Q1

1 $x(t) = Ae^{-\frac{bt}{2m}} \cos(\omega t)$

a $\tilde{A}(t) = Ae^{-bt/2m}$ $\qquad E(t) = U(t) + K(t)$

$E_0 = \frac{1}{2} k A^2$ $\qquad U(t) = \frac{1}{2} k (x)^2$

$E(t) = E_0 e^{-bt/m}$ $\qquad K(t) = \frac{1}{2} m v^2$

show this

$v(t) = \frac{dx}{dt} = A\left[-\frac{b}{2m} e^{-\frac{bt}{2m}} \cos(\omega t) + \omega \sin(\omega t) \cdot e^{-\frac{bt}{2m}}\right]$

$= Ae^{-\frac{bt}{2m}}\left[-\frac{b}{2m}\cos(\omega t) - \omega \sin(\omega t)\right]$

$= \tilde{A}(t)\left[-\frac{b}{2m}\cos(\omega t) - \omega\sin(\omega t)\right]$

$U(t) = \cancel{x}\frac{1}{2} k (\tilde{A}(t) \cos(\omega t))^2$

$K(t) = \frac{1}{2} m \left[\tilde{A}(t)\left[-\frac{b}{2m}\cos(\omega t) - \omega \sin(\omega t)\right]\right]^2$

$= \frac{1}{2} m (\tilde{A}(t))^2 \left[-\frac{b}{2m}\cancel{\cos(\omega t)} - \omega\sin(\omega t)\right]^2$ $\quad 0$

b is small so

$\approx \frac{1}{2} m (\tilde{A}(t))^2 \left[\omega^2 \sin^2(\omega t)\right]$

$U(t) + K(t) = \frac{1}{2}\tilde{A}(t)^2\left[k\cos^2(\omega t) + m\omega^2 \sin^2(\omega t)\right]$

$\omega^2 = \frac{k}{m}$ $\qquad = \frac{1}{2}\tilde{A}(t)^2 [k] = \frac{1}{2} k A^2 e^{-bt/m}$

$E(t) = E_0 e^{-bt/m} = \frac{1}{2} k A^2 e^{-bt/m}$ ✓

b small Δt,

$$\Delta E \approx \left|\frac{dE}{dt}\right| \Delta t$$

show: $\Delta E = \frac{bE}{mw}$ one radian of oscill

$$\frac{dE}{dt} = -\frac{b}{m} E_0 e^{-bt/m} = -\frac{b}{m} E$$

take abs val

$$\Delta E \approx \frac{b}{m} E_0 e^{-\frac{bt}{m}} \Delta t$$

one radian $\rightarrow$ $w = 2\pi f = \frac{2\pi}{T}$ let $t_0 = 0$ so $t = \frac{1}{2\pi}$ as well

$$\cancel{\frac{b}{m} E_0 e^{-\frac{b}{2\pi m}}} \cdot \frac{1}{2\pi}$$

$$\Delta E \approx -\frac{b}{m} E \cdot \frac{1}{w} = \boxed{-\frac{bE}{mw}}$$

$$\frac{T}{2\pi} = \frac{\text{time for one cycle}}{2\pi \text{ rad / cycle}} = \text{time for one radian}$$

$$\frac{T}{2\pi} = \frac{1}{w}$$

Q1

c

$$Q = \frac{E}{\text{en in one rad}}$$

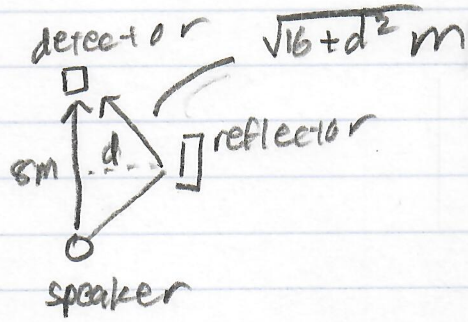
$$= \frac{E_0 e^{-\frac{b}{2\pi m}}}{bE/m\omega}$$

