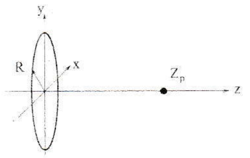


- 1a) (5 points) If the circle shown represents a uniform **ring** of charge Q and radius R, what is the resultant electric field at z_p?

If you remember this, you don't need to derive it...

$$\boxed{\vec{E} = \frac{Qz}{4\pi\epsilon_0 (R^2+z_p^2)^{3/2}}\,\hat{k}}$$

- 1b) (5 points) If the circle shown represents a uniform **disk** of charge Q and radius R, what is the resultant electric field at z_p?

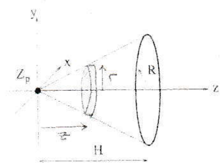
Again, if you remember this correctly (not as likely), you don't have to derive it. If you don't, build the thing out of infinitesimal rings... $R \to r$ $Q \to dq \to \sigma\, dA$

$$d\vec{E} = \frac{\sigma 2\pi r dr\, z}{4\pi\epsilon_0 (r^2+z_p^2)^{3/2}}\,\hat{k}$$

$$\int d\vec{E} = \frac{\sigma}{4\epsilon_0}\int_0^R \frac{2r\,dr}{(r^2+z_p^2)^{3/2}}\,\hat{k}$$

$$\vec{E} = \frac{\sigma}{2\epsilon_0}\left[1-\frac{1}{\sqrt{1+R^2/z^2}}\right]\hat{k} \to \boxed{\vec{E} = \frac{Q}{2\pi\epsilon_0 R^2}\left[1-\frac{1}{\sqrt{1+R^2/z^2}}\right]\hat{k}}$$

$$\sigma = \frac{Q}{\pi R^2}$$



$$\frac{r}{z} = \frac{R}{H}$$

- 1c) (20 points) A charge distribution occupies the volume of a right circular cone of base-radius R and height H that is oriented so that its apex is on the origin and its longitudinal symmetry axis lies along the $+z$-axis with the base intersecting $z = H$, as shown. Assuming the charge distribution has a volume density given by

$$\rho(z) = \frac{(n+3)\,Q}{\pi R^2 H^{n+1}}\, z^n$$

find the (vector) electric field at the origin.

Build the cone from infinitesimal disks. $R \to r$ $Q \to dq \to \rho\, dv$

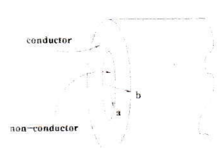
$$d\vec{E} = \frac{\rho(z)\,\pi r^2 dz}{2\pi\epsilon_0 r^2}\left[1-\frac{1}{\sqrt{1+(r/z)^2}}\right](-\hat{k})$$

we're on the left side of the distribution

$$\int d\vec{E} = \frac{1}{2\epsilon_0}\left[1-\frac{1}{\sqrt{1+(R/H)^2}}\right]\int_0^H \rho(z)\,dz\ (-\hat{k})$$

$$\boxed{\vec{E} = \frac{Q}{2\pi\epsilon_0 R^2}\,\frac{n+3}{n+1}\left[1-\frac{1}{\sqrt{1+(R/H)^2}}\right](-\hat{k})}$$

15



2) An infinitely-long, non-conducting cylinder of radius a carries a uniform volume charge-density ρ. The non-conducting cylinder is, in turn, surrounded by a concentric, neutral conducting cylinder of inner-radius a and outer-radius b.

- 2a) (10 points) Find the amount of charge contained in a concentric cylinder of radius r, length L, for $r < a$, $a < r < b$ and $r > b$.

$(r<a)$	$q_{in}(r) = \rho\pi r^2 L$
$(a<r<b)$	$q_{in}(r) = 0$
$(b<r)$	$q_{in}(r) = \rho\pi a^2 L$

Gauss (cylindrical symmetry) $\vec{E} = \frac{q_{in}(r)/L}{2\pi\epsilon_0 r}\,\hat{r}$

$\vec{E} = 0$ inside the conductor

The conductor is neutral, all the induced charge is contained within the volume and so only the non-conducting cylinder contributes

- 2b) (10 points) Find the electric field (vector) at all points inside and outside the distribution, as a function of distance from the longitudinal symmetry axis.

