1B-1 Fall 2020: FINAL

Show all your work and use proper units throughout. This final is open-book but not open-Chegg and must be completed without help. Please write your answers into the boxes. If you submit your work with your own formatting please try to submit the same number of pages as the template. You have 24 hours to submit your answers.

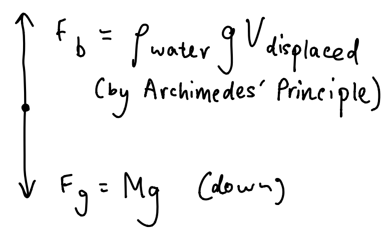
1. Please complete part A of this final on KUDU!

2. A boat floats on a lake and carries a large boulder. When you throw the boulder overboard into the water so that it sinks, will the water level of the lake sink, rise, or stay the same?
EXPLAIN your answer in detail using the appropriate equations. *[10 points]*

The water level sinks.

We're interested in using Archidemes Principle to compare the volume displaced by the 1) boat with the rock in it and 2) the boat and rock separately.

Let the boat have mass m_b, and rock have mass m_r and density ρ_{rock}. Consider the FBD of the boat:



The boat is in equilibrium, so $|F_b| = |F_g|$, or

$$\rho_{water} V_{displaced} = M$$

1) If the rock is in the boat, $M = m_r + m_b$. Then,

$$V_{displaced} = \frac{m_r + m_b}{\rho_{water}}. \quad (V_1)$$

2) Once we drop the rock in, $M = m_b$. So, the volume displaced by the boat decreases to $V_{boat} = \frac{m_b}{\rho_{water}}$. However, the rock also displaces some water equal to its volume, as it's fully submerged. So, $V_{Rock} = \frac{M_r}{\rho_{rock}}$.

Then, $V_{displaced} = \frac{m_b}{\rho_{water}} + \frac{m_r}{\rho_{rock}}. \quad (V_2)$

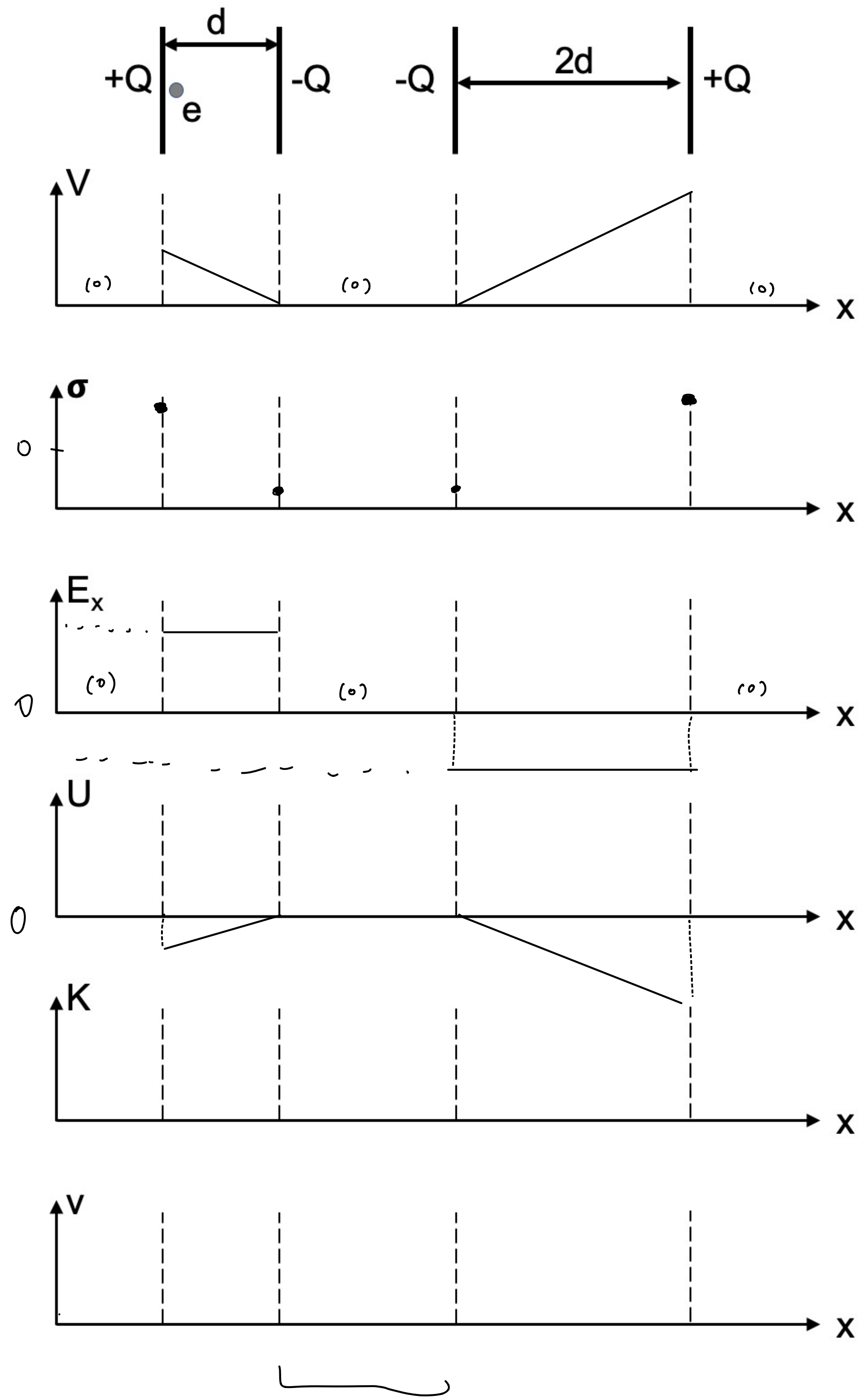
We can subtract $V_1 - V_2$ to get that the difference in displaced water is

