1BSUM20 Final Exam

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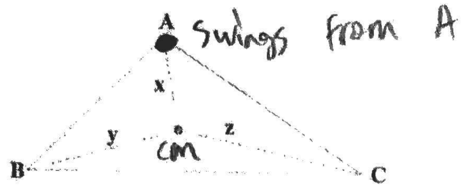
- The exam is open-book and open notes. You will probably do better to limit yourself to a single page of notes you prepared well in advance.
- **All work must be your own.** You are not allowed to collaborate with anyone else, you are not allowed to discuss the exam with anyone until all the exams have been submitted (after the close of the submissions window for the exam).
- **You have 150 minutes** to complete the exam and more than sufficient time to scan the exam and upload it to GradeScope. The exam *must* be uploaded to GradeScope within the time alloted (that is, by the end of the 3-hour finals slot). We will only except submissions through GradeScope and will not accept any exam submitted after the submission window closes (CAE students must contact Corbin for instructions).
- **Given the limits of GradeScope, you must fit your work for each part into the space provided.** You may work on scratch paper, but you will not be able to upload the work you do on scratch paper, so it is essential that you copy your complete solution onto the exam form for final submission. We can only consider the work you submit on your exam form.
- **For full credit the grader must be able to follow your solution from first principles to your final answer.** ***There is a valid penalty for confusing the grader.***
- It is YOUR responsibility to make sure the exam is scanned correctly and uploaded before the end of the submission window. The graders may refuse to grade pages that are significantly blurred, solutions to problems that are not written in the correct place, pages submitted in landscape mode and/or work that is otherwise illegible - if any of this occurs, you may not receive *any* credit for the affected parts.
- Focus on the concepts involved in the problem, the tools to be used, and the set-up. If you get these right, all that's left is algebra.
- **Have Fun!**

The following must be signed before you submit your exam:

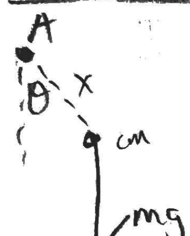
By my signature below, I hereby certify that all of the work on this exam was my own, that I did not collaborate with anyone else, nor did I discuss the exam with anyone while I was taking it.

Signature Kevin Wang



Exam 1) A non-uniform triangular sheet (of mass m) is found to balance at a point a distance x from vertex A, y from vertex B and z from vertex C as shown. It is mounted parallel to a frictionless vertical wall so that it is free to swing (in the vertical plane defined by the wall) around vertex A. When released at rest from a small angular displacement from equilibrium, it is found to oscillate with a period T_A.

- 1a) (15 points) Estimate the rotational inertia of the triangle around an axis perpendicular to the triangular sheet that passes through the triangle's center-of-mass. I_{cm}



$$\vec{\tau}_g = I\alpha \qquad I = I_{cm} + mx^2$$

$$-mgx\sin\theta = I\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} + \frac{mgx}{I}\sin\theta = 0$$

$$\omega^2 = \frac{mgx}{I} \qquad \omega = \sqrt{\frac{mgx}{I}} \qquad \frac{T_A^2}{4\pi^2} = \frac{I_{cm}+mx^2}{mgx}$$

$$T = \frac{2\pi}{\omega} \qquad T_A = 2\pi\sqrt{\frac{I}{mgx}}$$

$$T_A = 2\pi\sqrt{\frac{I_{cm}+mx^2}{mgx}} \qquad \boxed{I_{cm} = \frac{mgxT_A^2}{4\pi^2} - mx^2}$$

- 1b) (10 points) Having done a quick calculation on the side, you discover the natural frequency for oscillations about the A apex, T_{A0} differs slightly from the measured value of the period, T_A. How many complete oscillations will the triangle make before it loses half of its initial energy?

Damped Oscillations?

