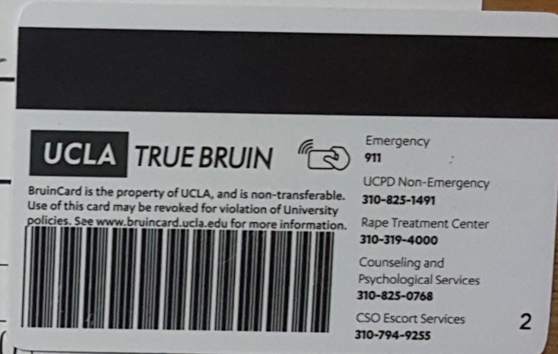
MT1 Physics 1B W16

Full Name (Printed) Sahil Gandhi

Full Name (Signature) SGandhi.

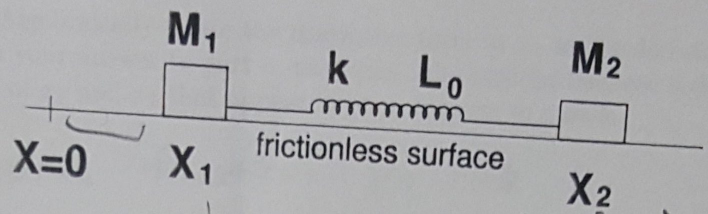
Student ID Number 704-

Seat Number



Problem	Grade
1	08/30
2	25 /30
3	21 /30
Total	54 /90

- Do not peek at the exam until you are told to begin. You will have approximately 50 minutes to complete the exam.
- Don't spend too much time on any one problem. Solve 'easy' problems first. Go for partial credit!
- **HINT:** Focus on the concepts involved in the problem, the tools to be used, and the set-up. If you get these right, all that's left is algebra.
- **Have Fun!**



2 ← 3 → 4

1) A pair of masses, M_1 and M_2 are joined by a spring of constant k and natural (unstretched) length L_0.

+5

- 1a) (5 points) Find the amount the spring is extended or compressed (Δx) as a function of the position of each mass (x_1 and x_2, respectively). Make sure Δx is positive when the spring is extended and negative when it's compressed.

$f(m_1) = x_1 - 0$

$f(m_2) = (x_2 - x_1)$

$x_1 - 0 =$ Position of M_1,

$x_2 - x_1 = L_0$

$(3-1) = L_0$

$(4-1) - 2 = \quad 3-2=1 \to L$

$(2-1) - 2 = \quad -1$

$\rightarrow \quad \boxed{\Delta x = (x_2 - x_1) - L_0}$

+0

- 1b) (5 points) Use Newton's laws to obtain differential equations (written in terms of x_1 and x_2) that describe the motion of each mass. Note that these are 'coupled' equations - they will each depend on both x_1 and x_2 - but not to worry, we'll address that later. Make sure the derivative term in each equation has the correct sign when the spring is extended and compressed!

$F_{net} = m a_{net} \rightarrow a = \frac{\partial^2 x}{\partial t^2}$

$F_{net} = -k \cdot \Delta x = k \cdot ((x_2 - x_1) - L_0)$

$-k\,(x_2 - x_1) - L_0) = (M_1 + M_2)\,\frac{\partial^2 x}{\partial t^2}$

$$\boxed{\underbrace{\frac{\partial^2 x}{\partial t^2}}_{a} + \left(\frac{k}{M_1 + M_2}\right)\underbrace{\left((x_2 - x_1) - L_0\right)}_{x} = 0}$$

+0

- 1c) (5 points) Algebraically relate the derivative term in x_1 to the derivative term in x_2. Use this result, along with your answer to part a, to obtain relationships between a derivative of Δx and each of the derivatives of x_1 and x_2 that appear in your answers to part b.

$x_1 =$ some function. $x_2 \rightarrow$

derivative of $\Delta x \rightarrow$

+1

- 1d) (10 points) Now it's time to put it all together - rewrite the differential equations from part b in terms of Δx. The results should look familiar. Find the solution for Δx as a function of time (make sure you evaluate, in terms of given information, any constants that are determined by the construction of the system).

Δx as function of time =

$$\boxed{\Delta x(t) = L_0 \cdot \cos\left(\sqrt{\frac{k}{M_1+M_2}} \cdot t\right)}$$

+2

- 1e) (5 points) Suppose we were to tie one end of the spring off to a wall, and the other end of the spring to a mass M. What value would M have to have in order for this new system to oscillate with the same period as the two-mass system we've been working on? This value, known as the *reduced mass*, is used to simplify the discussion of binary systems (diatomic molecules, for instance) in oscillation.