$$\frac{m_r}{\rho_{water}} - \frac{m_r}{\rho_{rock}}$$

The rock sank, so $\rho_{rock} > \rho_{water}$. Then, $\frac{m_r}{\rho_{water}} > \frac{m_r}{\rho_{rock}}$,

so $V_1 - V_2$ is positive, and it follows that less water was displaced after throwing the rock off — the water level sinks.

3. Consider two infinite parallel plate capacitors as shown. Sketch the variation of the following parameters as a function of position x inside and outside the capacitors: (a) electric potential, (b) charge density, (c) electric field E_x, (d) electric potential energy of an electron, (e) kinetic energy of the electron, (f) velocity of the electron. Assume that the electron starts from rest at a point of your choice and moves only in the horizontal direction. It can pass through the plates without losing energy. For partial credit write down the appropriate equations instead. *[10 points]*



$U = qV$

$q < 0$

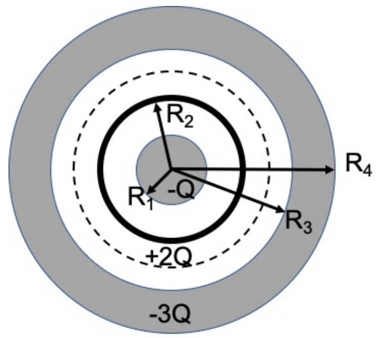
$KE = \frac{1}{2}mv^2$

KE + U stays constant for a given electrons

Since E is constant, F and thus a are also constant.

Also for this region (↑), E is constant and thus will never have any velocity.

4. Consider the spherically symmetric charge configuration shown below. A conducting sphere of radius R_1 and charge -Q is surrounded by air, which is surrounded by a conducting thin shell of radius R_2 and charge +2Q. The shell is surrounded by air and a conducting solid spherical shell of inner radius R_3, outer radius R_4 and charge -3Q. *[10 points]*



a. What charges reside on the inner and the outer surfaces of the outer shell (R_3 to R_4)? *[2 points]*