using formulas from befor

$$\cancel{E} \cdot \frac{m\omega}{b\cancel{E}} = \frac{m\omega}{b}$$

$$\boxed{Q = \frac{m\omega}{b}}$$

Q 2



$v = 344\ m/s$

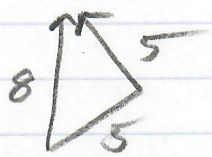
a $d = 3m$, $\lambda = ?$ $f = ?$

min dist for construc interf. ie exactly one wavelength apart

$$\Delta\varphi = k\Delta x = \frac{2\pi}{\lambda}\Delta x$$

$\Delta\varphi = $ "1λ" $= 2\pi$ $\qquad \cancel{2\pi} = \frac{\cancel{2\pi}}{\lambda}\Delta x$

$$\Delta x = 1\lambda = \boxed{2m}$$



$\sqrt{16+3^2} = \cancel{\sqrt{16+9} = 5}$

$\Delta x = 10 - 8 = 2$ $\qquad \cancel{\lambda = \sqrt{2\pi(2)} = \sqrt{4\pi}}$

$\cancel{= 2\sqrt{\pi} = 3.545\,m}$

$\lambda = \frac{v}{f}$ $\quad f = \frac{v}{\lambda} = \frac{344}{2} = \boxed{172\ Hz}$

b move refl. to right. first value d @ which intensity is minimum - destructive

half wavelength apart

increasing d so Δx must increase.

next possib Δx is when $\Delta x = \frac{3\lambda}{2}$

$\frac{3\lambda}{2} = \frac{3(2)}{2} = 3m$ $\qquad 2\sqrt{16+d^2} - 8 = 3$

$$2\sqrt{16+d^2} = 11$$

$$\sqrt{16+d^2} = \frac{11}{2}$$

$$16+d^2 = \frac{121}{4}$$

$$d^2 = \frac{121}{4} - 16$$

$$\boxed{d = 3.775\,m}$$

Q3

a

$$V(x,y,z) = V_0 - I_0 z + \frac{E_0 a^3 z}{(x^2+y^2+z^2)^{3/2}}$$

$E_z = ?$ as $x^2+y^2+z^2 \to \infty$, $V \to 0$

$$\Delta V_{i \to f} = -\int_{\vec{r}_i}^{\vec{r}_f} \vec{E} \cdot d\vec{l}$$

$E_0 a^3 z$

$$\frac{\partial V}{\partial z} = -E_0 + \frac{E_0 a^3 (x^2+y^2+z^2)^{3/2} - \frac{3}{2}(x^2+y^2+z^2)^{1/2} \cdot 2z}{(x^2+y^2+z^2)^3}$$

$$= -E_0 + \frac{E_0 a^3 (x^2+y^2+z^2) - 3z(E_0 a^3 z)}{(x^2+y^2+z^2)^{5/2}}$$

$$= -E_0 + E_0 a^3 \left(\frac{x^2+y^2+z^2-3z^2}{(x^2+y^2+z^2)^{5/2}} \right)$$

$$\boxed{\vec{E}(z) = -E_0 + E_0 a^3 \left(\frac{x^2+y^2-2z^2}{(x^2+y^2+z^2)^{5/2}} \right)}$$

$$V(x,y,z) = -\int_{\vec{r}}^{\infty} \vec{E} \cdot d\vec{l} = V(\vec{r}) - 0 = V(\vec{r})$$

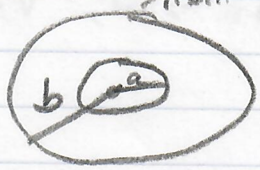
so take $\frac{\partial}{\partial z}$

b

$$V(z) = \frac{kQ}{\sqrt{r^2+z^2}}$$

use:
$$\int \frac{x\,dx}{\sqrt{x^2+A^2}} = \sqrt{x^2+A^2}$$

idea: take small rings and add together



$V(z) = ?$ for solid ring

$$\frac{\partial V}{\partial z} = \frac{0 - \frac{1}{2}(r^2+z^2)^{-1/2} \cdot 2z \cdot kQ}{r^2+z^2} = \frac{-kQ(r^2+z^2)^{-1/2}}{r^2+z^2}$$

$$E_z = \frac{-kQ}{(r^2+z^2)^{3/2}}$$ not needed

1) find V of infinitesimally ~~small~~ thin ring (given)

