowed.

- Scientific Calculators allowed, no computers or smartphones, please put books and notebooks in your backpacks.

- If a problem is ambiguous, notify the instructor. Clarifications will be written on the blackboard. Check the board occasionally.

- Time for exam: 60 minutes

- There are 4 questions, check that your exam has all 13 pages.

Good Luck !!

-additional space for calculation- Please denote exactly which problem you are working on

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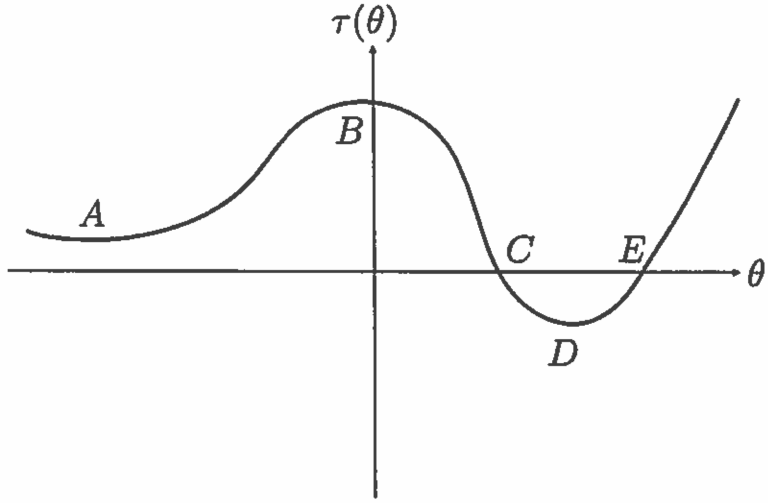
Problem 1: [15pts] Concept questions

a) [5pts] A physical torsion pendulum generates a torque $\tau(\theta)$ that depends on the angular displacement θ in a complicated way, as plotted below,



Around which points can the torsion pendulum execute simple harmonic motion?

Circle all that apply:

~~A~~

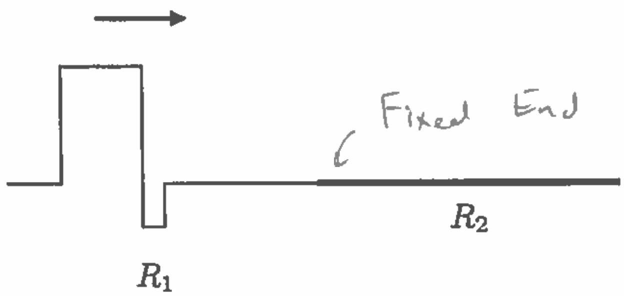
~~B~~

C $\quad \theta > C \Rightarrow \tau < 0$

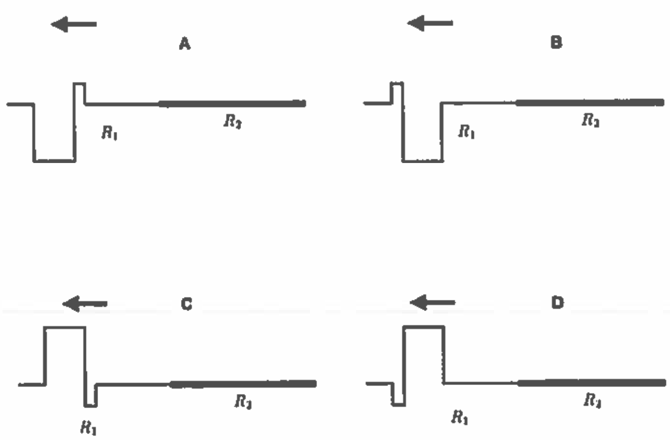
~~D~~ $\quad \theta < C \Rightarrow \tau > 0$

~~E~~

b) [5pts] Consider two ropes R_1 and R_2 that are made from the same material. The first rope R_1 is much thinner than the second rope R_2. The tail end of R_1 is attached to the tip of R_2 to form one long string, which is kept under constant tension. Assume that a wave pulse is traveling from left to right, along R_1 and towards R_2, as in the figure below,



When the pulse hits the junction, most of it is reflected back toward R_1. (Ignore the small pulse that is transmitted along R_2.) Which image below best represents the reflected wave?



