

## PHYS1B-1 Winter 2018 – 1st Midterm

Name: [REDACTED]

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Discussion session: 1D

- Length: 90 mins.
- Closed book.
- Simple calculators are allowed.
- A formula sheet is allowed.

Problem 1: 7 /10

Problem 2: 10 /10

Problem 3: 10 /10

Problem 4: 8.5 /10

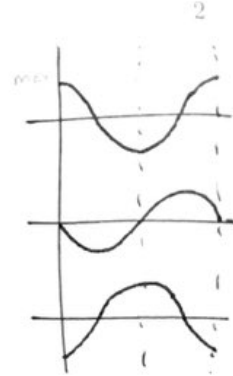
Problem 5: 10 /10

Total: 45.5 /50

### Problem 1

(a) What is true about the acceleration of a simple harmonic oscillation?

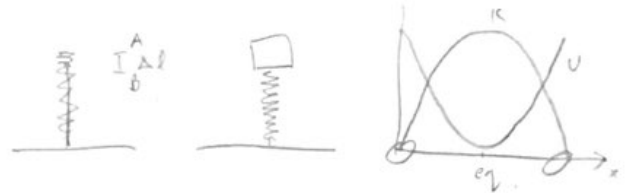
- A) The ~~acceleration~~ is a maximum when the displacement is a maximum.
- B) The ~~acceleration~~ is a maximum when the displacement is zero.
- C) The ~~acceleration~~ is a maximum when the speed is a maximum.
- D) The ~~acceleration~~ is zero when the object is instantaneously at rest.
- E) None of the above.



major A, prob E

(b) An object is attached to a vertical spring and bobs up and down between the two points A and B. When the kinetic energy is a minimum, the object is located:

- A) midway between A and B.
- B) 1/2 of the distance from A to B.
- C)  $1/\sqrt{2}$  times the distance from A to B.
- D) At either A or B.
- E) None of the above.



(c) A wave is traveling along a string. We can double the wave power by

- A) increasing the amplitude of the wave by a factor of 4.
- B) increasing the amplitude of the wave by a factor of 2.
- C) increasing the amplitude of the wave by a factor of  $\sqrt{2}$ .
- D) reducing the amplitude of the wave by a factor of 2.
- E) None of the above.

$$P = 2\sqrt{\mu F} \omega^2 A^2$$

$4\mu$   
 $4F$   
 $\sqrt{2}\omega$   
 $\sqrt{2}A$

(d) Consider the wave on a vibrating guitar string and the sound wave the guitar produces in the air. The string wave and the sound wave must have the same

- A) wavelength.
- B) frequency.
- C) velocity.
- D) amplitude.
- E) More than one of the above is true.

frequency

$$\lambda = \frac{v}{f}$$

(e) Observer A is a distance  $r$  away from a light bulb and observer B is  $4r$  away from the same bulb. If observer B sees a light intensity  $I$ , observer A will see a light intensity of:

- A)  $I$ .
- B)  $4I$ .
- C)  $16I$ .
- D)  $I/4$ .
- E)  $I/16$ .

$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$

$r_1$  A  
 $r_2$  B

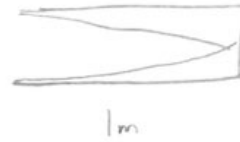
$$\frac{I_1}{I_2} = \frac{(4r)^2}{r^2} = 16$$

$$I_1 = 16I_2$$

(f) A stopped pipe (with one-end open) is 1 m long and has a fundamental frequency 10 Hz.

What is the sound wave speed in it?:

- A) 10 ms<sup>-1</sup>.
- B) 20 ms<sup>-1</sup>.
- C) 30 ms<sup>-1</sup>.
- D) 40 ms<sup>-1</sup>.
- E) Not enough information to compute.



$$L = \frac{\lambda}{4}$$

$$\lambda = 4L = 4 \text{ m}$$

$$v = 40$$

$$f_1 = 10 \text{ Hz}$$

(g) Which one of the following is true about the sound intensity level  $\beta$  and intensity  $I$ ?

