1B SUM20 QUIZ 1

Full Name (Printed) ______________________________

Full Name (Signature) ____________________________

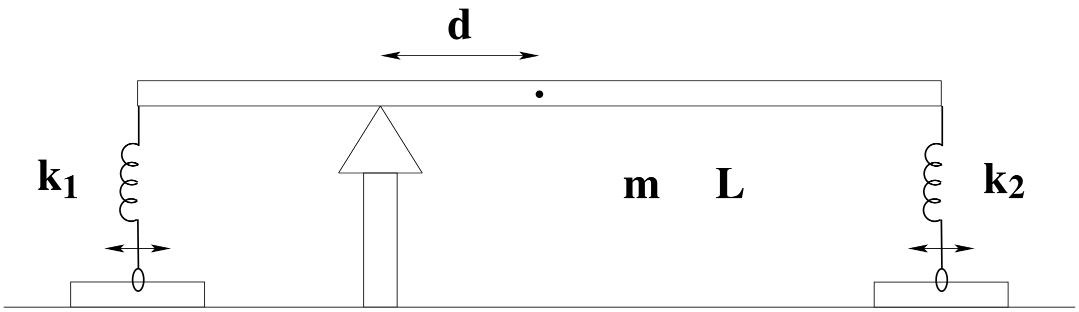
Student ID Number ______________________________

- The exam is open-book and open notes. You will probably do better to limit yourself to a single page of notes you prepared well in advance.
- **All work must be your own.** You are not allowed to collaborate with anyone else, you are not allowed to discuss the exam with anyone until all the exams have been submitted (after the close of the submissions window for the exam).
- You have **30 minutes** to complete the exam and sufficient time to scan the exam and upload it to GradeScope. The exam *must* be uploaded to GradeScope within the time alloted (that is, by 12:30 pm PDT). We will only except submissions through GradeScope and will not accept any exam submitted after the submission window closes (CAE students must contact Corbin for instructions).
- **Given the limits of GradeScope, you must fit your work for each part into the space provided.** You may work on scratch paper, but you will not be able to upload the work you do on scratch paper, so it is essential that you copy your complete solution onto the exam form for final submission. We can only consider the work you submit on your exam form.
- **For full credit the grader must be able to follow your solution from first principles to your final answer.** ***There is a valid penalty for confusing the grader.***
- It is **YOUR** responsibility to make sure the exam is scanned correctly and uploaded before the end of the submission window. The graders may refuse to grade pages that are significantly blurred, solutions to problems that are not written in the correct place, pages submitted in landscape mode and/or work that is otherwise illegible - if any of this occurs, you may not receive *any* credit for the affected parts.
- Focus on the concepts involved in the problem, the tools to be used, and the set-up. If you get these right, all that's left is algebra.
- **Have Fun!**

The following must be signed before you submit your exam:

By my signature below, I hereby certify that all of the work on this exam was my own, that I did not collaborate with anyone else, nor did I discuss the exam with anyone while I was taking it.

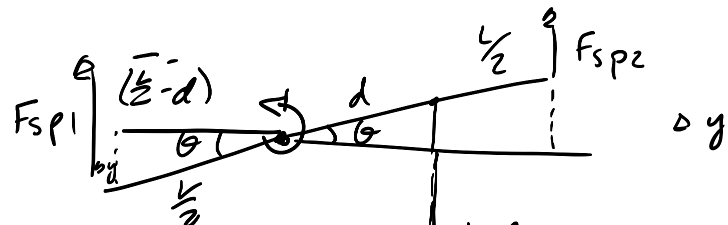
Signature ____



Quiz 1) A uniform rod of mass m and length L pivots around a point located a distance d from its center. On the ends are mounted springs of constants k_1 and k_2, both of which are cut so that they are unstretched when the rod is horizontal. The bottom of each spring is mounted on a frictionless rail so that the springs remain vertically-oriented when the rod is displaced from a horizontal position.

- 1a) (10 points) From first principles, derive the differential equation of motion for the system. Do not make any approximations.