Beats

$$E_{total} = \tfrac{1}{2}kA^2$$

$$E_{total} = \tfrac{1}{2}kx^2 + \tfrac{1}{2}mv^2$$

$$T_{A0} < T_A$$

$$E = E_0 e^{-2\beta t}$$

$$\frac{E}{E_0} = e^{-2\beta t}$$

$$\ln\left(\tfrac{1}{2}\right) = -2\beta t$$

$$t = \frac{-\ln\left(\frac{1}{2}\right)}{2\beta}$$

$$\text{Oscillations} = \frac{\text{time}}{\text{Period}} = \frac{-\ln\left(\frac{1}{2}\right)}{2\beta T_A}$$

- **1c) (5 points)** **Evaluate your answer to part *b* in the limit that $T_A \to T_{A0}$ and discuss.**

$$\lim_{T_A \to T_{A0}} = \frac{-\ln(\frac{1}{2})}{2\beta T_A}$$

When T_A nears T_{A0} there won't be any energy loss because the natural frequency equals the oscillation frequency

Exam 2) Deep under water, submersible craft rely on sonar pings (loud blasts of sound) to maintain situational awareness. By measuring how long it takes for sound to return from an object they can deduce the distance to that object; by comparing the frequency of the sound when it returns to the frequency it had when it left, it can deduce the speed with which the object is moving.

Beats

For this problem, you may assume that everything is happening in one dimension (that is, objects move toward or away from each other along an axis defined by the direction that the pings are propagating) and you may assume that the speed of sound at this depth (and temperature) is given by v_{snd}.

In this problem, a submersible (A) on surveillance-duty is in pursuit of an intelligence-collection drone (B). The submersible pings its its surroundings with sound emitted at the frequency f_A.

- 2a) (15 points) To gauge its own speed, the submersible first pings a stationary object wedged in the ocean floor. If the pings return with a frequency f_S, how fast is the submersible traveling?

$V_s = V_{submarine}$

sub $\leftarrow$ $\frac{f_{obs}}{f_{src}(f_A)} = \frac{V_{snd}}{V_{snd} - V_s}$

$f_{obs} = \frac{V_{snd}}{V_{snd} - V_s} f_A$ — Frequency stationary object receives + emits

$$\frac{f_{obs}(f_s)}{\frac{V_{snd}}{V_{snd} - V_s} f_A} = \frac{V_{snd} + V_s}{V_{snd}}$$

$$f_s = \frac{V_{snd} + V_s}{V_{snd}} \left(\frac{V_{snd}}{V_{snd} - V_s} \right) f_A$$

$$f_s = \frac{V_{snd} + V_s}{V_{snd} - V_s} f_A$$

$$f_s V_{snd} - f_s V_s = f_A V_{snd} + f_A V_s$$

$$f_s V_{snd} - f_A V_{snd} = f_A V_s + f_s V_s$$

$$V_{snd}(f_s - f_A) = V_s(f_A + f_s)$$

$$\boxed{V_s = \frac{V_{snd}(f_s - f_A)}{(f_A + f_s)}}$$

- **2b) (15 points)** Next, the crew of the submersible pings the drone (B). If these pings return with a frequency f_B, how fast is the drone moving (in terms of f_A, f_S, f_B, and/or v_{snd})?

$$V_S = \frac{V_{snd}(f_S - f_A)}{(f_A + f_S)}$$

$$\frac{f_{obs}}{f_{src}(f_A)} = \frac{V_{snd} - V_d}{V_{snd} - V_S}$$

$$f_{obs} = \frac{V_{snd} - V_d}{V_{snd} - V_S} f_A$$ = Sound emitted by drone

$$\frac{f_{obs}(f_b)}{\frac{V_{snd} - V_d}{V_{snd} - V_S} f_A} = \frac{V_{snd} + V_S}{V_{snd} + V_d}$$

$$f_b = \frac{V_{snd} + V_S}{V_{snd} + V_d} \frac{V_{snd} - V_d}{V_{snd} - V_S} f_A$$