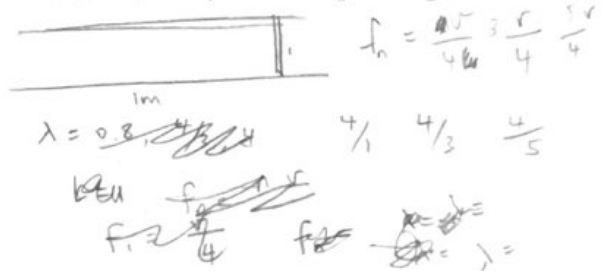
- A) Both of them obey inverse-square distance laws.
- B) Both of them can be negative.
- C) Both of them can never be negative.
- D)  $\beta$  obeys the inverse-square distance law but  $I$  does not.
- E)  $I$  can never be negative but  $\beta$  can be negative.

$$\beta = 10 \log\left(\frac{I}{I_0}\right)$$

$$= 10 [\log(I) - \log(I_0)]$$

(h) A 1 m long pipe can produce sound of wavelengths 0.8 m, 4/3 m, 4 m (no wavelengths longer than these). This pipe is

- A) both ends open.
- B) both ends closed.
- C) one end open.
- D) We cannot judge since the speed is unknown.
- E) None of the above.



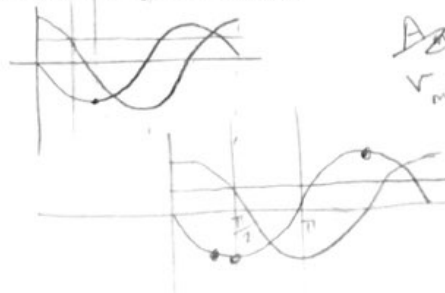
(i) Two pure tones are sounded together and a beat frequency  $f_{\text{beat}}$  is heard. What happens to  $f_{\text{beat}}$  if the frequency of one of the tones is increased?

- A) It increases.
- B) It decreases.
- C) It remains unchanged.
- D) It vanishes.
- E) Not enough information to judge.

$$f_{\text{beat}} = f_a - f_b$$

(j) A simple harmonic oscillator has a maximum amplitude  $A$  and a maximum speed of  $v$ . When the displacement is  $A/2$ , the speed becomes?

- A) ~~2v~~
- B) ~~v/2~~
- C)  ~~$\sqrt{3}v/2$~~
- D)  $\sqrt{2}v/3$
- E)  ~~$\sqrt{2}v$~~



$$v_{\text{max}} = \omega A$$

max speed is  $v$

$$v = -A\omega$$

$$x = 0.523$$

$$A = 1$$

$$v =$$

## Problem 2

A transverse string wave is traveling along the x-axis (towards +v.e. x), with speed  $v$ , amplitude  $A$  and wavelength  $\lambda$ . At  $x = t = 0$ , the displacement is upward, i.e.  $y(x = t = 0) = A$ . Express your answers in terms of  $v$ ,  $A$ ,  $\lambda$ . (a) What are the wave number  $k$  and angular frequency  $\omega$ ? (b) Write down the wave function  $y(x, t)$ . (c) What is the maximum magnitudes of transverse velocity and acceleration? (d) When  $|y| = A/3$ , what is the transverse acceleration magnitude? (e) What is the conditions for  $x$  and  $t$  at which  $y(x, t) = A$ ? (f) If the wave reverses its propagation direction, which of the above answers (a-e) remain(s) unchanged? (g) If the initial condition is changed to  $y(x = t = 0) = 0$  instead, which of the above answers (a-e) remain(s) unchanged?

## Problem 3

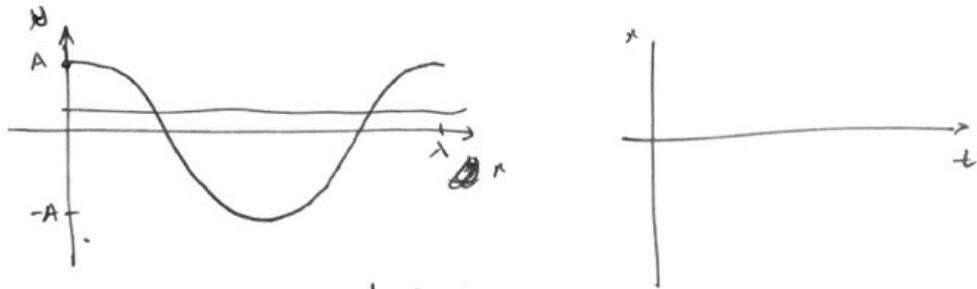
A simple harmonic oscillator is characterized by mass  $m$ , spring constant  $k$  and amplitude  $A$ . Suppose we have an initial displacement  $y(t = 0) = A$ . (a) Write down the expressions for the kinetic energy  $E_{KE}(t)$  and potential energy  $E_{PE}(t)$ . Plot them as a function of time. (b) At  $t = t_0$ ,  $E_{KE}(t_0) = E_{PE}(t_0)$ . Find the smallest  $t_0$ . What is corresponding displacement magnitude? (c) When  $y(t) = A/2$ , what is the ratio of  $E_{KE}(t)$  to  $E_{PE}(t)$ ?

