MT2 Physics 1B W16

Full Name (Printed) [redacted]

Full Name (Signature) [redacted]

Student ID Number [redacted]

Seat Number ______

Problem	Grade
1	23 /30
2	19 /30
3	13 /30
Total	55 /90

$$\frac{V_{snd} + V_{snd}V}{2(V_{snd} - V)}$$

$$x = \frac{V_{snd}V}{2(V_{snd} - V)}$$

- Do not peek at the exam until you are told to begin. You will have approximately 50 minutes to complete the exam.
- Don't spend too much time on any one problem. Solve 'easy' problems first. Go for partial credit!
- **HINT:** Focus on the concepts involved in the problem, the tools to be used, and the set-up. If you get these right, all that's left is algebra.
- **Have Fun!**

$$\cancel{V_{snd}^2} - \frac{V_{snd}V}{2} = \cancel{V_{snd}^2} - V_{snd}V + xV_{snd} - xV$$

$$\frac{V_{snd}V}{2} = x(V_{snd} - V)$$

$\cancel{x}_2$ $V/2$

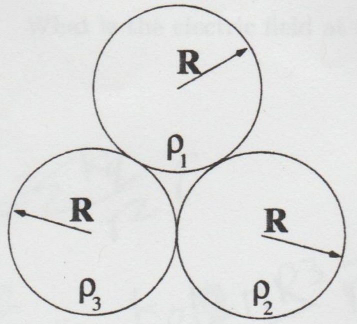
E ⟶ O

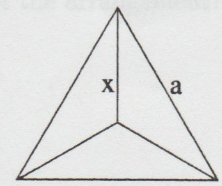
$$f_L = \frac{V_{snd}}{V_{snd} - V} f_E$$

$$\frac{20+x}{20-5} = \frac{20}{10} \quad 2$$

$$f_L = \frac{V_{snd} + x}{V_{snd} - V/2} f_E$$

$$\frac{V_{snd} + x}{V_{snd} - V/2} = \frac{V_{snd}}{V_{snd} - V}$$





$a^2 = 3x^2$

1) Three spheres of identical radius R (but different, uniform charge densities ρ_1, ρ_2 and ρ_3) are arranged so that they touch one another as shown in the diagram above. [The diagram at the right is a hint for later].

- 1a) (10 points) How much work will it take to assemble these spheres into the arrangement shown? [Assume the spheres themselves have already been assembled - that is, neglect the self-energy of each sphere].

+8 From outside, point charge

$q_1 = \rho_1 \frac{4}{3}\pi R^3$

$q_2 = \rho_2 \frac{4}{3}\pi R^3$

$q_3 = \rho_3 \frac{4}{3}\pi R^3$

$$W_{assemble} = k\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

$$= k\left(\frac{16\rho_1\rho_2\pi^2 R^3}{9R} + \frac{16\rho_1\rho_3\pi^2 R^3}{9R} + \frac{16\rho_2\rho_3\pi^2 R^3}{9R}\right)$$

$$= \frac{4\pi^2 k R^2}{3}\left(\rho_1\rho_2 + \rho_1\rho_3 + \rho_2\rho_3\right)$$

5

- +8 1b) (10 points) What is the electric potential at the center of the arrangement?

$$V = \sum \frac{k q_i}{r_i}$$

$R^2 = 3x^2$

$x = \frac{R}{\sqrt{3}}$

$$= \frac{\sqrt{3} k \rho_1 4\pi R^2}{3} + \frac{\sqrt{3} k \rho_2 4\pi R^2}{3} + \frac{\sqrt{3} k \rho_3 4\pi R^2}{3}$$

$$= \frac{4\sqrt{3} k\pi R^2}{3}\left(\rho_1 + \rho_2 + \rho_3\right)$$

+7

• 1c) (10 points) What is the electric field at the the center of the arrangement?

$$E_{tot} = \sum \frac{kq}{r^2}\hat{r}$$

$$\vec{E}_1 = \frac{k\rho_1 \cancel{2}\,12\pi R^3}{R^2}\hat{r} \qquad \hat{r} = -\hat{j}$$

