PHYSICS 1B

MIDTERM 1

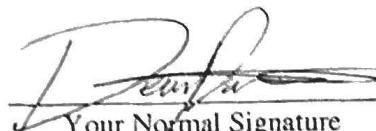Spring, 2017   Dr. Coroniti

There are 100 points on the exam, and you have 50 minutes. To receive full credit, show all you work and reasoning. No credit will be given for answers that simply "appear". The exam is closed notes and closed book. You do not need calculators, so please put them, and all cell phones, away. If you need more space, use the backside of the page.

DevenPatel
Your **Full Name** - Printed Clearly

104 766 465
Your Student ID Number

Your Normal Signature

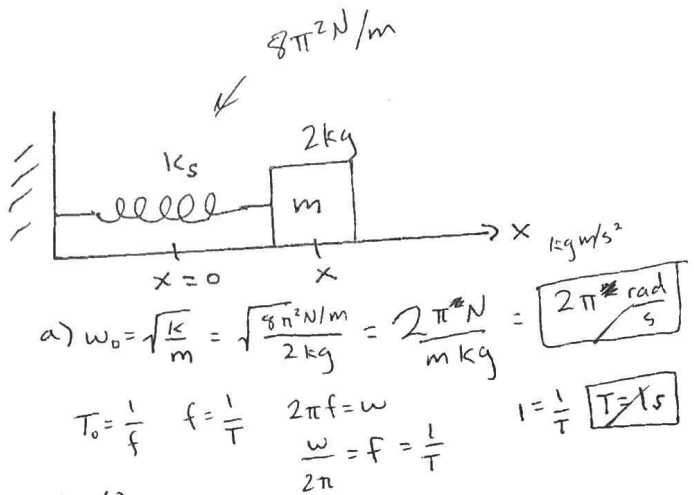| Problem	Score
1	19
2	16
3	21
4	21
Total	77

(30 Pts)

1. A block with mass $m = 2$ kg is attached to a spring with a stiffness constant $k_s = 8\pi^2$ N/m, and is free to oscillate on a smooth horizontal surface about the spring's equilibrium position $x = 0$. At time $t = 0$, the mass is observed to located at $x(0) = 2$ m, and to be moving with a speed $v(0) = 4\pi$ m/s toward $x = 0$. Leave answers in terms of π and whole numbers.

(4) a. Find the angular frequency (ω_0) for oscillations about $x = 0$, and the oscillation period.

4

(10) b. Find the function $x(t)$ that describes the position of the mass as a function of time $t \geq 0$.

7

(8) c. Find the velocity of the mass for the first time that it passes through $x = 0$. [Recall: $\sin(\pi/4) = \cos(\pi/4) = \sqrt{2}/2$.]

4

(8) d. Calculate the total mechanical energy E of the system, and the maximum displacement amplitude. [Recall the relation between E and the amplitude.]

4

$8\pi^2$ N/m

2 kg

k_s

ⅿ

$x = 0$ x

$\rightarrow x$ kg·m/s²

a) $\omega_0 = \sqrt{\frac{k}{m}} = \sqrt{\frac{8\pi^2 N/m}{2kg}} = 2\frac{\pi N}{m kg} = \boxed{2\pi \; \text{rad} / s}$

$T_0 = \frac{1}{f}$ $f = \frac{1}{T}$ $2\pi f = \omega$ $1 = \frac{1}{T}$ $\boxed{T = 1s}$

$\frac{\omega}{2\pi} = f = \frac{1}{T}$

b) $x(t) = A\cos(\omega t + \phi)$
$= A\cos(2\pi t + \phi)$

$v(t) = -\omega A \sin(\omega t + \phi)$
$x(t) = A\cos(\omega t + \phi)$

$= A\cos(2\pi t - \frac{\pi}{4})$

$\arctan\left(\frac{v_0}{-x\omega_0}\right) = \phi = \arctan\left(\frac{4\pi}{-4\pi}\right) = \arctan(+1) = \phi = \frac{\pi}{4}$