## Problem 4

(a) Four identical sound sources are placed along the x-axis at  $x = 0, x_0, 2x_0, 3x_0$  and each of them produces unidirectional sound with amplitude  $A$  and wavelength  $\lambda$ . What is the net wave amplitude if the separation (i)  $x_0 = 2\lambda$ , (ii)  $x_0 = \lambda$ , (iii)  $x_0 = \lambda/2$ , (iv)  $x_0 = \lambda/4$ ?  
(b) Now remove the sound source at  $x = 3x_0$ . What is the net wave amplitude if (i)  $x_0 = 2\lambda$ , (ii)  $x_0 = \lambda$ , (iii)  $x_0 = \lambda/2$ , (iv)  $x_0 = \lambda/4$ ?

## Problem 5

You are driving at velocity  $v_{me} = v/5$ , where  $v$  is the sound speed. A police car is approaching you from behind and you hear a siren frequency  $f_1$ . You are then relieved as the police car continues past you, after which you hear another frequency  $f_2 = f_1/2$ . Assuming that all velocities are constant. (a) What is the speed of the police car  $v_p$  (in terms of  $v$ )? (b) What is the siren frequency  $f_p$  heard by the police (in terms of  $f_1$ )?



travels w/ speed  $v$ ,  $A$ ,  $\lambda$

a.  ~~$y(x,t) = A \cos(kx - \omega t)$~~  in terms of  $v, A, \lambda$

$$k = \frac{2\pi}{\lambda}$$

$$\omega = vk = \frac{v \cdot 2\pi}{\lambda}$$

b.  $y(x,t) = A \cos(kx - \omega t)$   
 $= A \cos\left(\frac{2\pi}{\lambda}x - \frac{2\pi v}{\lambda}t\right)$

c. maximum transverse velocity:  
 $v_y(x,t) = -\omega A \sin(kx - \omega t)$   
 $v_{y,max} = \omega A = \frac{2\pi A v}{\lambda}$

maximum transverse acceleration:  
 $a_y(x,t) = \omega^2 A \cos(kx - \omega t)$   
 $a_{y,max} = \omega^2 A = \left(\frac{2\pi v}{\lambda}\right)^2 A$

d.  $|y| = \frac{A}{3}$ :  
 ~~$\frac{A}{3} = \omega^2 A \cos(kx - \omega t)$~~   $\frac{A}{3} = A \cos(kx - \omega t)$   
 ~~$\frac{1}{3\omega^2} = \cos(kx - \omega t)$~~   $\frac{1}{3} = \cos(kx - \omega t)$

$$a_y(x,t) = \omega^2 A \cos(kx - \omega t)$$

$$a_y = \omega^2 A \cdot \frac{1}{3}$$

$$= \frac{1}{3} \left(\frac{2\pi v}{\lambda}\right)^2 \cdot A = \frac{4\pi^2}{3} \left(\frac{v^2}{\lambda^2}\right) A$$

e.  $y(x,t) = A$   
 when:  $\cos(kx - \omega t) = 1$   
 $kx - \omega t = 0, 2\pi, 4\pi \dots$   
 $\left(\frac{2\pi}{\lambda}\right)x - \left(\frac{2\pi v}{\lambda}\right)t = 0, 2\pi, 4\pi \dots = 2n\pi$  for  $n=0, 1, 2, \dots$

f. unchanged: a, c, d, /

g. unchanged: a, c, d, /



$$y(t=0) = A$$

a.  $E_{KE} = \frac{1}{2} m v(t)^2$        $E_{PE} = \frac{1}{2} k y(t)^2$

since we know  $m, k, A$ :

