PHYSICS 1B
FIRST MIDTERM

Fall, 2008 Dr. Coroniti

There are 100 points on the exam, and you have 50 minutes. To receive full credit, show all your work and reasoning. No credit will be given for answers that simply "appear". The exam is closed book and closed notes. You do not need calculators, so please put them away. If you need extra space, use the backside of the page.

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Your Full Name – Printed Your normal Signature

Problem	**Score**
1	23
2	21
3	17
4	19
Total	80

Useful Fomulae

1. Power $P = -T_0 \dfrac{\partial y}{\partial x}\dfrac{\partial y}{\partial t}$

2. $$\int_0^{2\pi} \sin^2\theta \, d\theta = \frac{\theta}{2} - \frac{\sin(2\theta)}{4}$$

$$\int_0^{2\pi} \cos^2\theta \, d\theta = \frac{\theta}{2} + \frac{\sin(2\theta)}{4}$$

3. $\sin(-\theta) = -\sin\theta$

$\cos(-\theta) = \cos\theta$

$\langle \sin^2\theta \rangle = \frac{1}{2}$

$\langle \cos^2\theta \rangle = \frac{1}{2}$

(25 Pts) $kg\,m/s^2/m$

1. A massless spring with stiffness constant $k_S = 27\pi^2$ N/m is attached to a block with a mass of $m = 3$ kg. The block slides without friction on a horizontal surface. At time $t = 0$, the block is observed to be momentarily at rest at a distance of 0.5 m from its equilibrium position $x = 0$.

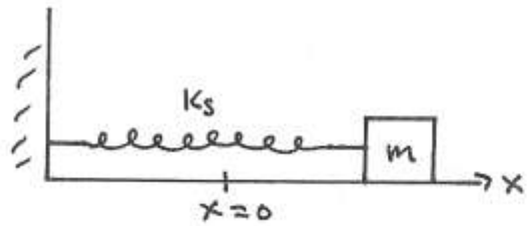
(5) a. What is the oscillation <u>period</u> of the mass about its equilibrium position?

(5) b. What is the position $x(t)$ of the mass for $t > 0$?

(5) c. What is the magnitude of the block's maximum acceleration?

(5) d. Now suppose that the block started at the equilibrium position at $t = 0$ with an initial speed v_0 moving to the right. For what value of v_0 will the mass reach the same maximum amplitude as above?

(5) e. For each of the above initial conditions, what is the total mechanical energy of the system?



a) $\omega = \sqrt{\frac{k}{m}} \Rightarrow \nu = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \rightarrow T = 2\pi\sqrt{\frac{m}{k}} = 2\pi\sqrt{\frac{3}{27\pi^2}} = 2\pi\sqrt{\frac{1}{9\pi^2}} = \boxed{\frac{2}{3}\text{ s}}$ ✓

b) $x(t) = A\cos(\omega t + \phi)$

$= B\cos(\omega t) + C\sin(\omega t)$

$0.5 = B\cos 0 + C\sin 0$

$0.5 = B$

$v(t) = -B\omega\sin(\omega t) + C\omega\cos(\omega t)$

$v(t) = -0.5\,\omega\sin(\omega t) + C\omega\cos(\omega t)$

$0 = -0.5\,\omega\sin 0 + C\omega\cos 0$

$C\omega = 0 \rightarrow C = 0$

$x(t) = 0.5\cos\left(\sqrt{\frac{k}{m}}\,t\right) = 0.5\cos\left(\sqrt{\frac{27\pi^2}{3}}\,t\right) = \boxed{0.5\cos(3\pi t)}$ 4

c) $a(t) = \frac{d^2}{dt^2}(A\cos\omega t) = \frac{d}{dt}(-A\omega\sin\omega t) = -\omega^2 A\cos\omega t = -\omega^2 x(t)$

$a_{max} = |-\omega^2 A| = \frac{27\pi^2}{3}(0.5) = \boxed{\frac{9}{2}\pi^2\ \frac{m}{s^2}}$ ✓

d) $x(0) = 0 \qquad v(0) = v_0$

$x(t) = A\cos(\omega t) = B\cos(\omega t) + C\sin(\omega t)$, $x(0) = 0 \Rightarrow B = 0$

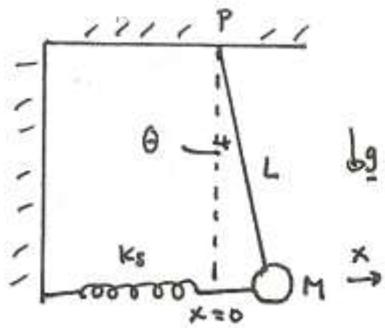
$v(t) = C\omega\cos(\omega t)$, $v(0) = v_0 \rightarrow C = \frac{v_0}{\omega}$

