PHYSICS 1B

MIDTERM 2

Spring, 2017 Dr. Coroniti

There are 100 points on the exam, and you have 50 minutes. To receive full credit, show all you work and reasoning. No credit will be given for answers that simply "appear". The exam is closed notes and closed book. You do not need calculators, so please put them, and all cell phones, away. If you need more space, use the backside of the page.

Amit Mondal
Your **Full Name** - Printed Clearly

Your Normal Signature

804746916
Your Student ID Number

Problem	**Score**
1	19
2	18
3	16
4	23
Total	76

FORMULAE

1. $\frac{1}{(1+x)^{1/2}} \approx 1 - \frac{x}{2}$; $x \ll 1$

2. $\frac{1}{(1 \pm x)^{3/2}} \approx 1 \mp \frac{3}{2}x$

3. $(1+x)^{1/2} \approx 1 + \frac{x}{2} - \frac{x^2}{8}$

4. $\int \frac{x\,dx}{(x^2+a^2)^{3/2}} = -\frac{1}{(x^2+a^2)^{\frac{1}{2}}}$

5. $\int \frac{x\,dx}{(x^2+a^2)^{1/2}} = (x^2+a^2)^{1/2}$

6. $\underline{E} = \frac{Q\hat{r}}{4\pi\epsilon_0 r^2}$

7. $\underline{dE} = \frac{dQ\,\hat{r}}{4\pi\epsilon_0 r^2}$

8. $\oiint \underline{E} \cdot \underline{dA} = \frac{Q}{\epsilon_0}$

9. $dV = -\underline{E} \cdot \underline{dr}$

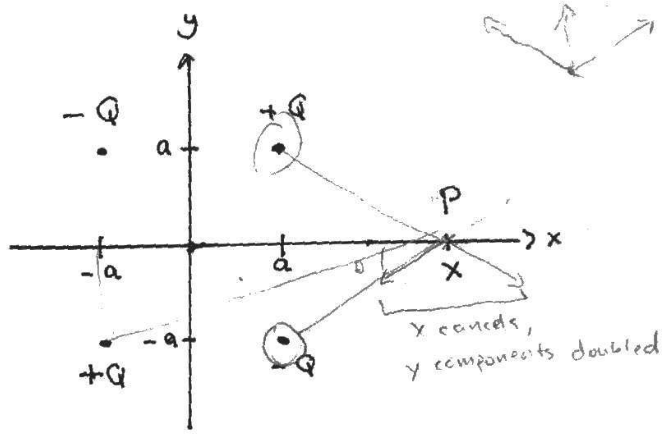
10. $dV = \frac{dQ}{4\pi\epsilon_0 r}$

(25 Pts)

1. Two positive and two negative point charges with magnitude Q are located at the following (x, y) coordinates as shown:

   (1) +Q (a, a) (3) -Q (- a, a)

   (2) -Q (a, - a) (4) +Q (- a, - a)



(15) a. For the pair of charges at $x = a, y = \pm a$, find the direction of the electric field that the charges produce at Point P (x, y = 0), and show that the magnitude of the electric field $|\underline{E}|$ is

$$|\underline{E}| = \frac{2Qa}{4\pi\epsilon_0}\frac{1}{[(x-a)^2+a^2]^{3/2}}$$

(5) b. Find the total electric field at Point P that is produced by the four charges.

(5) c. Show that if x >> a, the magnitude of the total electric field is approximately given by

$$|\underline{E}| \approx \frac{12Qa^2}{4\pi\epsilon_0 x^4}$$

What type of electric field does this represent?