2) use integral to add up rings

$$V_{tot} = \int_a^b V(r)\,dr = \int_a^b \frac{kQ}{\sqrt{r^2+z^2}}\,dr$$

$$= kQ \int_a^b \frac{1}{\sqrt{r^2+z^2}}\,dr$$

$$kQ \int_a^b \frac{r}{r^2+z^2}\,dr$$

$$= kQ\sqrt{r^2+z^2}$$

Q4

fixed P, variable ΔV. carried by current w/ lines resist R

a

1) station generates current

$$I = \frac{P}{\Delta V}$$

2) current encounters resistance

$$P_R = I^2 R \qquad (P_{loss})$$

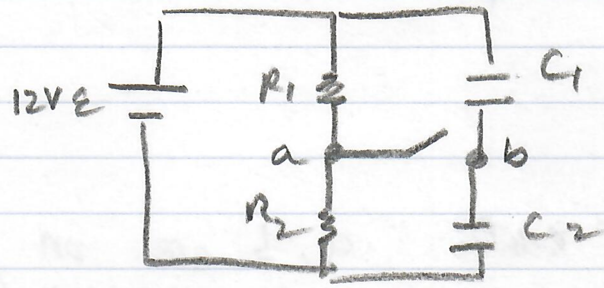
$$= \frac{P^2}{\Delta V^2} R \qquad \boxed{P_{loss} = \frac{P^2}{\Delta V^2} R}$$

$$\Rightarrow \frac{(750 \cdot 10^3)^2}{14000^2} \cdot 3$$

$$= 8609.69 \text{ W}$$

b when voltage increases, current decreases. when current is lower, the power lost will decrease as well, thus increasing efficiency.

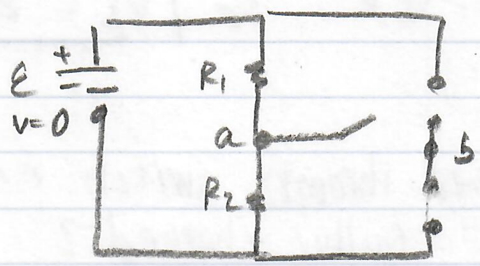
Q5



$R_1 = 1\Omega$
$R_2 = 2\Omega$
$C_1 = 8\,mF$
$C_2 = 4\,mF$

a switch open, fully charged. potential diff is?
$V_b - V_a = ?$

fully charged means capac is like an opening



b receives no current.
$V_b = 0$.

$$\varepsilon - IR_1 - IR_2 = 0$$
$$I(R_1 + R_2) = \varepsilon$$
$$I = \frac{\varepsilon}{R_1 + R_2} = \frac{12}{3} = 4\text{ A}$$

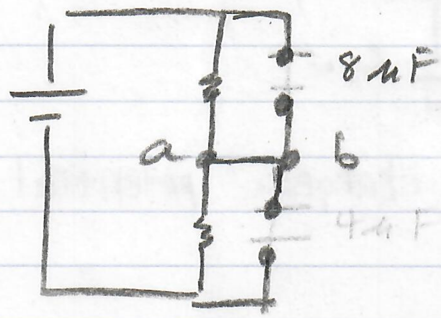
gain ε, drop IR_1 to reach a

$$V_a = \varepsilon - IR_1 = 12 - 4(1) = 8\text{ V}$$

$$V_b - V_a = 0 - 8 = \boxed{-8\text{ V}}$$

b close switch, capac fully charged

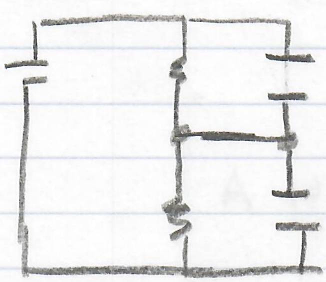
$V_b = ?$



a, b are on an equipotential so $V_a = V_b$

from part a, $V_a = 8V$ so $\boxed{V_b = 8V}$

c what max charge flowed through switch from time closed to time fully charged?



