1B SUM20 QUIZ 4

Full Name (Printed) ______________________________

Full Name (Signature) ______________________________

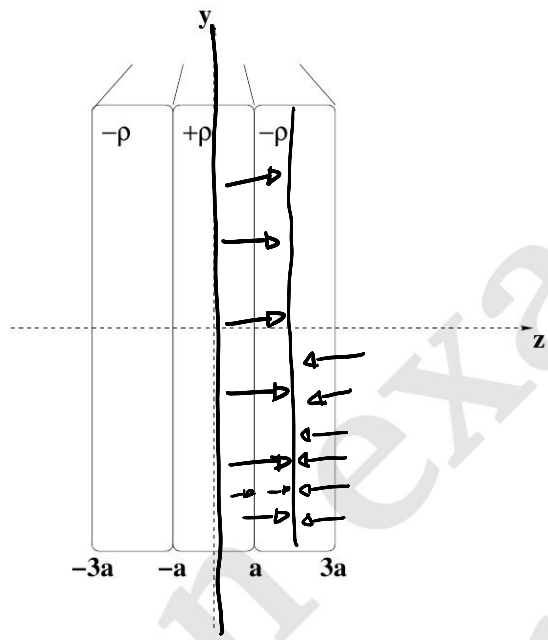
Student ID Number ______________________________

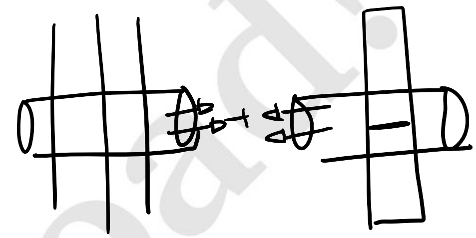
- The exam is open-book and open notes. You will probably do better to limit yourself to a single page of notes you prepared well in advance.
- **All work must be your own.** You are not allowed to collaborate with anyone else, you are not allowed to discuss the exam with anyone until all the exams have been submitted (after the close of the submissions window for the exam).
- You have **30 minutes** to complete the exam and sufficient time to scan the exam and upload it to GradeScope. The exam *must* be uploaded to GradeScope within the time alloted (that is, by 12:30 pm PDT). We will only except submissions through GradeScope and will not accept any exam submitted after the submission window closes (CAE students must contact Corbin for instructions).
- **Given the limits of GradeScope, you must fit your work for each part into the space provided.** You may work on scratch paper, but you will not be able to upload the work you do on scratch paper, so it is essential that you copy your complete solution onto the exam form for final submission. We can only consider the work you submit on your exam form.
- **For full credit the grader must be able to follow your solution from first principles to your final answer.** ***There is a valid penalty for confusing the grader.***
- It is **YOUR** responsibility to make sure the exam is scanned correctly and uploaded before the end of the submission window. The graders may refuse to grade pages that are significantly blurred, solutions to problems that are not written in the correct place, pages submitted in landscape mode and/or work that is otherwise illegible - if any of this occurs, you may not receive *any* credit for the affected parts.
- Focus on the concepts involved in the problem, the tools to be used, and the set-up. If you get these right, all that's left is algebra.
- **Have Fun!**

The following must be signed before you submit your exam:

By my signature below, I hereby certify that all of the work on this exam was my own, that I did not collaborate with anyone else, nor did I discuss the exam with anyone while I was taking it.

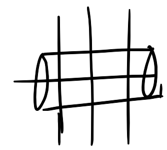
Signature ______________________________





Quiz 4) **Dielectric Sandwich:** A large, thick, non-conducting plane of uniform volume charge density $+\rho$ is sandwiched between two large, thick, uniform non-conducting planes of charge density $-\rho$. The arrangement is oriented parallel to the x, y-plane, centered on the origin. The positive plane extends (in thickness) from $z = -a$ to $z = +a$. The negative planes extend (in thickness) from $z = -3a$ to $z = -a$ and from $z = +a$ to $z = +3a$, respectively.