Physical Pendelum



$\sum \tau =$

$$\left(d+\frac{L}{2}\right)\cdot\cos(\theta)\cdot F_{sp2} - (d)(\cos\theta)\cdot mg - \left(\frac{L}{2}-d\right)\cdot\cos(\theta)\cdot F_{sp1}$$

$$F_{sp2} = -\left(\frac{L}{2}+d\right)\cdot\sin(\theta)\,k_2$$

$$F_{sp1} = -\left(\frac{L}{2}-d\right)\cdot\sin(\theta)\cdot k_1$$

$$\tau = -\left(d+\frac{L}{2}\right)^2\cdot\cos(\theta)\sin(\theta)\cdot k_2 - (d)\cos(\theta)\cdot mg$$

$$-\left(\frac{L}{2}-d\right)^2\cos(\theta)\cdot\sin(\theta)\,k_1 = I\alpha$$

↑ to account for sin(θ) direction

$$I = \frac{1}{12}mL^2 + md^2$$

$$\left(\frac{1}{12}mL^2 + md^2\right)\frac{d^2\theta}{dt^2} = -\left(d+\frac{L}{2}\right)^2\cos(\theta)\sin(\theta)\cdot k_2 - d\cos(\theta)\cdot mg - \left(\frac{L}{2}-d\right)^2\cos(\theta)\cdot\sin(\theta)\,k_1$$

1

- 1b) (5 points) Under what condition(s) will the system execute simple harmonic motion? Re-write the differential equation of motion assuming that condition (or those conditions) hold.

If small angles...

$$\left(\frac{1}{12}mL^2+md^2\right)\frac{d^2\theta}{dt^2} = -\left(d+\frac{L}{2}\right)^2\theta\cdot k_2 - \underline{dmg} - \left(\frac{L}{2}-d\right)^2\theta k_1$$

$$\left(\frac{1}{12}mL^2+md^2\right)\frac{d^2\theta}{dt^2} + \left(\left(d+\frac{L}{2}\right)^2k_2 + \left(\frac{L}{2}-d\right)^2k_1\right)\theta = -dmg$$

$$\frac{d^2\theta}{dt^2} + \frac{\left(\left(d+\frac{L}{2}\right)^2k_2 + \left(\frac{L}{2}-d\right)^2k_1\right)\theta}{\left(\frac{1}{12}mL^2+md^2\right)} = \frac{-d\,g}{\left(\frac{1}{12}L^2+d^2\right)}$$

if it rotates in small angles and the distance to the center is not great... $\frac{d^2\theta}{dt^2} + \frac{\frac{L^2}{4}k_2 - \frac{L^2}{4}k_1}{\frac{1}{12}mL^2}\theta = 0$ $\quad \omega^2 = \frac{3k_2+3k_1}{m}$

- 1c) (10 points) Assuming there is no damping and the proper conditions are met, what will be the period of the oscillator's simple harmonic motion? What is the orientation of the rod when it is in equilibrium?

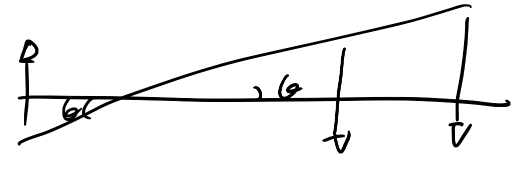
$$T = \frac{2\pi}{\omega_0} = \frac{2\pi}{\sqrt{(3k_2+3k_1)/m}}$$

at equilibrium if d is small

$$\sqrt{\frac{3k_2+3k_1}{m}}\,\theta = 0$$

$$0 = -\left(d+\frac{L}{2}\right)^2\theta\cdot k_2 - dmg - \left(\frac{L}{2}-d^2\right)\theta k_1$$

$$\theta = \frac{dmg}{\left(-\left(d+\frac{L}{2}\right)^2k_2 - \left(\frac{L}{2}-d^2\right)^2k_1\right)}$$



θ will be positive
the rod will dip on left