$= \boxed{\sqrt{5} \cos(2\pi t + \frac{\pi}{4})}$ −3

$A = \sqrt{x_0^2 + \frac{v_0^2}{\omega_0^2}}$ ← from energy $= \sqrt{4 + \frac{4\pi^2}{4\pi^2}} = \boxed{\sqrt{5}}$

$A = \sqrt{8}$

c) $\theta = \sqrt{5}\cos(2\pi t + \frac{\pi}{4})$ V_{max} is at 0

$V_{max} = -\omega A$ since $\sin$ max is 1

$= \boxed{-2\pi\sqrt{5}}$
error carried

d) $E_{tot} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 = \frac{1}{2}kA^2$

$= \frac{1}{2} \cdot 2kg(-2\pi\sqrt{5})^2 + 0 = \frac{1}{2}kA^2$ error carried

$\neq \boxed{20\pi^2 = E_{tot}}$

$20\pi^2 = \frac{1}{2}kA^2$

$40\pi^2 = kA^2$ ✓

$5 = A^2$

$\boxed{A = \sqrt{5}}$

(20 Pts)

2. A pendulum with bob mass m and massless rod of length L hangs under gravity. The mass is attached to two horizontal springs with stiffness constants k_1 and k_2 that are unstretched when the mass hangs straight down. The mass is displaced by a small angle θ, and released to oscillate about the origin x = 0. Since $\theta \ll 1$, $\sin\theta \approx \theta$, $x \approx L\theta$, and the motion essentially occurs only in the x-direction.

(12) a. Recalling the $\underline{\tau} = \underline{r} \times \underline{F}$, $\tau = I\alpha$, $\alpha = d^2\theta/dt^2$, and $I = mL^2$ about the pivot point P, or just using $\underline{F} = m\,\underline{a}$ with $\underline{F}$ being only along x, prove that the angular frequency of small oscillations about x = 0 is

$$\omega_0 = \left[\frac{k_1 + k_2}{m} + \frac{g}{L}\right]^{1/2}$$
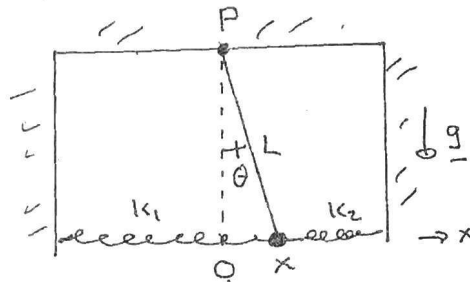
(8) b. Now suppose that the horizontal oscillations are driven by a force $F(t) = F_0 \sin(\omega t)$, and that the mass moves through water and experiences a force of friction $f = -b\,v$. Write the equation that now describes the motion (Newton's Second Law or its torque equivalent). If the driving frequency $\omega = \omega_0$, find x(t).

$F = -kx - mg \sin\phi \qquad k_E = k_1 + k_2$

a) $\gamma = Id = rF \qquad -k x r \sin\phi - mgr \sin\phi$

$\ddot{\phi} - \omega^2\phi = -\frac{k\ell}{I}\phi - mg\frac{\ell}{I}\phi \qquad \sin\phi = \phi$

$\omega^2 = \frac{k\ell}{I} + \frac{mg\ell}{I} = \left[\frac{k_1 + k_2}{m} + \frac{g}{\ell}\right]^{1/2}$

since we said $k_e = k_1 + k_2$
there is 2 ℓs upstairs that would cancel?

b) $\ddot{x}m = -k_e x - b\dot{v} \pm F_0 \sin(\omega t)$?

$\ddot{x} + \omega^2 x + \frac{b\dot{x}}{m} = \frac{F_0 \sin(\omega t)}{m}$

$\ddot{x} \propto \omega^2$
$\dot{x} \propto \omega$

$\omega^2 x = F_0 \sin(\omega t)$ same

$x = \frac{F_0 \sin(\omega t)}{m\omega^2}$??