$$f_b V_{snd} + f_b V_d = \left(\frac{V_{snd} + V_S}{V_{snd} - V_S}\right) V_{snd} - V_d \, f_A$$

$$V_{snd} + \frac{V_{snd}(f_S - f_A)}{f_S + f_A} \qquad V_{snd} - \frac{V_{snd}(f_S - f_A)}{f_A + f_S}$$

$$\frac{V_{snd}(f_S + f_A)}{f_S + f_A} \qquad \frac{V_{snd}(f_S + f_A)}{f_S + f_A}$$

$$\frac{2 V_{snd} f_S}{f_S + f_A} \qquad \frac{2 V_{snd} f_A}{f_S + f_A}$$

$$\cancel{\frac{2 V_{snd} f_S}{f_S + f_a} \cdot \frac{f_S + f_a}{2 V_{snd} f_A}}$$

$$\frac{f_S}{f_A} = \frac{V_{snd} + V_S}{V_{snd} - V_S}$$

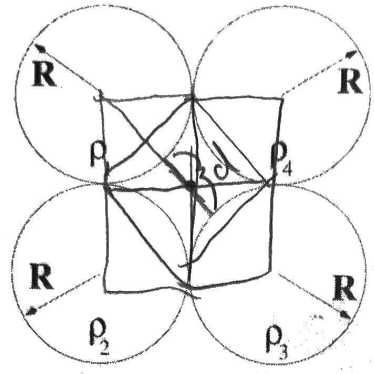
$$\frac{f_S}{\cancel{f_A}} (V_{snd} - V_d) \cancel{f_A}$$

$$f_B V_{snd} + f_B V_d = f_S V_{snd} - f_S V_d$$

$$f_B V_d + f_S V_d = f_S V_{snd} - f_B V_{snd}$$

$$V_d (f_B + f_S) = V_{snd}(f_S - f_B)$$

$$\boxed{V_d = \frac{V_{snd}(f_S - f_B)}{f_B + f_S}}$$



Exam 3) Four spheres of identical radius R (but different, uniform charge densities ρ_1, ρ_2 and ρ_3 and ρ_4) are arranged so that their centers all lay in a common plane, equidistant from a common central point. Each sphere touches two neighboring spheres as shown.

- **3a) (5 points)** How much work will be required to assemble each of these spheres in isolation (that is, before they are brought together in an assembly)?

$W_{ext} = q\Delta V$

$dW_{ext} = dq\, V(r)$

$q_{in}(r) = \int_0^r \rho\, dV = \rho \frac{4}{3}\pi r^3$

$W_{ext} = \int dq \frac{q_{in}(r)}{4\pi\varepsilon_0 r}$

$\frac{dq}{dr} = 4\rho\pi r^2$

$W_{ext} = \int_0^R \frac{\cancel{4\rho\pi r^2}\, \cancel{\rho \frac{4}{3}\pi r^3}}{4\pi\varepsilon_0 r} dr$

$dq = 4\rho\pi r^2 dr$

Replace ρ for $\rho_1, \rho_2, \rho_3, \rho_4$

$$\boxed{W_{ext} = \frac{4\rho^2 R^5 \pi}{15\varepsilon_0}}$$

$W_{ext} = \int_0^R \frac{\frac{4}{3}\rho^2 r^4 \pi}{\varepsilon_0} dr = \frac{\frac{4}{3}\rho^2\pi}{\varepsilon_0}\int_0^R r^4 dr \quad \left[\frac{1}{5}r^5\right]_0^R$

- **3b) (5 points)** Having already constructed the spheres, how much work will it take to assemble those spheres into the arrangement shown? [Again, assume the spheres themselves have already been assembled - that is, neglect the self-energy of each sphere.]