- 4a) (10 points) Find the (vector) electric field at every point on the $+z$-axis.



$\int \rho \cdot \partial V = \int \sigma \, \partial A$

$\pi r^2 \cdot z \cdot \rho = \sigma \cdot \pi r^2$

$\sigma = z\rho$

at $z = 0$

$E = 0$

at $0 < z < 2a$

$$\frac{z\rho}{2\epsilon_0}\hat{z} + \frac{-\rho(z-2a)}{2\epsilon_0}\hat{z} = E(r)$$

at $2a < z < 3a$

$$E(r) = \frac{-\rho(z-2a)}{2\epsilon_0}\hat{z}$$

at $z > 3a$

$$E(r) = \frac{-\rho a}{2\epsilon_0}\hat{z}$$

- 4b) (10 points) The electric potential at the center of the distribution is given as V_0. Find the electric potential at every point on the $+z$-axis.

at $z=0$

$E=0$ so $V=0$

at $0<z<2a$

$$\Delta V = -\int_0^z E \cdot \partial z$$

$$= -\int_0^z \frac{z\rho}{2\epsilon_0} - \int_0^z \frac{\rho(2a-z)}{2\epsilon_0}$$

$$= -\frac{\rho}{2\epsilon_0}\left(\frac{1}{2}z^2\right) - \frac{\rho}{2\epsilon_0}\left(2az - \frac{1}{2}r^2\right)$$

$$= -\frac{\rho}{2\epsilon_0}\left(\frac{1}{2}r^2\right) - \frac{\rho}{2\epsilon_0}\left(2az - \frac{1}{2}r^2\right)$$

$$= -\frac{\rho}{2\epsilon_0}\cdot 2aZ$$

at $2a<z<3a$

$$\Delta V = -\int_0^z E(r)\, \partial z$$

$$= -\int_0^{2a} E(r) - \int_{2a}^{z} E(r)\, \partial z$$

$$= \frac{-\rho}{2\epsilon_0}4a^2 - \int_{2a}^{z} \frac{\rho(2a-z)}{2\epsilon_0}\, \partial z$$

$$= -\frac{\rho}{2\epsilon_0}4a^2 - \left(\frac{\rho}{2\epsilon_0}\left(2az - \frac{1}{2}z^2\right)\Big|_{2a}^{z}\right)$$

$$= \frac{-\rho}{2\epsilon_0}4a^2 - \frac{\rho}{2\epsilon_0}\left[2az - \frac{1}{2}z^2 - 4a^2 + \frac{1}{2}4a^2\right]$$

$$= \frac{-\rho}{2\epsilon_0}4a^2 - \frac{\rho}{2\epsilon_0}\left[2az - \frac{1}{2}z^2 - 2a^2\right]$$

$$= \frac{-\rho}{2\epsilon_0}2a^2 - \frac{\rho}{2\epsilon_0}\left[2az - \frac{1}{2}z^2\right]$$

for $z>3a$

$$= -\frac{\rho}{2\epsilon_0}2a^2 - \frac{\rho}{2\epsilon_0}\left[6a^2 - \frac{1}{2}(9a^2)\right]$$

- 4c) (5 points) A particle of charge q and mass m sits in unstable equilibrium at the origin. It is given a very slight nudge in the $+z$ direction. Estimate its speed as it passes through $z=a$, and again at $z=3a$. Assume the planes are relatively diffuse so that mechanical drag on the particle may be neglected.

at a

$$E = \frac{\rho a}{\epsilon_0}$$

$$V(a) = -\frac{\rho a^2}{\epsilon_0}$$

$$U = -q\cdot\frac{\rho a^2}{\epsilon_0} = \frac{1}{2}mv^2$$

$$v = \sqrt{\frac{2(q)\rho a^2}{m\epsilon_0}}$$

at $3a$

$$\frac{\rho(a)}{2\epsilon_0}$$

so

$$v = \sqrt{\frac{(q)\rho a^2}{m\epsilon_0}}$$