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Problem 1

(a) A positive charge q_1 and a negative charge $-q_2$ ($\neq -q_1$) are placed along the x-axis separated by a distance. A third charge q_3 is to be placed along the x-axis in order that there is no net electric force on it.

A) There is only one point to place q_3.
B) There are two points to place q_3.
C) It is impossible.
D) It depends on the sign of q_3.
E) None of the above.

(b) A positive charge Q is fixed at the origin. A second positive charge q is released from rest near Q and is free to move. Which of the following about q is true?

A) Its speed will be greatest just after it is released.
B) Its acceleration is zero just after it is released.
C) As it moves farther from Q, its acceleration will keep increasing.
D) As it moves farther from Q, its speed will keep decreasing.
E) As it moves farther from Q, its speed will keep increasing.

(c) A sphere of radius R carries a charge Q distributed uniformly throughout its volume. At a distance d from the center, the electric field reach a value equal to half of its maximum. Which of the following is true?

A) $d < R$.
B) $d = R$.
C) $d > R$.
D) There are two solutions. One with $d < R$ and one with $d > R$.
E) None of the above.

(d) Under electrostatic conditions, the electric field just outside the surface of a conductor

A) is always zero.
B) is always parallel to the conducting surface.
C) is always perpendicular to the conducting surface.
D) is perpendicular to the surface only if the surface is flat.
E) can be either parallel or perpendicular to the surface depending on the surface charge.

(e) A nonconducting sphere is uniformly charged. Which statement about the potential magnitude V is true? The reference is set to infinity.

A) V is highest at the center of the sphere.
B) V is highest at the surface of the sphere.
C) V at the center of the sphere is zero.
D) V at the center of the sphere is the same as the V at the surface.
E) V at the surface is higher than the V at the center.

(f) A negative charge is moved from point A to B along an equipotential surface.

A) The negative charge performs work in moving from point A to B.
B) Work is required to move the charge from point A to B.
C) No work is required for the move.
D) The work done on the charge depends on the path of motion.
E) The work done on the charge depends on the distance between A and B.

(g) A parallel plate capacitor with charge Q is connected to a battery. The parallel plates are pulled apart such that the separation is doubled. The capacitor now carries a charge of

A) $4Q$.
B) $2Q$.
C) Q.
D) $Q/2$.
E) $Q/4$.

$$\frac{Q}{V} = \frac{\varepsilon_0 A}{d}$$

(h) A charged parallel-plate capacitor has round plates and an energy density u_0. All geometric parameters of the capacitor (plate diameter and separation) are doubled. The energy density becomes:

A) $16u_0$.
B) $4u_0$.
C) u_0.
D) $u_0/4$.
E) $u_0/16$.

$$u = \frac{\frac{1}{2}\frac{\varepsilon_0 A}{d}V^2}{Ad} = \frac{1}{2}\frac{\varepsilon_0 V^2}{d^2}$$

(i) Which of the following statement about Gauss's law is true?

A) Gauss's law is valid only for symmetric charge distributions such as spheres and cylinders.
B) If there is no charge inside a Gauss surface, the electric field must be zero on that surface.
C) Only charge enclosed by a Gauss surface can produce an electric field on that surface.
D) For a Gauss surface inside a conductor, the electric field must be zero at all points on that surface.
E) Electric field through a Gauss surface depends only on the charge enclosed, not on the surface shape.

(j) A conductor carries a charge of $-2\ C$ and has a hollow cavity inside. A positive charge of $1\ C$ is placed inside the cavity. Which statement is true about the charge on the inner (q_{in}) and outer (q_{out}) surfaces of the conductor?

A) $q_{in} = 0$ C and $q_{out} = -2$ C.
B) $q_{in} = -2$ C and $q_{out} = 0$ C.
C) $q_{in} = -2$ C and $q_{out} = +2$ C.
D) $q_{in} = -1$ C and $q_{out} = +1$ C.
E) $q_{in} = -1$ C and $q_{out} = -1$ C.

Problem 2

(a) A nonconducting line is bent into a semicircular arc (characterized by θ) with radius R. A charge Q is distributed uniformly on it. We set the reference point to infinity. Find the potential at the center when (i) $\theta = 2\pi$ (i.e. a circle), (ii) $\theta = \pi$ (i.e. the arc of a half circle), (iii) $\theta = \pi/3$.
(b) We now bend it into a circle ($\theta = 2\pi$) and charge it with different nonuniform densities (i) $\lambda \cos\phi$, (ii) $\lambda \cos^2\phi$, (iii) $\lambda \cos^3\phi$. Find the corresponding potentials at the center.