$$\boxed{\frac{4\rho^2 R^5 \pi}{15\varepsilon_0}}$$

$W_{ext} = q\Delta V$

$W_{ext,\ sphere1} = 0$

$W_{ext,\ sphere2} = \rho_2 V\left(\frac{\rho_1 V}{4\pi\varepsilon_0 2R}\right)$

$W_{ext,\ sphere3} = \rho_3 V\left(\frac{\rho_1 V}{4\pi\varepsilon_0 (2R+d)} + \frac{\rho_2 V}{4\pi\varepsilon_0 2R}\right)$

$W_{ext,\ sphere4} = \rho_4 V\left(\frac{\rho_1 V}{4\pi\varepsilon_0 2R} + \frac{\rho_2 V}{4\pi\varepsilon (2R+d)} + \frac{\rho_3 V}{4\pi\varepsilon_0 2R}\right)$

$W_{ext} = W_{ext,\ sphere2} + W_{ext,\ sphere3} + W_{ext,\ sphere4}$

- **3c) (10 points)** What is the electric field at the the center of the arrangement?

Efield = Efield due to each sphere added together

4 gaussian surfaces / $R = R + \frac{d}{2}$

$\hat{r}$

$$\text{Sphere 1} = \frac{p_1 V}{4\pi\varepsilon_0 (R+\frac{d}{2})^2}\hat{r} = \frac{p_1 V}{4\pi\varepsilon_0 (R+\frac{d}{2})^2}(\cos\theta\hat{x} - \sin\theta\hat{y})$$

$$\text{Sphere 2} = \frac{p_2 V}{4\pi\varepsilon_0 (R+\frac{d}{2})^2}(\cos\theta\hat{x} + \sin\theta\hat{y})$$

$$\text{Sphere 3} = \frac{p_3 V}{4\pi\varepsilon_0 (R+\frac{d}{2})^2}(-\cos\theta\hat{x} + \sin\theta\hat{y})$$

$$\text{Sphere 4} = \frac{p_4 V}{4\pi\varepsilon_0 (R+\frac{d}{2})^2}(\cancel{-}\cos\theta\hat{x} - \sin\theta\hat{y})$$

- **3d) (10 points)** How much work would one have to do to drag a test-charge q to the center of the arrangement from a point infinitely-distant?

$$W_{ext} = q\Delta V$$

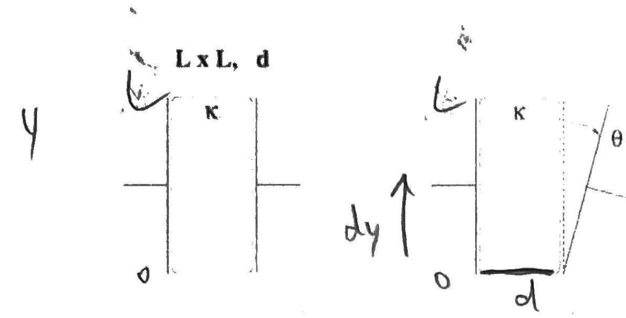
All spheres $\rightarrow$ One charged particle

$$dV = \int \frac{k\,dq}{(R+\frac{d}{2})}$$

$$V = \frac{k(Q)}{R+\frac{d}{2}} = \frac{k(p_1V + p_2V + p_3V + p_4V)}{(R+\frac{d}{2})}$$

$$W_{ext} = q\left(\frac{kV(p_1+p_2+p_3+p_4)}{(R+\frac{d}{2})}\right)$$

$$W_{ext} = \frac{qk(\frac{4}{3}\pi R^3(p_1+p_2+p_3+p_4)}{R+\frac{d}{2}}$$



Exam 4) A parallel-plate capacitor is formed by placing two square conducting plates (of area L^2) a distance d from one another. The region in-between the plates is completely filled with a dielectric of constant κ. At some point, one of the plates comes loose, and begins to tilt with a small angle θ.

- **4a) (10 points)** What is the capacitance of the new (tilted) arrangement?