Circle the correct answer:

A

B

C

D

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c) **[5pts]** Circle all statements that are true of a transverse mechanical standing wave:

~~A~~ Mechanical energy is everywhere the same along the string

B No mechanical energy is transported along the string

~~C~~ At any instant in time, kinetic energy is minimal at the displacement anti-nodes

~~D~~ Mechanical energy is zero at displacement anti-nodes

E Mechanical energy is zero at displacement nodes

Problem 2: [30pts]

Consider a long organ pipe, which extends from $x = 0$ to the left (i.e. $x \leq 0$). A loudspeaker is placed over the right end of the pipe at $x = 0$. The vibrations of the loudspeaker cause a sinusoidal displacement $u(t)$ of the air at this end, which is given by

$$u(t) = u_0 \sin(\omega t) ,$$

with $u_0 = 1.0\,\text{cm}$ and $\omega = 2000\,\text{Hz}$. This results in a traveling sound wave in the pipe.

a) [10pts] What is the frequency and the wavelength of the traveling wave. (Assume that the speed of sound in air is $v = 340\text{m/s}$.) Is it longitudinal or transverse?

is $\boxed{\text{Longitudal Wave}}$

$\omega = 2\pi f$

$f = \frac{\omega}{2\pi} = \frac{2000\,\text{Hz}}{2\pi} = \boxed{f = 318.3\,\text{Hz}}$

$v = \lambda f$

$\lambda = \frac{v}{f} = \frac{340\,\text{m/s}}{318.3\,\text{Hz}} = \boxed{\lambda = 1.068\,\text{m}}$

b) [10pts] Write down the displacement $u(x,t)$ of the traveling wave.

$u(x,t) = u_0 \sin\left(\frac{2\pi}{\lambda} x + \omega t\right)$ (+) b/c going left

$= (1.0\,\text{cm}) \sin\left(\frac{2\pi}{1.068\,\text{m}} x + (4000\,\text{Hz})t\right)$

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c) [10pts] Imagine we start observing the wave at time $t = 0$. What is the first moment in time $t > 0$ when the pressure $P(x,t)$ of the wave at $x = -1.0\,\mathrm{m}$ vanishes?

$$p(x,t) = -B\frac{\partial}{\partial x} U(x,t) = -Bu_0 \frac{2\pi}{\lambda} \cos\left(\frac{2\pi}{\lambda} x + \omega t\right)$$

$$p(-1.0\,\mathrm{m}, t) \overset{\text{set}}{=} 0 = -Bu_0 \frac{2\pi}{\lambda} \cos(\omega t - 0.588\ \mathrm{Rad})$$

$$\Rightarrow \omega t - 0.588 \cdot \mathrm{Rad} = \frac{\pi}{2}$$

$$t = \frac{\frac{\pi}{2} + 0.588\ \mathrm{Rad}}{2000\ \mathrm{Hz}}$$

$$= \boxed{0.00108\ \mathrm{s}}$$

Problem 3: [30pts]

A violin string of unknown length L is made from steel (with density $\rho = 7.85\,\text{g/cm}^3$). Its cross-section is approximately round, with radius 0.3 mm.

a) [10pts] If the tension of the string is 250 N, what is the speed of transverse waves traveling on the string?

$$N = \frac{m}{L} = \frac{\rho A L}{L} = \rho A = \rho \pi r^2 = \left(7.85 \frac{g}{cm^3}\right)\left(\pi (0.03\,cm)^2\right)$$

$$= 2.2195 \times 10^{-5} \frac{kg}{cm} = 0.0022195 \frac{kg}{m}$$

$$V = \sqrt{\frac{T}{N}} = \sqrt{\frac{250\,N}{0.0022195 \frac{kg}{m}}} = \boxed{V = 335.6\,m/s}$$

b) [10pts] Imagine you are exciting the string using a sine wave generator. Sweeping out all frequencies starting at zero, resonant standing waves are found at $f_1 = 440\text{Hz}$ and $f_2 = 880\text{Hz}$, but at no other frequencies between 0 and f_2. What kind of boundary conditions can give rise to this behavior (justify your reasoning)? Find the length L of the string.

The string must be fixed at both ends (it can't be open at both ends because the string has tension). It can't be open at one end because then the 2nd harmonic would have to be 3 times f_1, not simply double. Only odd multipes can give rise to asymmetric node anti-node pattern found.

$$V = \lambda f \Rightarrow \lambda = \frac{V}{f} = \frac{335.6\,m/s}{440\,Hz} = 0.763\,m$$

$$L = 2\lambda = \boxed{1.5255\,m}$$

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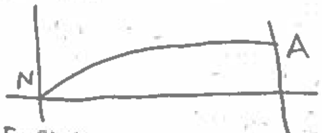
c) [10pts] If you change the boundary conditions at one end of the string (either from fixed to free or vice versa), and you again sweep out frequencies starting at zero, what are the first two frequencies f_1^{new} and f_2^{new} (with $0 < f_1^{\text{new}} < f_2^{\text{new}}$) at which you observe resonance using your new setup?