Gauss' Law. Cylindrical Symmetry. $\vec{E} = \frac{q_{in}(r)/L}{2\pi\epsilon_0 r}\,\hat{r}$

$$\boxed{\vec{E} = \begin{cases} \frac{\rho}{2\epsilon_0} r\,\hat{r} & (r<a) \\ 0 & (a<r<b) \\ \frac{\rho a^2}{2\epsilon_0 r}\,\hat{r} & (b<r) \end{cases}}$$

- 2c) (10 points) If the conductor is found to have a potential V_a with respect to some reference point, find the potential at all points inside and outside the distribution as a function of distance from the longitudinal symmetry axis

$$\Delta V(\vec{r}_{ref}, \vec{r}) = -\int_{r_{ref}}^{r} \vec{E}\cdot d\vec{r}$$

$$V(\vec{r}) = V(\vec{r}_{ref}) - \int_{r_{ref}}^{r} E_r\, dr \quad \text{(radial field)}$$

$(r \le a)$ $V(r) = V_a - \int_a^r \frac{\rho r}{2\epsilon_0}\,dr$

$V(r) = V_a - \frac{\rho}{4\epsilon_0}(r^2-a^2)$

$(a \le r \le b)$ $V(r) = V_a - \int_a^r 0\,dr$

$V(r) = V_a$

$(b \le r)$ $V(r) = V_a - \int_a^b 0\,dr - \int_b^r \frac{\rho a^2}{2\epsilon_0 r}\,dr$

$V(r) = V_a - \frac{\rho a^2}{2\epsilon_0}\ln(r/b)$

$$\boxed{V(r) = \begin{cases} V_a - \frac{\rho}{4\epsilon_0}(r^2-a^2) & r \le a \\ V_a & (a \le r \le b) \\ V_a - \frac{\rho a^2}{2\epsilon_0}\ln\left(\frac{r}{b}\right) & (b \le r) \end{cases}}$$

15

3) A charged particle of mass m_1 and charge q_1 is shot directly towards a charge of mass m_2 and charge q_2 from an infinite distance away. If $q_1 q_2 > 0$, the initial speed of m_1 (measured in the frame in which m_2 is initially at rest) is v_0 and m_2 is free to move...

- 3a) (5 points) Assuming the system is correctly defined, which mechanical quantities are conserved and why?

$\Rightarrow$ "Correctly defined" = The system consists of m_1 & M_2

$$\boxed{\begin{array}{ll} \Sigma \vec{F}_{ext} = 0 & \text{So linear Momentum is Conserved} \\ W_{NC} = 0 & \text{So Mechanical energy is Conserved} \end{array}}$$

- 3b) (5 points) How fast is each particle moving when they get as close to one another as they can get?

At that instant, $\vec{V}_1 = \vec{V}_2$ (Why? :))

$$\Sigma P_{xi} = \Sigma P_{xf}$$

$$m_1 V_0 = (M_1 + M_2) V_{1x} \Rightarrow V_{1x} = \frac{M_1}{M_1 + M_2} V_0$$

$$\boxed{\text{Both masses have a speed } V_1 = \frac{M_1}{M_1 + M_2} V_0}$$

a

- 3c) (20 points) How close will the particles get to one another?

$$\Delta E = W_{NC} \Rightarrow E_i = E_f$$

$$\frac{1}{2} m_1 V_0^2 = \frac{1}{2}(m_1 + M_2) V_1^2 + \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}} - \frac{G M_1 M_2}{r_{12}}$$

$$\frac{1}{2} \frac{M_1 M_2}{M_1 + M_2} V_0^2 = \left(\frac{q_1 q_2}{4\pi\epsilon_0} - G M_1 M_2\right) \frac{1}{r_{12}}$$

$$\boxed{r_{12} = \frac{2(m_1 + M_2)}{M_1 M_2 V_0^2} \left(\frac{q_1 q_2}{4\pi\epsilon_0} - G M_1 M_2\right)}$$

$G \lll \frac{1}{4\pi\epsilon_0}$, and in all likelihood the masses aren't that large... it's probably ok to neglect gravity in this case...

it's also fun to note the appearance of reduced mass $\left(\mu \equiv \frac{M_1 M_2}{M_1 + M_2}\right)$.

Note also that the distance of Closest approach is $\propto 1/V_0^2$ - does that seem reasonable?

$\rightsquigarrow$ What happens when $\frac{q_1 q_2}{4\pi\epsilon_0} \to G M_1 M_2$?? :)