$dA = L\,dy$

$$C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0\, dA}{d} = \frac{\varepsilon_0 L\, dy}{d + \theta y}$$ when θ is small

$$C = \int_0^L \frac{\varepsilon_0 L\, dy}{d + \theta y}$$

$u = d + \theta y$ $\quad \frac{du}{dy} = \theta$

$$C = \varepsilon_0 L \int_0^L \frac{1}{d + \theta y}\, dy$$

x is small

$\ln(1+x) \approx x - \frac{x^2}{2}$

$$C = \frac{\varepsilon_0 L}{\theta}\left[\ln\left(1 + \frac{y\theta}{d}\right)\right]_0^L = \frac{L\varepsilon_0}{\theta}\ln\left(1 + \frac{L\theta}{d}\right)$$

$$C = \frac{L\varepsilon_0}{\theta}\left(\frac{\theta L}{d} - \frac{\theta^2 L^2}{2d^2}\right)$$

$$C = L\varepsilon_0\left(\frac{L}{d} - \frac{\theta L^2}{2d^2}\right)$$

- **4b) (10 points)** If the capacitor is held at a constant charge Q, how much work would have to be done to put the plate back in its proper place? Is that work positive or negative? Explain.

+ −

The work is ngative because the negatively charged plates needs to be held back from moving toward the positively charged plate

Constant charge, changing C

$$W_{ext} = \Delta U = \frac{1}{2}Q^2\left(\frac{1}{C_f} - \frac{1}{C_i}\right) = \frac{1}{2}Q^2\left(\frac{d}{\varepsilon_0 L^2} - \frac{1}{L\varepsilon_0\left(\frac{L}{d} - \frac{\theta L^2}{2d^2}\right)}\right)$$

$$= -\frac{1}{2}Q^2\left(\frac{1}{L\varepsilon_0\left(\frac{L}{d} - \frac{\theta L^2}{2d^2}\right)} - \frac{d}{\varepsilon_0 L^2}\right)$$

- **4c) (5 points)** Check your answers to the first two parts by evaluating them in the limit $\theta \to 0$.

When $\theta \to 0$ $C = \frac{\varepsilon_0 L^2}{d}$

$$\lim_{\theta \to 0} C = L \varepsilon_0 \left(\frac{L}{d} - \frac{\theta L^2}{2d^2} \right) = L \varepsilon_0 \left(\frac{L}{d} \right) = \frac{L^2 \varepsilon_0}{d}$$

$$\downarrow$$

$$\frac{\varepsilon_0 A}{d}$$

- **4d) (5 points)** The capacitor is charged to Q, the plates are tilted as shown. Find the surface charge density on the plates as a function of height (vertical distance from the bottom of the plates, as drawn). σ

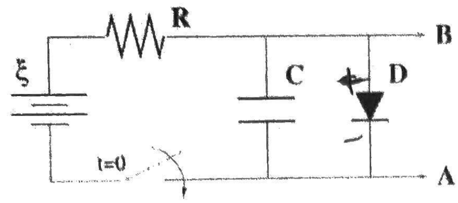
σ on plate $1 = \boxed{\frac{Q}{L^2}} = \sigma_1$

$d\sigma$ on plate $2 = \frac{Q}{dA}$ $\quad Q = ?$

$$d\sigma = \frac{Q}{dA}$$

$$\boxed{\sigma_2 = \int_0^L \frac{Q}{L\,dy}} \; ?$$

$Q = CV$

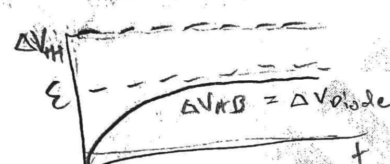


$V = \frac{Q}{C}$

Exam 5) In the circuit shown above, we know all the component values, the switch has been open a long time, and the capacitor is initially uncharged. The new component, D, is a semiconducting diode. If it is forward-biased (high-potential to low points in the direction of the arrow), it has no resistance when the potential difference across it exceeds the threshold voltage for the device (ΔV_{th}). When the potential difference across the device is less than the threshold voltage, it has infinite resistance.