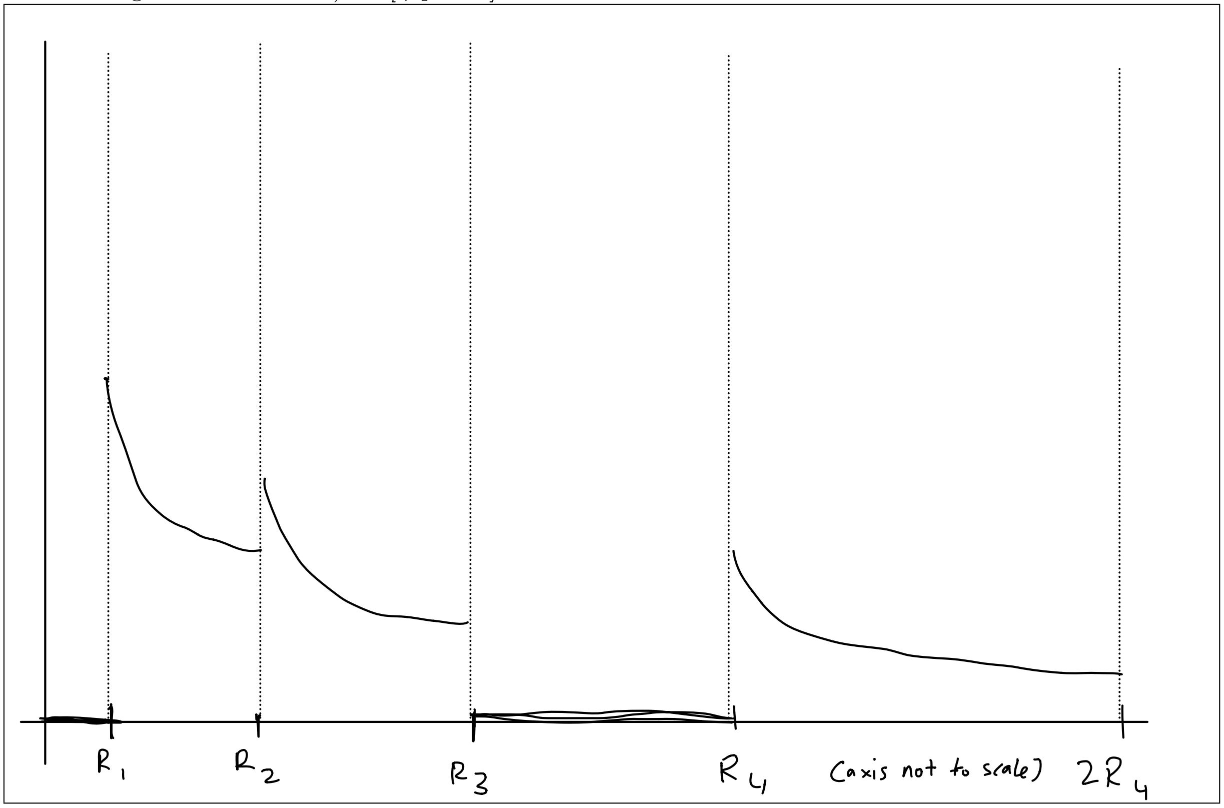
We'll use the fact that electric field inside a conductor is 0. So, any Gaussian spherical surface with $R_3 \le r \le R_4$ must necessarily enclose 0 charge. So, the inner surface of the sphere must have charge $-Q$.

There is no charge in the interior of the outer conductor, so the remaining charge must be on the outer surface. Thus the outer surface has charge $-2Q$.

b. What is the electric flux through the dashed Gaussian surface shown? *[2 points]*

$$\Phi = \frac{Q_{encl}}{\varepsilon_0} = \frac{Q}{\varepsilon_0} \ V \cdot m$$

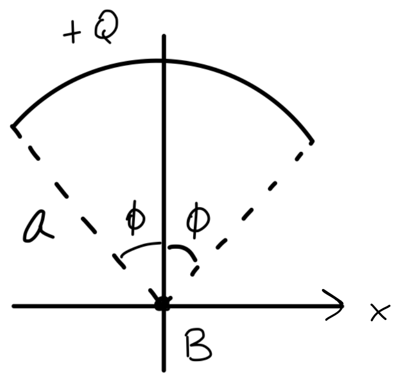
c. Qualitatively correct, sketch the magnitude of the electric field as a function of radius everywhere, that is in the region $0 < r < 2R_4$)   *[4 points]*



d. What charge resides on the outermost surface (R_4) after you ground the outer conductor?   *[2 points]*

If we ground the outermost surface, there will be 0 charge left on the outermost surface.

5. A uniformly charged line of charge Q is shaped into a circular segment with radius a that subtends and angle of 2Φ at the center of the circle. Find the electric field at point B and show all your work. *[10 points]*



We want to sum up $d\vec{E}$'s contributed by the circle segment. Note that all contributions in the x-direction is cancelled by the reflecting piece across the y-axis. To get the correct component of $d\vec{E}$, we just multiply by $\cos\theta$. So, we can just set up the following integral

$$d\vec{E} = \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{a^2}\right)\cos\theta\, dq \implies E = \int_{\theta=-\phi}^{\phi} \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{a^2}\right)(\cos\theta)\, dq .$$

Note that $dq = \frac{Q}{L}\, d\ell = \frac{Q}{2\phi a}\, d\ell$, and $d\ell = d\theta \cdot a$, so $dq = \frac{Q}{2\phi}\, d\theta$

$$\implies E = \int_{\theta=-\phi}^{\phi} \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{a^2}\right)(\cos\theta)\frac{Q}{2\phi}\, d\theta$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{a^2}\right)\frac{Q}{2\phi}\int_{\theta=-\phi}^{\phi} \cos\theta\, d\theta$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{a^2}\right)\frac{Q}{2\phi}(2\sin\phi)$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{1}{a^2}\right)\frac{Q}{\phi}\sin\phi \ \text{N/C}$$ in the downward y direction