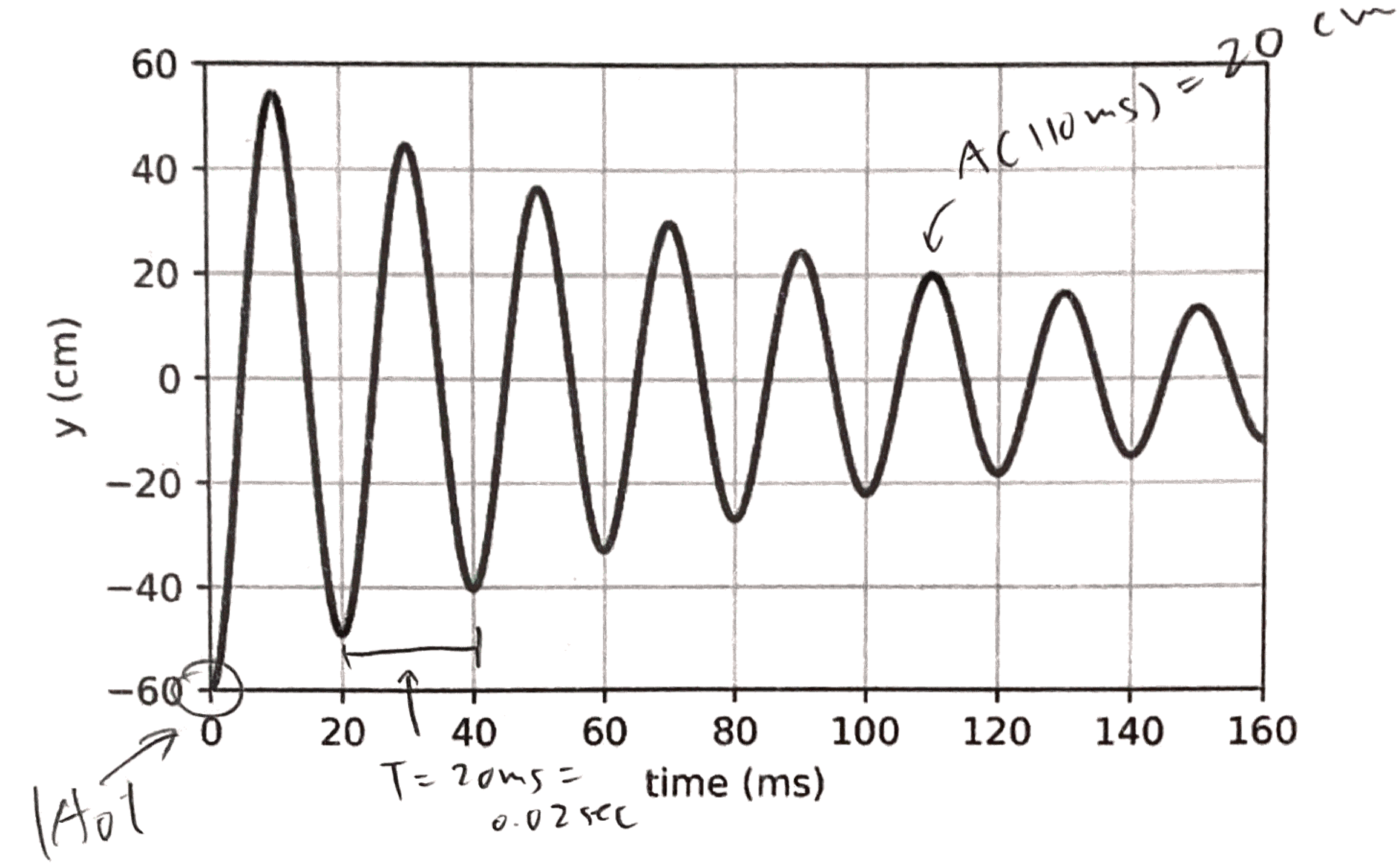
1B-1 Fall 2020: Quiz 2B

Show all your work and use proper units throughout. This quiz is open-book but not open-Chegg and must be completed without help. Please write your answers into the boxes. If you submit your work with your own formatting please try to submit one problem per page.

Consider the damped oscillation as shown in the figure below:



a) Estimate the initial amplitude, the period, and the decay constant α from the plot. *[5 points]*

$A_0 = 60$ cm $= 0.6$ m

$T = 20$ ms $= 0.02$ sec

$A(t) = A_0 e^{-\alpha t}$

$A(110 \text{ ms}) = 20$ cm

$0.6 e^{-\alpha \cdot 0.11} = 0.2$

$e^{-\alpha \cdot 0.11} = \frac{1}{3}$

$-\alpha \cdot 0.11 = \ln(\frac{1}{3})$ * ln is natural logarithm

$\alpha = \frac{-\ln(\frac{1}{3})}{0.11}$

$= 9.99 \text{ s}^{-1}$

b) Write down the mathematic formula describing this oscillation and substitute the proper numbers and units for all relevant variables *[5 points]*

$Y = A_0 e^{-\alpha t} \cos(\omega' t + \phi)$ $\omega' = \frac{2\pi}{T} = 100\pi = 314.16\ sec^{-1}$

$Y = (0.6\ m)\, e^{-(9.99\ s^{-1})(t\ sec)} \cos((314.16\ s^{-1})\cdot(t\ sec) + \pi)$

c) Using your decay constant from a) calculate the amplitude at t=250 ms. *[4 points]*

$A(t) = A_o e^{-\alpha t}$

$A(0.25) = 0.6 e^{-9.99(0.25)} = 0.0494 \text{ m}$

$= \boxed{4.94 \text{ cm}}$

d) What damping regime is this oscillation in and what decay constant would be required so that this oscillation becomes critically damped? Explain your reasoning in your own words and using the relevant equations. *[2 points]*

This is underdamped, as it is easy to observe a sinusoidal oscillation like in SHO, except with exponentially decaying amplitude such that $\lim_{t \to \infty} A = 0$.

For critical damping, we need $\omega' = 0$, so $\sqrt{\omega_0^2 - \alpha^2} = 0$,
so $\omega_0 = \alpha$.

To find ω_0, use the formula and the fact we know ω' and α:

$$\omega' = \sqrt{\omega_0^2 - \alpha^2}$$

$$100\pi = \sqrt{\omega_0^2 - 9.99^2}$$

$$(100\pi)^2 = \omega_0^2 - 9.99^2$$

$$\omega_0 = \sqrt{(100\pi)^2 + 9.99^2}$$

$$= 314.32 \text{ s}^{-1}$$

Therefore, for critical damping, $\alpha = \omega_0 = \boxed{314.32 \text{ s}^{-1}}$.

Then, we would have $\omega' = \sqrt{314.32^2 - 314.32^2} \Rightarrow \omega' = 0$ and a function of the form $x = Ae^{-\alpha t}$.