2 different x(t)?

$\omega = \omega_0$
$\ddot{x} + \omega^2 x = 0.$
$b\dot{x} = \frac{F_0 \sin\omega t}{m}$
$\dot{x} = \left(\frac{F_0 \sin\omega t}{b}\right.$
$x = \frac{-F_0 \cos\omega_0 t}{\omega_0 b}$

plug in

(25 Pts)

3. A very long stretched string with equilibrium tension T_0 consists of a piece to the left $(x < 0)$ of the origin with mass per unit length μ_1, and a piece to the right $(x > 0)$ of the origin with a mass per unit length μ_2. At the origin $(x = 0)$, the string is attached to a motor that drives the string in the y-direction with a displacement $y(0, t') = A \sin(\omega t')$.

(6) a. Write (do NOT derive), the partial differential equation (the wave equation) that governs the space and time dependence of the displacement $y(x, t)$ of the string.
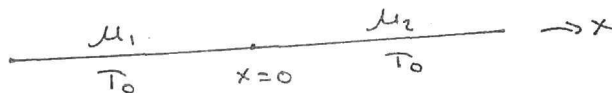
(10) b. Find $y(x, t)$ for the waves which are driven by the motor to travel in the **positive** and the **negative** x-directions.

(9) c. Calculate the **time-averaged** power that is transported by each wave. Of the two waves, which transports the higher power? [Recall: power $= \underline{F} \cdot \underline{v}$, and $F_y = -T_0 \, \partial y/\partial x$, $v = \partial y/\partial t$; $\langle \sin^2\theta \rangle = \langle \cos^2\theta \rangle = \frac{1}{2}$.]

a)
$$\boxed{\frac{\partial^2 y}{\partial t^2} = \frac{T_0}{\mu} \frac{\partial^2 y}{\partial x^2}}$$

(same) $\quad \underline{\underline{or}} \quad \dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{c^2} \dfrac{\partial^2 y}{\partial t^2}$ ④

b) $\quad y(0, t') = A \sin(\omega t')$

$x - x' = c(t - t')$

$t - \dfrac{x^*}{c} = t'$

$\left(c^2\right) \dfrac{\partial^2 y}{\partial \xi^2} = \dfrac{T_0}{\mu} \dfrac{\partial^2 y}{\partial t^2} \quad c = \sqrt{\dfrac{T_0}{\mu}} \qquad \omega = ck$

pos
$$y(x,t) = A \sin\left(\omega t - \frac{x\omega}{c}\right) = A \sin(\omega t - kx)$$

in neg. direction

$A \sin(\omega t + kx)$

$\boxed{A \sin\left(\omega t + \dfrac{\omega}{\sqrt{\frac{T_0}{\mu_1}}} x\right)}$

where k is

$\dfrac{\omega}{c} = \dfrac{\omega}{\sqrt{\frac{T_0}{\mu}}}$

$\boxed{A \sin\left(\omega t - \dfrac{\omega}{\sqrt{\frac{T_0}{\mu_2}}} x\right)}$

neg

c) $\langle P \rangle = \underline{F} \cdot \underline{v}$

$= -T_0 \dfrac{\partial y}{\partial x} \dfrac{\partial y}{\partial t}$

$\frac{1}{2}$

$= -T_0 A^2 \cos\left(\omega t + \frac{\omega}{\sqrt{\frac{T_0}{\mu}}} x\right)\left(\frac{\omega}{\sqrt{\frac{T_0}{\mu}}}\right) \cos\left(\omega t + \frac{\omega}{\sqrt{\frac{T_0}{\mu}}} x\right)(\omega)$

pos

$+ T_0 A^2 \cos^2\left(\omega t - \frac{\omega}{\sqrt{\frac{T_0}{\mu}}} x\right)\left(+\frac{\omega}{\sqrt{\frac{T_0}{\mu}}}\right)(\omega)$ $\times \frac{1}{2}$