since  $y(t=0) = A$ :  
 $\phi = 0$

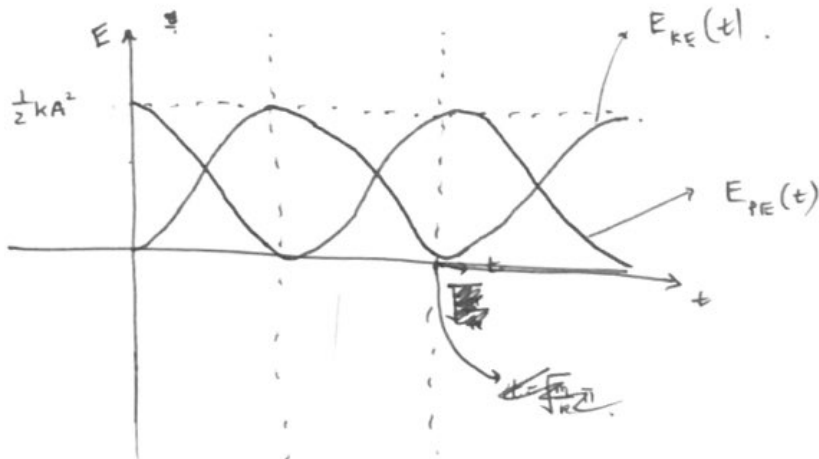
~~$x(t) = A \cos(\omega t)$~~        $\omega = \sqrt{\frac{k}{m}}$

$$y(t) = A \cos\left(\sqrt{\frac{k}{m}} t\right) \quad v(t) = -A \sqrt{\frac{k}{m}} \sin\left(\sqrt{\frac{k}{m}} t\right)$$

$$E_{KE}(t) = \frac{1}{2} m A^2 \frac{k}{m} \sin^2\left(\sqrt{\frac{k}{m}} t\right)$$

$$= \left(\frac{1}{2} k A^2\right) \sin^2\left(\sqrt{\frac{k}{m}} t\right)$$

$$E_{PE}(t) = \left(\frac{1}{2} k A^2\right) \cos^2\left(\sqrt{\frac{k}{m}} t\right)$$



b.  $t = t_0$  when plots intersect.       $t = \sqrt{\frac{m}{k}} \pi$

$$E_{KE}(t) = E_{PE}(t)$$

$$\left(\frac{1}{2} k A^2\right) \sin^2\left(\sqrt{\frac{k}{m}} t\right) = \left(\frac{1}{2} k A^2\right) \cos^2\left(\sqrt{\frac{k}{m}} t\right)$$

$$\sin\left(\sqrt{\frac{k}{m}} t\right) = \cos\left(\sqrt{\frac{k}{m}} t\right)$$

$$\tan\left(\sqrt{\frac{k}{m}} t\right) = 1 \implies t_0 = \sqrt{\frac{m}{k}} \arctan(1)$$

$$= \frac{\pi}{4} \sqrt{\frac{m}{k}}$$

c.  $y(t_0) = A \cos\left(\sqrt{\frac{k}{m}} \cdot \sqrt{\frac{m}{k}} \frac{\pi}{4}\right)$

$$= A \cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2} A$$

c.  $y(t) = \frac{A}{2} = A \cos\left(\sqrt{\frac{k}{m}} t\right)$ .

$$\frac{1}{2} = \cos\left(\sqrt{\frac{k}{m}} t\right)$$

$$\frac{\pi}{3} = \sqrt{\frac{k}{m}} t$$

$$\frac{\pi}{3} \sqrt{\frac{m}{k}} = t$$

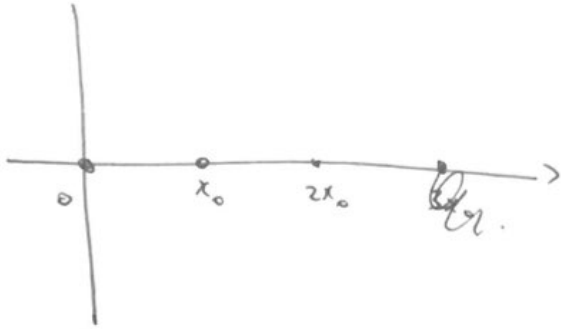
$$\begin{aligned} \frac{E_{KE}(t)}{E_{PE}(t)} &= \frac{\left(\frac{1}{2} k A^2\right) \sin^2\left(\sqrt{\frac{k}{m}} \sqrt{\frac{m}{k}} \frac{\pi}{3}\right)}{\left(\frac{1}{2} k A^2\right) \cos^2\left(\sqrt{\frac{k}{m}} \sqrt{\frac{m}{k}} \frac{\pi}{3}\right)} \\ &= \frac{\sin^2\left(\frac{\pi}{3}\right)}{\cos^2\left(\frac{\pi}{3}\right)} \end{aligned}$$

$$= \underline{\underline{3}}$$

3



4 .