$\Sigma E_x = 0$

(a) $E_y = \frac{kq}{r^2}\sin\theta$ $\sin\theta = \frac{a}{\sqrt{(x-a)^2+a^2}}$ $\Sigma E_y = k\left(\frac{Q}{((x-a)^2+a^2)}\frac{a}{((x-a)^2+a^2)^{1/2}} + \frac{Q}{((x-a)^2+a^2)}\frac{a}{((x-a)^2+a^2)^{1/2}}\right) =$

$2Qak\left(\frac{1}{((x-a)^2+a^2)^{3/2}}\right) = \frac{2Qa}{4\pi\varepsilon_0}\frac{1}{((x-a)^2+a^2)^{3/2}}$ ~~15~~ 13

(b) $\sin\theta = \frac{a}{((x+a)^2+a^2)}$ $\Sigma E_y = \frac{2Qa}{4\pi\varepsilon_0}\left(\frac{1}{((x+a)^2+a^2)}\cdot\frac{1}{((x+a^2)+a^2)^{1/2}}\right)$ 5

$\frac{2Qa}{4\pi\varepsilon_0}\frac{1}{((x+a)^2+a^2)^{3/2}}$

$\boxed{\frac{2Qa}{4\pi\varepsilon_0}\left(\frac{1}{((x+a)^2+a^2)^{3/2}} - \frac{1}{((x-a)^2+a^2)^{3/2}}\right)}$

(c) For $x >> a$ $\frac{2Qa}{4\pi\varepsilon_0}\left[\frac{1}{\cancel{a^{4/3}}((x+a)^2/a^2+1)^{3/2}} - \frac{1}{a^{4/3}((x-a)^2/a^2+1)^{3/2}}\right]$ $\frac{1}{(1\pm y)^{3/2}} \approx 1\mp\frac{3}{2}x$

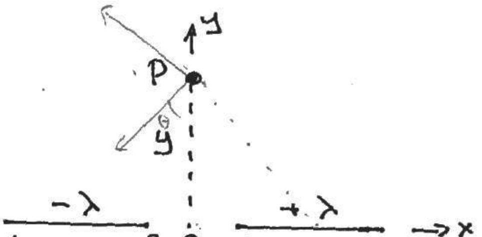
$\frac{2Qa}{a^{4/3}4\pi\varepsilon_0}\left[1-\frac{3(x+a)^2}{2a^2} - 1 + \frac{3(x-a)^2}{2a^2}\right] \approx \frac{12Qa^2}{4\pi\varepsilon_0 x^4}$ 1

Electric quadrupole

(20 Pts)

2. A thin rod with a uniform charge per unit length (dQ/dx) of $+\lambda$ extends along the x-axis from x = a to x = L. A second thin rod with dQ/dx $= -\lambda$ extends along the x-axis from x = - a to x = - L.

(15) a. Find the direction of the electric field at the Point P (x = 0, y) that is produced by the charged rods, and show that the magnitude of the electric field is given by



$$|\underline{E}| = \frac{2\lambda}{4\pi\epsilon_0}\left[\frac{1}{(y^2+a^2)^{\frac{1}{2}}} - \frac{1}{(y^2+L^2)^{\frac{1}{2}}}\right]$$

(5) b. If y >> a , and y >> L, show that the magnitude of the electric field is approximately given by

$$|\underline{E}| \approx \frac{\lambda(L^2-a^2)}{4\pi\epsilon_0 y^3}$$

What type of field does $|\underline{E}|$ represent? Note that $\lambda(L^2 - a^2) = 2\lambda(\frac{L+a}{2})(L-a)$; what quantity does this expression represent?