$v_0 = 0.5\,\omega = 0.5\sqrt{\frac{27\pi^2}{3}} = (0.5)(3\pi) = \boxed{\frac{3\pi}{2}\ \frac{m}{s}}$ ✓

e) $E = \frac{1}{2}kA^2 = \frac{1}{2}(27\pi^2)(0.5)^2 = \left(\frac{1}{2}\cdot\frac{1}{4}\right)(27\pi^2) = \boxed{\frac{27}{8}\pi^2\ \text{J}}$ 4

(25 Pts)

2. A pendulum with a massless rod of length L and bob mass M is suspended under gravity (acceleration = g). A massless spring with stiffness constant k_S is attached to M as shown. When the mass hangs vertically, the spring exerts no force on M; ie., $x = 0$ is the system's equilibrium position. The mass is displaced slightly from $x = 0$ by a small distance x such that $x/L \ll 1$ and $\sin\theta \approx \theta$; thus, the motion of the mass is essentially only in the x-direction.

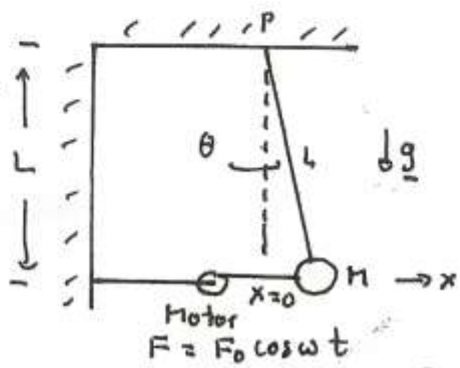


(6) a. Find the equation of motion for M that describes the small oscillations of x(t) about the equilibrium position. [Recall: torque $\underline{\tau} = \underline{r} \times \underline{F}$; $I\alpha = \tau$]

(9) b. Show that M will oscillate about $x = 0$ with the angular frequency

$$\omega_0 = [\, k_S/M \; + \; g/L \,]^{1/2}$$

(10) c. The spring is removed, and is replaced by a motor located a distance L below the pivot point P and attached to the rod as shown. The motor exerts a driving force $F(t) = F_0 \cos(\omega t)$ on the rod in the x-direction. Assuming that θ stays small ($\cos\theta \approx 1$), show that the steady solution for x(t) is given by



$$x(t) = \frac{F_0/M \cos(\omega t)}{\omega_0^2 - \omega^2}$$

Explain in physical terms the behavior of the the system for the cases: i) $\omega \ll \omega_0$, ii) $\omega \gg \omega_0$, iii) $\omega \approx \omega_0$.

a) $\Sigma\tau = I\frac{\partial^2\theta}{dt^2}$ $L\sin\theta \approx L\theta$

$-MgL\theta - k_S L^2\theta = mL^2 \frac{\partial^2\theta}{\partial t^2}$ => in terms of x

(−1)

b) $\frac{-(mgL - k_S L^2)\theta}{mL^2} = -\omega^2\theta$

$\omega = \sqrt{\frac{mgL + k_S L^2}{mL^2}} = \sqrt{\frac{k_S}{m} + \frac{g}{L}}$ ✓ ✓

c) $F = ma$ $m\ddot{x} = -kx + F_0\cos\omega t$

$\ddot{x} + \omega_0^2 x = \frac{F_0}{m}\cos\omega t$

Try $x(t) = A\cos\omega t$

$\omega_0^2 A\cos\omega t - A\omega^2\cos\omega t \overset{?}{=} \frac{F_0}{m}\cos\omega t$

$A = \frac{F_0/m}{\omega_0^2 - \omega^2}$ => $\boxed{x = A\cos\omega t}$

$x(t) = \frac{F_0/m}{\omega_0^2 - \omega^2}$ uhh, close?

(−3)

Phase?

if $\omega_0 \approx \omega$,
energy continues to come into the system without damping, infinitely large peak

$\omega \gg \omega_0$
Too low of a frequency to see meaningful oscillations

$\omega \ll \omega_0$
Frequency too high; inertia of string restricts motion

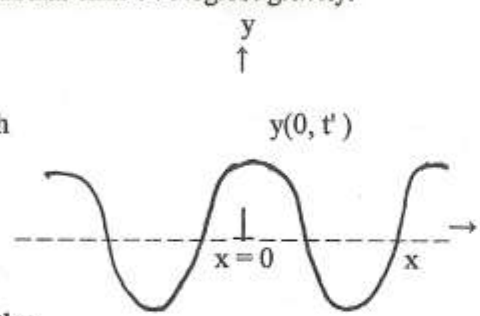
(25 Pts)

3. A very long string with an uniform mass per unit length μ is stretched in the horizontal x-direction with an equilibrium tension T_0. At the origin $x = 0$, the string is continuously displaced in the vertical y-direction by a source (a motor) so that $y(0, t') = A \sin(\omega t')$ as shown for some particular time t'. Neglect gravity.

(5) a. Write (do not derive) the equation that describes the displacement of the string $y(x, t)$ about its equilibrium position x at time t, and identify the speed at which disturbances travel along the string.