$\boxed{= -T_0 A^2 \dfrac{\dfrac{\omega^2}{\sqrt{\frac{T_0}{\mu_1}}}}{2}}$

⑧

pos is
higher

$\boxed{= T_0 A^2 \dfrac{\left(\dfrac{\omega^2}{\sqrt{\frac{T_0}{\mu_2}}}\right)}{2}}$

(25 Pts)

4. A pipe of length L and uniform cross-sectional area is open to the atmosphere at one end x = 0, and is closed at the other end x = L. The speed of sound in air is C_s, and is related to the compressibility (bulk modulus) $\beta = \rho_0 C_s^2$ where ρ_0 is the equilibrium mass density.

(8)   a. State and physically explain the boundary conditions that the longitudinal displacement y (x,t) and the perturbed pressure δP (x,t) must satisfy at the open and closed ends of the pipe. [Recall: $\delta P = - \beta \, \partial y/\partial x$.]
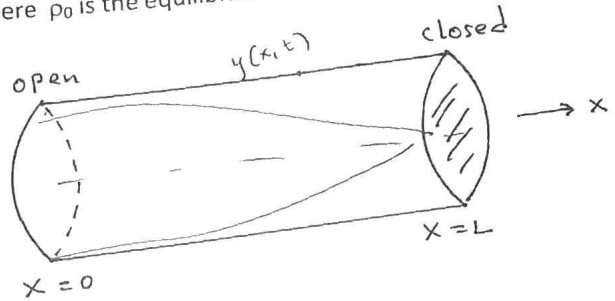
(12) b. Find the solution for y (x,t) for the waves that stand in the pipe (the normal modes), and for the wavenumber k_n, angular frequency ω_n, and ordinary frequency υ_n for the standing waves.

(5)   c. The length of the pipe is increased by a small distance $\Delta x \ll L$. The frequency of the nth harmonic is observed to decrease by $\Delta \upsilon$. Show that $\Delta \upsilon$ is approximately given by $\Delta \upsilon \approx \upsilon_n \, \Delta x/L$.

open   y(x,t)   closed
→ x
x = L
x = 0

a) displacement is 0 at the closed end and max at the open since there's no space to be displaced at the closed end, pushing all the displacement is where air meets the atmosphere.

$y(L,t) = 0$

perturbed Pressure is 0 at the open end and max at the closed since if it was max at open there would be it would be "hemorrhaging energy" so it is max at the closed where all the pressure builds up and molecules collide the most.

b) $y(x,t) = \tilde{y} \sin kx \cos(\omega t + \phi)$
$y(L,t) = \tilde{y} \sin kL \cos(\omega t + \phi)$

$\cos$

$kL = 0$
$\# @ \; k = (2n-1)\frac{\pi}{2}$

$\rightarrow y(x,t) = \tilde{y} \cos(kL) \cos(\omega t + \phi)$

$\tilde{y} = -\frac{\delta P}{\beta k}$
$= -\frac{\delta P}{\rho_0 C_s^2}$

$SP = -\beta \frac{\partial y}{\partial x} = -\beta \tilde{y} k \cos kx \cos \omega t + \phi$

$\cos kL = max$
$\cos k(0) = 0$
$\cos(kL) = 0$

$k_n = \frac{(2n-1)\pi}{2L}$

$\omega = kc$
$\omega = \frac{(2n-1)\pi c}{2L}$

$f = \frac{(2n-1)c}{4L}$
$f_n = \frac{(2n-1)c}{4L}$

$\omega = 2\pi f$
$f_n = \frac{\omega_n}{2\pi} =$

c) $f_n = \frac{(2n-1)c}{4L}$ n - same

when n is lower
f_n is lower

$\Delta f = f \frac{\Delta y}{L}$