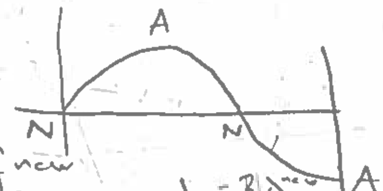
Now, one fixed end one open end



f_1^{new} $L = \frac{\lambda_1^{new}}{4}$

$\Rightarrow \lambda_1^{new} = 4L$

$f_1^{new} = \frac{v}{\lambda_1^{new}} = \boxed{55.0 \text{ Hz}}$



f_2^{new} $L = \frac{3\lambda_2^{new}}{4}$

$\Rightarrow \lambda_2^{new} = \frac{4L}{3}$

$f_2^{new} = \frac{v}{\lambda_2^{new}} = \boxed{165.0 \text{ Hz}}$

Problem 4: [30pts] The vibrations of a molecule can be modeled using an ideal spring with spring constant $k = 1.0\,\mathrm{N/m}$ with an attached mass $m = 6.6 \times 10^{-26}$kg. Denote the displacement of the spring from equilibrium by x. At $t = 0$, the string is stretched so that $x = 7.0 \times 10^{-10}$m, and it is given a push so that its initial velocity is $v = 2.0$m/s. The ensuing time dependence of x is given by

$$x(t) = R\cos(\omega t + \phi_0)\ , \qquad R > 0, \qquad -\pi < \phi_0 \leq \pi\ .$$

a) [10pts] Find ω, R, and ϕ_0.

$$\omega = \sqrt{\frac{K}{m}} = \sqrt{\frac{1.0\,\frac{N}{m}}{6.6\times10^{-26}\,\mathrm{Kg}}} = \boxed{3.892\times10^{12}\ \mathrm{Hz}}$$

$$R = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}} = \sqrt{(7.0\times10^{-10}\,\mathrm{m})^2 + \frac{(2.0\,\mathrm{m/s})^2}{(3.892\times10^{12}\,\mathrm{Hz})^2}} = \boxed{7.000\times10^{-10}\,\mathrm{m}}$$

$$\phi_0 = \tan^{-1}\left(-\frac{v_0}{\omega x_0}\right) = \tan^{-1}\left(\frac{2.0\,\mathrm{m/s}}{(3.892\times10^{12}\,\mathrm{Hz})(7.0\times10^{-10}\,\mathrm{m})}\right)$$

$(x_0 > 0)$

$$= \boxed{-7.340\times10^{-4}\ \mathrm{Rad}}$$

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b) [10pts] At what time $t > 0$ does the mass reach its highest speed for the first time? [If you could not do a), assume that $\phi_0 = -3.0 \times 10^{-15}$ rad.

$v(t) = x'(t) = -\omega R \sin(\omega t + \phi_0)$

Since $\phi \leq \frac{\pi}{2}$, first time $v(t)$ is maximized is when $\omega t + \phi_0 = \frac{\pi}{2}$

$$t = \frac{\frac{\pi}{2} - \phi_0}{\omega} = \frac{\frac{\pi}{2} + 7.340 \times 10^{-4}}{3.892 \times 10^{12}\ Hz}$$

$$= \boxed{4.037 \times 10^{-13}\ s}$$

c) **[10pts]** More realistically, the spring that describes the small vibrations $x(t)$ above has the following potential energy function,

$$U(\ell) = U_0 \left(\left(\frac{4.0 \times 10^{-10}\mathrm{m}}{\ell} \right)^{12} - 2 \left(\frac{4.0 \times 10^{-10}\mathrm{m}}{\ell} \right)^{6} \right) , \qquad U_0 > 0 \, .$$

Here $\ell > 0$ is the length of the spring. Find the location of stable equilibrium and calculate the spring constant k for simple harmonic motion around that point. By comparing to the value $k = 1.0\,\mathrm{N/m}$ given above, determine U_0.

$$U(\ell) \overset{\text{set}}{=} 0 = U_0 \left(\left(\frac{4.0 \times 10^{-10} m}{\ell} \right)^{12} - 2 \left(\frac{4.0 \times 10^{-10} m}{\ell} \right)^{6} \right)$$

$$\Rightarrow 0 = \frac{1}{\ell^6} \left(\frac{(4.0 \times 10^{-10} m)^{12}}{\ell^6} - 2(4.0 \times 10^{-10} m)^6 \right)$$

$$\ell^6 = \frac{(4.0 \times 10^{-10} m)^{12}}{2(4.0 \times 10^{-10} m)^6}$$

$$\Rightarrow \ell = 3.564 \times 10^{-10} m \, , \quad \text{stable equilibrium}$$

$$F = -\frac{d}{d\ell} U = U_0 \left[+ \frac{(4.0 \times 10^{-10} m)^{12}}{\ell^{13}} - \frac{(4.0 \times 10^{-10} m)^6}{\ell^7} \right]$$

$$F(\ell = 3.564 \times 10^{-10} m) = U_0 \cdot \underbrace{-0.432 \times 10^{-10}}_{A}$$

$$= -k\ell = U_0 \cdot A$$

$$\Rightarrow \boxed{U_0 = 6.032 \text{ J}}$$

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