(10) b. Find the displacement $y(x, t)$ for <u>both</u> $x > 0$ and for $x < 0$.

(10) c. Calculate the <u>time-averaged</u> total power that the source must supply in order to maintain the traveling waves. [Caution: the local power is a <u>signed</u> quantity signifying the direction in which energy flows. The total power is just the magnitude.]



a) $\frac{\partial^2 y}{\partial x^2} = \frac{T_0}{\mu} \frac{\partial^2 y}{\partial t^2}$ ← wave equation

+5

speed = $c = \sqrt{\frac{T_0}{\mu}}$

b) $y(x,t) = A\sin(kx + \omega t)$

$x > 0$ ~~[illegible]~~ $x < 0$?

+3

$k = \frac{2\pi}{\lambda} = 2\pi$

c) $P = -T_0 \frac{\partial y}{\partial x} \frac{\partial y}{\partial t} = -T_0 (k\cos(kx + \omega t))(\omega\cos(kx + \omega t))$

$= -T_0 k \omega \cos^2(kx + \omega t)$

$\langle P \rangle = \frac{T_0 k \omega}{2}$ since $\langle \cos^2 x \rangle = \frac{1}{2}$

good, this is per side multiply by 2 to get total

+9

19

(25Pts)

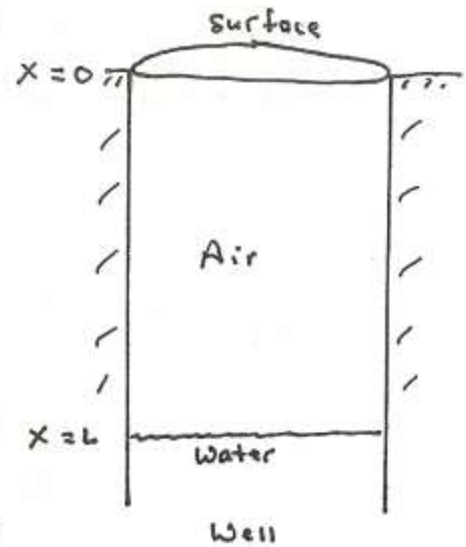
4. A water well extends from the surface ($x = 0$) to a depth L as shown. If the speed of sound in air C_S is known, the depth of the well can be determined by exciting the waves that stand in the well (the normal modes), and then measuring the frequency difference $\Delta\nu$ (in Hertz) between two successive harmonics (harmonic n+1 and n).

(4) a. If the longitudinal displacement of the air from its equilibrium position is $y(x, t)$, and the pressure perturbation of the air from the equilibrium pressure P_0 is $\delta P(x, t)$, state (do not derive) the general equation that determines $y(x, t)$ and $\delta P(x, t)$ for sound waves.

(5) b. State and physically explain the boundary conditions that both $y(x, t)$ and $\delta P(x, t)$ must satisfy at $x = 0$ (open end) and at $x = L$ (closed end, assuming that the sound waves are totally reflected by the water at the bottom of the well).

(12) c. Find the solutions for $y(x, t)$ and $\delta P(x, t)$, and find the wave numbers k_n and the angular frequencies ω_n for the sound waves that stand in the well. [Recall that $\delta P = -\beta\, \partial y/\partial x$ where β is the bulk modulus.]

(4) d. If the speed of sound is $C_S = 400$ m/s, and the frequency difference is $\Delta\nu = 50$ Hertz, how deep is the well?



4 a) $\frac{\partial^2 y}{\partial x^2} = \frac{\beta}{P_0}\frac{\partial^2 y}{dt^2}$ and $\frac{\partial^2 \delta P}{\partial x^2} : \frac{B}{P_0}\frac{\partial^2 \delta P}{\partial t^2}$

4 b) at $x=0$, $\delta P(0,t) = 0$ since pressure is at a node when the wave is unobstructed and $y(0,t)$ is at a ~~maximum~~ antinode

at $x=L$, $y(L,t) = 0$ since displacement is at a node when blocked and $\delta P(L,t)$ is at a ~~maximum~~ antinode (also means $\left(\frac{\partial \delta P}{\partial t}\right)_{L}=0$)

good!

10 c) $\delta P(x,t) = \beta k A \sin(kx)\cos(\omega t + \phi)$ integrate (since $y = -\frac{1}{\beta}\int \delta P(x,t)\,dx$)

$y(x,t) = A\cos(kx)\cos(\omega t + \phi)$

$k = (2n+1)\frac{\pi}{2}$, $n=0,1,2,3\ldots$ since we want $\cos kx = 0$ to satisfy $y(L,t)=0$

$k = \frac{2\pi}{\lambda} = 2\pi c\nu = c\omega$ $\quad k = \frac{(2n+1)\frac{\pi}{2}}{L}$ $n = 0,1,2,3,\ldots$

$\omega = \frac{(2n+1)\frac{\pi}{2}}{cL}$ $n=0,1,2,3,\ldots$

1 d) $c = \lambda\nu$