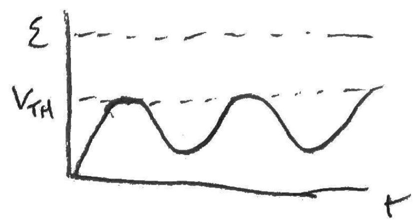
- **5a) (5 points)** How will the maximum potential difference across the capacitor compare to the potential difference across the battery? Suppose $\xi < \Delta V_{th}$. Find ΔV_{AB} as a function of time and plot it, qualitatively. Sketch ξ and ΔV_{th} on your plot.

Maximum potential across capacitor equals ε
The capacitor slowly builds up charge driven through the batteries potential until it is charge
$Q = C\varepsilon$



- **5b) (5 points)** Suppose $\xi > \Delta V_{th}$. Describe what happens now. Plot ΔV_{AB} (qualitatively) as a function of time. Sketch ξ and ΔV_{th} on your plot.

When potential difference across the capacitor reaches ΔV_{TH}, the capacitor isn't fully charged yet, However, because the diode has no resistance current prefers to flow through the diode instead of the capacitor. The capacitor discharges until ΔV_{cap} is lower than V_{TH} which causes the diode to become infinitely resistant and current to charge the capacitor once again.



- **5c) (10 points)** What is the period of the waveform you sketched in part *b*?

T= time for one full cycle = $\frac{2\pi}{w}$

find w ?

from $\Delta V_{cap} \rightarrow \Delta V_{TH}$

$$\mathcal{E} - IR - \frac{q}{C} = 0 \qquad \mathcal{E} - R\frac{dq}{dt} - \frac{q}{C} = 0$$

Try to convert to this form $\rightarrow$ $\frac{d^2q}{dt^2} + w^2 q = 0$

- **5d) (5 points)** Suppose you wanted the potential difference on the output across *A* and *B* to have a short period and a very linear rise. How would you choose your component values? Explain.

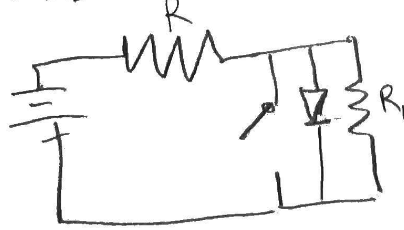
Linear rise means that $\mathcal{E}$ is much higher than ΔV_{TH} since if $\mathcal{E}$ is close to ΔV_{TH} the rise in ΔV_{cap} would level off. Therefore, choose a battery with high $\mathcal{E}$.

For a short period decrease R and C. With a lower time constant the capacitor more quickly approaches $\mathcal{E}$, and therefore ΔV_{TH}, and has a shorter period,

- **5e) (5 points)** In real life, some sort of load resistance will have to be connected across terminals *A* and *B*. If the circuit is to be used to generate the signal described in part *b*, how small can the load resistance be before the circuit fails to operate as desired?

Resistance should be large enough so current prefers to flow through the capacitor than the resistor

At $t \rightarrow 5\tau$



Capacitor is fully charged

$$\Delta V_{R_L} = \mathcal{E}\frac{R_L}{R+R_L}$$

$\Delta V_{R_L} \geq \Delta V_{TH}$ for wave to form

$$\Delta V_{TH} \leq \mathcal{E}\frac{R_L}{R+R_L} \qquad \frac{\Delta V_{TH}}{\mathcal{E}} \leq \frac{R_L}{R+R_L} \qquad \frac{R+R_L}{R_L} \leq \frac{\mathcal{E}}{\Delta V_{TH}}$$

$$R+R_L - \frac{\mathcal{E}}{V_{TH}}R_L \leq 0 \qquad R_L\left(1-\frac{\mathcal{E}}{V_{TH}}\right)+R \leq 0 \qquad \boxed{R_L \leq \frac{-R}{\left(1-\frac{\mathcal{E}}{V_{TH}}\right)}}$$