The reduced mass of the system is:

$$T = \frac{2\pi}{\omega} \Rightarrow \frac{2\pi}{\sqrt{\frac{k}{M_1+M_2}}} = \frac{2\pi}{\sqrt{\frac{k}{M}}} \rightarrow \boxed{M = M_1 + M_2}$$

2) Recall that the amplitude of a driven mass-spring system is given by

$$A(\Omega) = \frac{F_0/m}{\sqrt{(\omega_0^2 - \Omega^2)^2 + (\frac{b\Omega}{m})^2}}$$

- 2a) (10 points) Show that the following relationship holds true for a mass-spring system:

$$\omega_{damp}^2 = \frac{1}{2}\left(\Omega_{res}^2 + \omega_0^2\right)$$

$\omega^2_{damp} = \omega_0^2 - \left(\frac{b}{2m}\right)^2$

$\Omega_{res}^2 = \omega_0^2 - 2\left(\frac{b}{2m}\right)^2$ > plug into the equation above

$\omega_0^2 - \left(\frac{b}{2m}\right)^2 = \frac{1}{2}\left(\omega_0^2 - 2\left(\frac{b}{2m}\right)^2 + \omega_0^2\right)$

$\omega_0^2 - \left(\frac{b}{2m}\right)^2 = \frac{1}{2}\left(2\omega_0^2 - 2\left(\frac{b}{2m}\right)^2\right)$

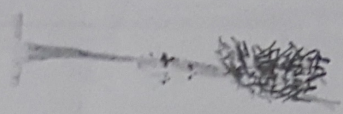
✓ $\boxed{\omega_0^2 - \left(\frac{b}{2m}\right)^2 = \omega_0^2 - \left(\frac{b}{2m}\right)^2 \rightarrow \therefore \omega^2_{damp} = \frac{1}{2}\left(\Omega_{res}^2 + \omega_0^2\right)}$

- 2b) (5 points) Convince the grader that the relationship in part a holds for all simple harmonic oscillators.

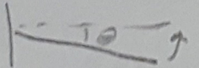
This relationship found in part a holds true for all simple harmonic osicillators because if they are not damped, then $b = 0$, so $\omega_0^2 = \frac{1}{2}(\omega_0^2 + \omega_0^2)$

$\rightarrow \omega_0^2 = \omega_0^2$

+3



- 2c) (10 points) On the way to class, you spot an abandoned bird's nest sitting in a low-hanging, more-or-less horizontal branch. You displace the end by some small amount and release it, and note that *a*) the tip makes about f_1 complete cycles every second, and *b*) it takes about N complete cycles for the amplitude of the branch's vibrations to drop to half its initial value. Find the natural frequency (f_0) for the branch/nest system. [*Careful.* These are <u>f's</u>, not ω's. <u>f's</u> are easier to observe directly.]



so N cycles to drop the amplitude to half, and f_1 complete cycles every ~~time~~ second

$\frac{1}{2}A = A_0 \cdot e^{-bt/2m}$ → time = $N \cdot T_{\text{complete cycles}}$ $T = 1/f_1$

$\frac{1}{2} = e^{-\frac{bN}{2mf_1}}$ $\omega = \sqrt{\omega_0^2 + \left(\frac{b}{2m}\right)^2}$ → $\left(\frac{2\pi}{f_1}\right)^2 - \left(\frac{f_1}{N} \cdot \ln\left(\frac{1}{2}\right)\right)^2 = \omega_0^2$

$-\frac{bN}{2mf} = \ln\left(\frac{1}{2}\right)$ → $\underbrace{\frac{b}{2m} = \frac{f_1}{N} \cdot \ln\left(\frac{1}{2}\right)}$ plugin $\omega_0 = \sqrt{\frac{4\pi^2}{f_1^2} - \left(\frac{f_1}{N}\left(\ln\frac{1}{2}\right)\right)^2}$

-9

$$\boxed{f_0 = \frac{2\pi}{\sqrt{\frac{4\pi^2}{f_1^2} - \frac{f_1}{N}\left(\ln\left(\frac{1}{2}\right)\right)^2}}}$$

$f_0 = \frac{2\pi}{\omega_0}$

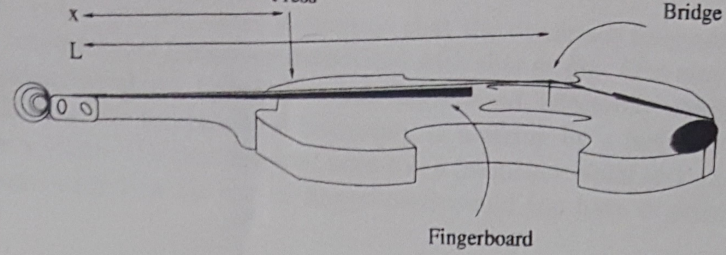
$f_0 = 2\pi\omega_0$

- 2d) (5 points) Suppose we want to knock that old nest out of the tree. What would be the most effective frequency to shake the branch at?