8-5

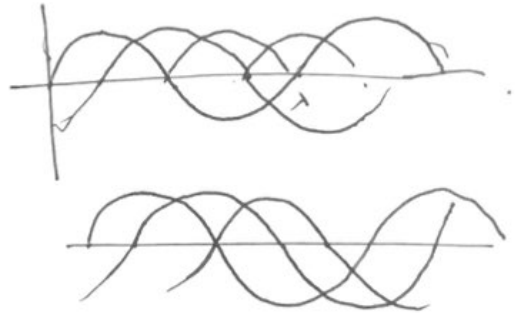
a. i. add to amplitude.

4A

ii. ~~4A~~ 4A

iii. 0

iv. 0

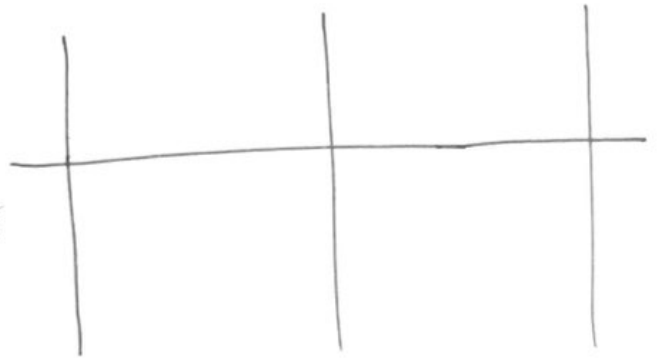


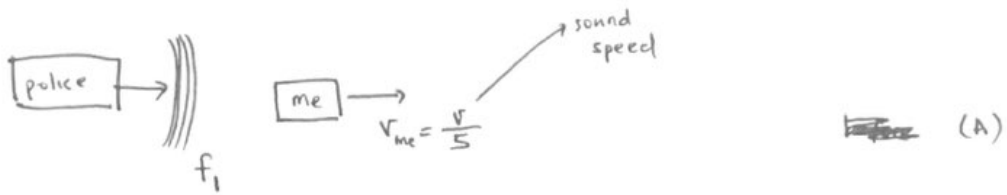
b. i. 3A

ii. 3A

iii. A

iv. ~~2A~~ 2A x -1.5





a. in (A):

$$f_L = \frac{v + v_L}{v + v_S} f_s$$

$$f_1 = \frac{v - \frac{v}{5}}{v - v_p} f_p \Rightarrow \boxed{f_1 = \frac{v - \frac{v}{5}}{v - v_p} f_p}$$

in (B):

$$f_L = \frac{v + v_{me}}{v + v_p} f_p = \frac{f_1}{2}$$

~~isolate~~  $\frac{v}{5}$

$$\Rightarrow \boxed{\frac{v + \frac{v}{5}}{v + v_p} f_p = \frac{f_1}{2}}$$

isolate  $v_p$ : using 2 eq.

$$\frac{f_1}{\left(\frac{f_1}{2}\right)} = \frac{\frac{v - \frac{v}{5}}{v - v_p}}{\frac{v + \frac{v}{5}}{v + v_p}} = \frac{v - \frac{v}{5}}{v - v_p} \cdot \frac{v + v_p}{v + \frac{v}{5}}$$

$$2 = \frac{4v}{5(v - v_p)} \cdot \frac{5(v + v_p)}{6v}$$

$$2 = \frac{20(v + v_p)}{30(v - v_p)}$$

$$3(v - v_p) = v + v_p$$

$$3v - 3v_p = v + v_p$$

$$2v = 4v_p \Rightarrow \boxed{v_p = \frac{1}{2}v}$$

b. isolate  $f_p$ :

$$f_1 = \frac{4v}{5} \cdot \frac{2}{\frac{v}{2}} f_p = \frac{4v}{5} \cdot \frac{2}{\frac{v}{2}} f_p = \frac{8}{5} \cdot f_p$$

$$\boxed{f_p = \frac{5f_1}{8}}$$