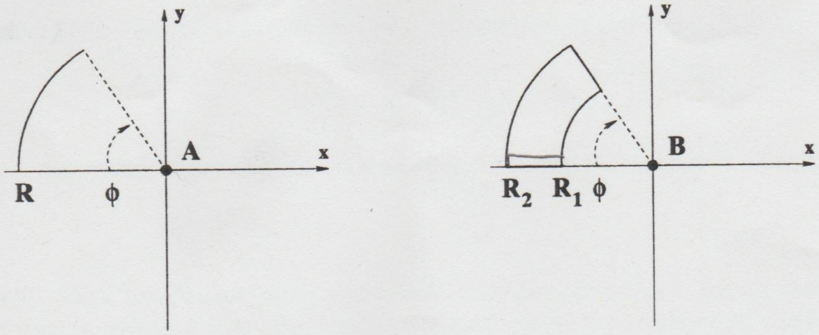
$$= -12k\rho_1 \pi R \hat{j}$$

$$\vec{E}_2 = 12k\rho_2 \pi R\left(\cos 30\,\hat{i} + \sin 30\,\hat{j}\right)$$

$$\vec{E}_3 = 12k\rho_3 \pi R\left(-\cos 30\,\hat{i} + \sin 30\,\hat{j}\right)$$

$$\vec{E}_x = 12k\pi R\cos 30\left(\rho_2 - \rho_3\right)$$

$$\vec{E}_y = 12k\sin 30\left(\rho_2 + \rho_3\right) - 12k\rho_1 \pi R$$



- 2a) (10 points) A thin nonconducting rod that carries an electric charge q (uniformly distributed) is bent to form a circular arc of radius R that subtends an angle ϕ as shown in the diagram on the left. Find the electric field (vector) at point A (located at the center of curvature of the arc).

$d\vec{E} = \frac{k\,dq}{r}\hat{r}$ $\qquad \hat{r} = \cos\phi\,\hat{i} - \sin\phi\,\hat{j}$

$\lambda = \frac{q}{R\phi}$ $\qquad dq = \lambda\,ds \qquad ds = R\,d\phi$

$\vec{E} = \frac{kq}{R}\left(\int_0^\phi \frac{\cos\phi}{\phi}\,d\phi\,\hat{i} - \int_0^\phi \frac{\sin\phi}{\phi}\,d\phi\,\hat{j}\right)$ $\qquad = \frac{q}{R\phi}R\,d\phi$

$= \frac{kq}{R}\left[(\cos\phi\ln\phi) - \int\right.$ $\qquad u = \cos\phi$ $\qquad = \frac{q}{\phi}d\phi$

$dv = \frac{d\phi}{\phi}$

$du = -\sin\phi\,d\phi$

$v = \ln\phi$

+8

- 2b) (10 points) Now consider the diagram shown on the right. Charge is spread over a wedge defined by the angle ϕ between the radial distances R_1 and R_2 (as shown) with an area charge density

$$\sigma(r) = \frac{4Q}{\phi(R_2^4 - R_1^4)}r^2$$

where r is the radial distance from the B (located at the center of curvature of the defining arcs). Find the electric field (vector) at B.

$d\vec{E} = \frac{k\,dq}{r^2}\hat{r}$ $\qquad \hat{r} = \cos\phi\,\hat{i} - \sin\phi\,\hat{j}$

$dq = \sigma(r)\cancel{(R_2 - R_1)}\,r\,d\phi$ dr

$= \frac{4kQr(R_2 - R_1)}{(R_2^4 - R_1^4)}\int_0^\phi \frac{\cos\phi\,\hat{i} - \sin\phi\,\hat{j}\,d\phi}{\phi}$

+6

- 2b) *(continued...)*

- 2c) (10 points) Find the electric potential produced by the wedge at point B relative to a point infinitely-distant from the wedge.

$$V = \int \frac{k\,dq}{r} \qquad dq = \sigma(r)\,(R_2 - R_1)\,r\,d\phi$$

$$= \frac{4kQr^2(R_2-R_1)}{(R_2^4-R_1^4)} \int_0^{\phi} \frac{d\phi}{\phi} \qquad +5$$

$$= \frac{4kQr^2(R_2-R_1)}{(R_2^4-R_1^4)} \Big[\ln\phi\Big]_0^{\phi}$$

$$= \frac{4kQr^2(R_2-R_1)\ln\phi}{(R_2^4-R_1^4)}$$

3) A spherical charge distribution of radius R carries a volume charge density

$$\rho(r) = \rho_0\,(1 - \frac{r^3}{R^3})$$

It is surrounded by a concentric spherical conducting shell that extends from $r = R$ to $r = 2R$ and carries an excess charge Q.