$R_{eq} = R_1 + R_2 = 3\Omega$

$C_{eq} = \frac{C_1 C_2}{C_1 + C_2} = \frac{8(4)}{12} = \frac{8}{3}\mu F$

$= 2.667 \mu F$

$Q(t) = C\mathcal{E}(1 - e^{-t/RC})$

max charge @ $t \to \infty$ $\lim_{t\to\infty} Q(t) = C\mathcal{E}$

$C\mathcal{E} = 2.667\times10^{-6} F (12) = \boxed{3.2\times10^{-5}\ C}$

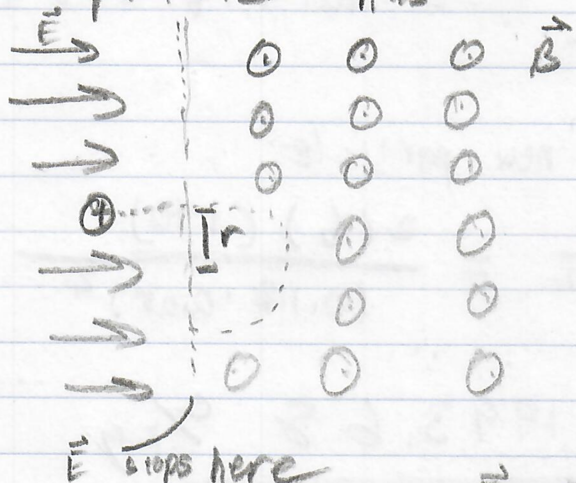
Q6

$m_p = 1.67 \times 10^{-27}$ kg

$v_i = 0$ $\quad |\vec{E}| = 140 \frac{V}{m}$ $\quad d = 6\,m$

then enters mag field $\perp$ to vel
$r = 8.35$ cm

new particle has $r = 11.8$ cm $\quad \frac{q}{m} = ?$



let $\vec{B}$ be directed $\odot$

step 1: find final v from e. field
step 2: use v to get r

elec

$\vec{F} = q\vec{E} = m\vec{a}$

$\vec{a} \parallel \vec{E}$ so $qE = ma$

$a = \frac{qE}{m}$

$v_f^2 = \cancel{v_i^2}^{\,0} + 2ad$

$v = \sqrt{2ad} = \sqrt{\frac{2dqE}{m}}$

mag

$R = \frac{mv}{|q|B}$

$\underline{\frac{|q|}{m} = \frac{v}{RB}}$

$\frac{|q|}{m} = \frac{1}{RB}\sqrt{\frac{2dqE}{m}}$

$\frac{q^2}{m^2} = \frac{1}{(rB)^2}\left(\frac{2dqE}{m}\right)$

$\frac{q}{m} = \frac{1}{(rB)^2}\left(2dE\right) = \frac{2dE}{(rB)^2}$

find B:

$$B = \frac{mv}{rq} \qquad v = \sqrt{\frac{2dqE}{m}}$$

$$= \frac{1.67\times10^{-27}(401446)}{0.0835(1.602\times10^{-19})} \qquad = \sqrt{\frac{2(6)(1.602\times10^{-19})(140)}{1.67\times10^{-27}}}$$

$$= 401446.4564 \text{ m/s}$$

$$= 0.05012 \text{ T}$$

find q/m for new particle:

$$\frac{q}{m} = \frac{2dE}{(rB)^2} = \frac{2(6)(140)}{(0.118 \cdot 0.05)^2}$$

$$= 48261993.68 \text{ C/kg}$$

$$\boxed{4.83\times10^{7} \text{ C/kg}}$$