The most effective frequency to shake branch would be at resonant frequency. -3

The resonant frequency of this system is calculated in part c above →

$$\boxed{\omega_0 = \sqrt{\frac{4\pi^2}{f_1^2} - \frac{f_1^2}{N^2} \cdot \left(\ln\frac{1}{2}\right)^2}}$$

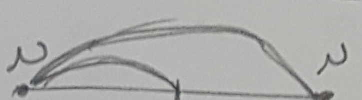


- 3a) (5 points) (Carefully) pluck a string on a violin. For a brief moment, the pluck generates noise - but that noise quickly gives way to music. Explain, in terms of physics, what is happening.

+1

When you pluck the violin string, you create traveling waves that upon reaching the other end, are flipped and returned. However, they are returned at a small phase difference. This occurs many times, and each phase difference corresponds to a slightly different frequency for that particular wave. The differences in these frequencies give way to beats which is the music we hear.

- 3b) (5 points) *Derive* the set of resonance frequencies for one of the open (that is, un-fingered) strings in terms of its effective length (L), the volume mass density of the material it is made of (ρ), the diameter of the string (D) and the tension in the string (T).



$$\text{mass} = \rho \cdot V = \rho \cdot \pi \cdot \left(\frac{D}{2}\right)^2 \cdot L$$

$$\mu = \frac{m}{L} = \frac{\rho \cdot \pi \cdot \left(\frac{D}{2}\right)^2 \cdot \cancel{L}}{\cancel{L}}$$

Like ends, so all n's

$$V_{string} = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{T}{\rho \cdot \pi \cdot \left(\frac{D}{2}\right)^2}}$$

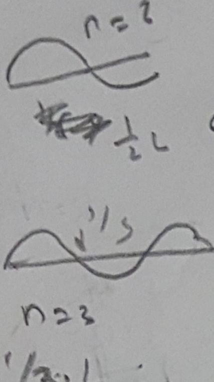
$$\lambda = \frac{2L}{n}$$

$$f_0 = \frac{V}{\lambda} = \frac{V}{\frac{2L}{n}} = \frac{nV}{2L} \rightarrow$$

+5

The set of resonance frequencies is given by $\dfrac{n \cdot \sqrt{\dfrac{T}{\rho \cdot \pi \cdot \left(\frac{D}{2}\right)^2}}}{2L}$ for $n = 1, 2, 3 \ldots$ all integer values.

• 3c) (5 points) It is not unreasonable to assume that the dominant frequency heard from an excited string will be the fundamental frequency associated with that string. One may change the fundamental frequency by pressing the string tightly into the fingerboard, effectively changing its length. Suppose you wanted to increase the fundamental frequency of a string by a factor F. Where (x) would you have to press? On a violin, the first fingered note has a frequency equal to 1.12 times the open-stringed frequency. Approximately how far up the fingerboard would you have to press to generate it?



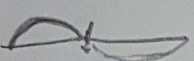
a) If you want to increase the fundamental by factor of F, need to f_0 V/λ → need to decrease λ by factor of F. To do that, $x = \frac{1}{F} \cdot L_0$ away from the edge.

b) To increase increase f_0 by a factor of 1.12, need to press it $x = \frac{1}{1.12} \cdot L_0$ away from the side with the curly strings (opposite of the bridge side).

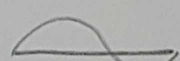
×3

* L_0 is the natural length of the string.

• 3d) (10 points) Another way to change the dominant frequency you hear is to **lightly** press the string in some magic spot. Since the string is not tightly pressed, it is still able to vibrate on either side of the finger - this has the effect of imposing an intermediate node on the system, emphasizing some harmonic over the fundamental. Find each of these magic spots (x) and the frequency associated with it (in terms of the fundamental).



These magic spots create every n^{th} multiple of the harmonic. Thus,

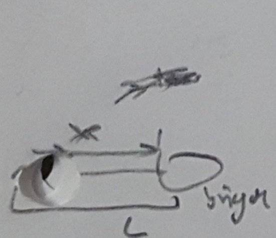


×10

n = 2 → magic spot is $\frac{1}{2}L$ away

$$\boxed{\text{The magic spots are } x = \tfrac{1}{n}L \text{ from the edge and create a frequency } f = \tfrac{n \cdot V}{2L} \text{ for every } n = 1, 2, 3 \ldots}$$

• 3e) (5 points) Bowing near the bridge can make a much nicer sound than bowing off towards the fingerboard. Explain in terms of physics why this is so.



Bowing near the bridge makes a nicer sound because the length from the ~~edge~~ finger board edge is much larger. When this length is larger, the frequencies produced are not as shrill and high as when the length is shortened by bowing closer to the finger board side.

×2