- 3a) (10 points) Find the charge inside a concentric sphere of radius r, for all values of r. Also, find the surface charge densities on the inner and outer surfaces of the conducting shell.

$(r < R)$

$$\int \rho\, dV = \rho_0 \int 1 - \frac{r^3}{R^3}\, dV$$

$dV = 4\pi r^2 dr$

$\left\{ \begin{array}{l} r \le R : \rho = \rho(r) \\ r > R : \rho = 0 \end{array} \right.$

$$= 4\pi\rho_0 \int_0^r r^2 - \frac{r^5}{R^3}\, dr$$

$$= 4\pi\rho_0 \left[\frac{r^3}{3} - \frac{r^6}{6R^3}\right]_0^R$$

$$= 4\pi\rho_0 \left(\frac{r^3}{3} - \frac{r^6}{6R^3}\right)$$

2

0

$(R < r < 2R)$

$$q_{in}(r > R) = q_{in}(R) + 0 = \frac{2\pi\rho_0 R^3}{3}$$

$(r > 2R)$

$$q_{in} = \frac{2\pi\rho_0 R^3}{3} + Q \quad 2$$

- 3b) (10 points) Find the electric field as a function of the radial distance from the center of the charge distribution (r) for all values of r.

$(r \le R)$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

$$E 4\pi r^2 = \frac{q_{in}}{\varepsilon_0}$$

$$E 4\pi r^2 = 4\pi\rho_0 \left(\frac{r^3}{3} - \frac{r^6}{6R^3}\right)$$

$$E = \rho_0 \left(\frac{r}{3} - \frac{r^4}{6R^3}\right) \quad 3$$

$(R < r < 2R)$

$$\int \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\varepsilon_0}$$

$$q_{in}(R) = \frac{2\pi\rho_0 R^3}{3}$$

0

$$E 4\pi r^2 = \frac{2\pi\rho_0 R^3}{3\varepsilon_0}$$

$$E = \frac{\rho_0 R^3}{6\varepsilon_0 r^2}$$

$(r > 2R)$

$$E 4\pi r^2 = \frac{2\pi\rho_0 R^3}{3\varepsilon_0} + \frac{Q}{\varepsilon_0}$$

$$E = \frac{\rho_0 R^3}{6\varepsilon_0 r^2} + \frac{Q}{4\pi\varepsilon_0 r^2}$$

3 1

- 3c) (10 points) If the electric potential within the conductor is given as V_0, find the potential as a function of the radial distance from the center of the charge distribution (r) for all values of r.

$(R<r<2R)$

$$V(r) = -\int_c^r E \cdot d\vec{r}$$

$$= -\int_c^r \frac{\rho_0 R^3}{6\varepsilon_0 r^2}\, dr$$

$$\frac{\rho_0 R^3}{6\varepsilon_0}\left[\frac{1}{r}\right]_c^r = \frac{\rho_0 R^3}{6\varepsilon_0 r} - V_0$$

$(r<R)$

$$V(r) = -\int_R^r E \cdot d\vec{r} \;+$$

$$= -\rho_0 \int_R^r \frac{r}{3} - \frac{r^4}{6R^3}\, dr \;+$$

$$= -\rho_0\left[\frac{r^2}{6} - \frac{r^5}{30R^3}\right]_R^r \;+$$

$$= -\rho_0\left(\frac{r^2}{6} - \frac{r^5}{30R^3} - \frac{R^2}{6} + \frac{R^2}{30}\right) +$$

$$= \rho_0\left(\frac{2R^2}{15} - \frac{r^2}{6} + \frac{r^5}{30R^3}\right) +$$

$(r>2R)$

$$\int_\infty^c$$