(a) Points in the $-x$ direction $\rightarrow \sum E_y = 0$

$E_x = \frac{k\,dq}{r^2}\sin\theta = \frac{k\,dq\,\sin\theta}{(l^2+y^2)}$ $\quad dq = dl\,\lambda \quad \sin\theta = \frac{l}{\sqrt{l^2+y^2}}$

$dE_x = \frac{k\lambda l\,dl}{(l^2+y^2)^{3/2}}$ justify: $2k\lambda \int_a^L \frac{l\,dl}{(l^2+y^2)^{3/2}} = 2k\lambda\left[-\frac{1}{(l^2+y^2)^{1/2}}\right]_a^L =$

$$\boxed{\frac{2\lambda}{4\pi\varepsilon_0}\left[\frac{1}{\sqrt{y^2+a^2}} - \frac{1}{\sqrt{y^2+L^2}}\right]}$$

(b) $\frac{2\lambda}{4\pi\varepsilon_0 y}\left[\frac{1}{\sqrt{1+a^2/y^2}} - \frac{1}{\sqrt{1+L^2/y^2}}\right] \rightarrow \frac{2\lambda}{4\pi\varepsilon_0 y}\left[1-\frac{a^2}{2y^2} - 1 + \frac{L^2}{2y^2}\right]$

Since y>>a and y>>L, $\frac{\cancel{2}\lambda}{4\pi\varepsilon_0 y}\frac{(L^2-a^2)}{\cancel{2}y^2} = \boxed{\frac{\lambda(L^2-a^2)}{4\pi\varepsilon_0 y^3}}$

Dipole

(28 Pts)

3. A spherical insulator with radius a carries a total charge of + Q that is uniformly distributed throughout its volume ($4\pi a^3/3$). The insulator sphere is surrounded by a concentric conducting spherical shell with an inner radius b (> a) and an outer radius c (> b) as shown. A total charge of - 2Q resides on the conducting shell.

(6) a. Sketch the electric field lines for this system of charges, and explain the distribution of the -2Q charge on the conducting shell.

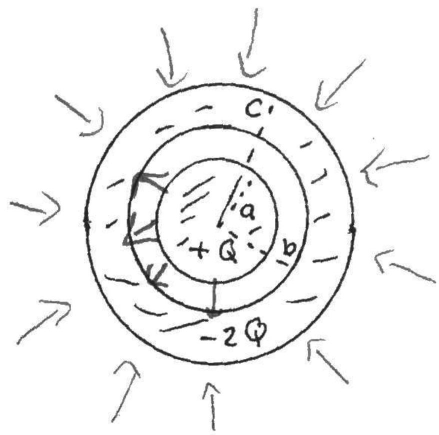
(12) b. Use Gauss's Law to find the electric field in the following regions:

(i) $0 < r < a$ (iii) $b < r < c$

(ii) $a < r < b$ (iv) $c < r$

(10) c. Take the zero of the electric potential V (r) to be at r = 0. Show that V (r) at the surface r = c of the conducting shell is given by

$$V(r = c) = -\frac{Q}{4\pi\epsilon_0}\left[\frac{3}{2a} - \frac{1}{b}\right]$$



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(a) See figure → field lines point inwards because of net negative charge

-Q is distributed on the inner surface, -Q is distributed on the outer

3

(b) $\frac{\frac{4}{3}\pi a^3}{\frac{4}{3}\pi r^3} = \frac{Q}{q_{enc}}$ $q_{enc} = \frac{Qr^3}{a^3}$ $E \cdot 4\pi r^2 = \frac{Qr^3}{\varepsilon_0 a^3}$; $\boxed{E = \frac{Qr}{4\pi\varepsilon_0 a^3}}$

(ii) $E \cdot 4\pi r^2 = \frac{Q}{\varepsilon_0}$ $E = \boxed{\frac{Q}{4\pi\varepsilon_0 r^2}}$ 3

(iii) $\frac{\frac{4\pi}{3}(c^3-b^3)}{\frac{4\pi}{3}(r^3-b^3)} = \frac{-2Q}{q_{enc}}$ $q_{enc} = \frac{-2Q(r^3-b^3)}{(c^3-b^3)} + Q$

$E \cdot 4\pi r^2 = \frac{\left(-\frac{2Q(r^3-b^3)}{(c^3-b^3)} + Q\right)}{\varepsilon_0}$ $E = \boxed{\frac{\left(-\frac{2Q(r^3-b^3)}{(c^3-b^3)} + Q\right)}{4\pi\varepsilon_0 r^2}}$

(iv) $\boxed{\frac{-Q}{4\pi\varepsilon_0 r^2}}$ 3

2

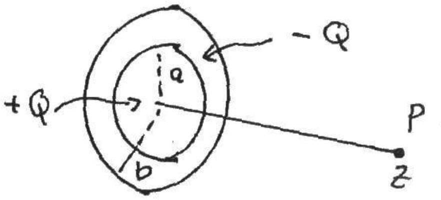
(c) $-\left(\int_0^a E\cdot dr + \int_a^b E\cdot dr + \int_b^c E\cdot dr\right)$:

$\int_0^a E\cdot dr = \frac{kQ}{a^3}$ $\int_a^b E\cdot dr = \frac{kQ}{b} - \frac{kQ}{a}$

(27 Pts)

4. A thin circular disk carries a charge of $+Q$ that is uniformly distributed over the radial region $0 < R < a$ (area = πa^2), and a charge of $-Q$ that is uniformly distributed over the annular radial range $a < R < b$ (area = $\pi(b^2 - a^2)$) as shown. Consider a Point P that is at a distance z along the axis of symmetry of the disk.

(15) a. Show that the electric potential at P which is produced by the positively charged inner disk is



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$$V_+ = \frac{Q}{2\pi\epsilon_0 a^2}\left[(z^2 + a^2)^{\frac{1}{2}} - z\right]$$

(7) b. Show that the electric potential at P which is produced by the negatively charged annulus is

$$V_- = -\frac{Q}{2\pi\epsilon_0(b^2 - a^2)}\left[(z^2 + b^2)^{\frac{1}{2}} - (z^2 + a^2)^{\frac{1}{2}}\right]$$

(5) c. For $z >> a$, and $z >> b$, show that the total electric potential $V = V_+ + V_-$ at P is approximately given by

$$V \approx \frac{Qb^2}{16\pi\epsilon_0 z^3}$$

Why does this charge distribution not produce a dipole contribution to the potential?

(a) $V_+ \to \sigma = \frac{Q}{\pi a^2}$ $\quad dq = 2\pi r(\sigma)dr \quad dV_+ = \frac{k(2\pi r\sigma)dr}{\sqrt{z^2+r^2}} = k(2\pi\sigma)\int_0^a \frac{r\,dr}{\sqrt{z^2+r^2}} =$

$k2\pi\sigma\left[(z^2+r^2)^{1/2}\right]_0^a = k2\pi\sigma\left[(z^2+a^2)^{1/2}-z\right] = \frac{1}{4\pi\epsilon_0}\cdot 2\pi\cdot\frac{Q}{\pi a^2}\left[(z^2+a^2)^{1/2}-z\right]$

$= \frac{Q}{2\pi\epsilon_0 a^2}\left[(z^2+a^2)^{1/2}-z\right]$ 15

(b) $\sigma = \frac{-Q}{\pi(b^2-a^2)}$ $\quad dq = 2\pi r\sigma dr \quad dV_- = \frac{k(2\pi r\sigma)dr}{\sqrt{z^2+r^2}} \quad k(2\pi\sigma)\int_a^b \frac{r\,dr}{\sqrt{z^2+r^2}} =$

$k2\pi\sigma\left[(z^2+b^2)^{1/2} - (z^2+a^2)^{1/2}\right] = \frac{1}{4\pi\epsilon_0}\cdot 2\pi\cdot\frac{-Q}{\pi(b^2-a^2)}\left[(z^2+b^2)^{1/2} - (z^2+a^2)^{1/2}\right] =$

$\boxed{-\frac{Q}{2\pi\epsilon_0(b^2-a^2)}\left[(z^2+b^2)^{1/2} - (z^2+a^2)^{1/2}\right]}$

(c)

The disks are concentric → no symmetry for dipole