Problem 3

(a) A capacitor (of capacitance C_1) is charged by a battery (of potential V_0). We remove the battery and connect C_1 with another uncharged capacitor (C_2). Calculate the charges on C_1 and C_2, respectively.
(b) We now charge a capacitor (C_1) by a battery (V_1) and another capacitor (C_2) by another battery (V_2). After removing the batteries, the two capacitors are connected such that terminals with the same charges join. Calculate the charges on C_1 and C_2, respectively. What are the results when $V_1 = V_2 = V$?

Problem 4

A large solid slab with thickness d is centered at the origin and parallel to the yz plane. It occupies the region $-d/2 \leq x \leq d/2$ and has a uniform charge density ρ. (a) Evaluate the electric field at $x = 0$, $x = d$ and $x = 2d$. (b) Plot $E_x(x)$ along the x-axis. (c) The reference point is set to the origin such that $V(x = 0) = 0$. Plot $|V(x \geq 0)|$. (d) Another identically charged slab is added and centered at $x = d$. Plot $E_x(x)$. (Remember to label all axes in the plots.)

Problem 5

On the x-axis, a solid sphere 1 of radius R is centered at $x = -R$ and has a uniform charge density ρ_1. Another solid sphere 2 of radius R is centered at $x = +R$ and has a uniform charge density ρ_2. At $x = -R/2$ on the x-axis, the net electric field turns out to be zero. (a) Compute ρ_1/ρ_2. (Derivation steps for the electric field are required.) (b) Along the x-axis, sketch $E_x(-R \leq x \leq R)$. No need to label any axis.

i. $E = \int_0^{2\pi} \frac{1}{4\pi\epsilon_0} \frac{\lambda \cdot R}{R^2} d\theta$ $\quad l = 2\pi R$ $\quad = \theta R$

$V = \int_\infty^b E \cdot dl$

$= \frac{1}{4\pi\epsilon_0} \int \frac{dq}{r} = \boxed{\frac{1}{4\pi\epsilon_0} \frac{Q}{R}}$

ii. $V = \frac{1}{4\pi\epsilon_0} \int_0^{\pi} \frac{dq}{r} = \boxed{\frac{1}{4\pi\epsilon_0} \frac{Q}{R}}$ $\quad dq = \lambda \cdot \phi R$

iii. $V = \frac{1}{4\pi\epsilon_0} \int_0^{\pi/3} \frac{dq}{r} = \boxed{\frac{1}{4\pi\epsilon_0} \frac{Q}{R}}$

b.

i.) $V = \frac{1}{4\pi\epsilon_0} \int_0^{2\pi} \frac{\lambda \cos\phi \, d\phi}{r} = \boxed{\frac{1}{4\pi\epsilon_0} \frac{\lambda \cdot}{R}}$ × -1

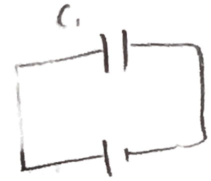
ii.) $V = \frac{1}{4\pi\epsilon_0} \int_0^{2\pi} \frac{\lambda \cos^2\phi}{r} d\phi = \boxed{\frac{\lambda}{4\pi\epsilon_0 R} \cdot \int_0^{2\pi} \cos^2\phi \, d\phi}$ $\quad -0.5$

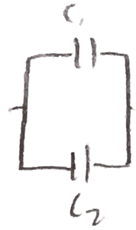
iii.) $V = \boxed{\frac{1}{4\pi\epsilon_0} \int_0^{2\pi} \frac{\lambda \cos^3\phi}{r} d\phi}$ $\quad -0.5$

(8)

3. a. When battery removed

C_1 stays same, total charge $= Q_0$





$Q_0 = C_1 V_0$ $\qquad Q_1 + Q_2 = Q_0$

voltages same

$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2}$$

$$Q_1 = Q_2 \cdot \frac{C_1}{C_2} = (C_1 V_0 - Q_1)\frac{C_1}{C_2}$$

$$\boxed{Q_2 = \frac{C_2 Q_1}{C_1}}$$

$$= \frac{C_1^2 V_0}{C_2} - \frac{C_1 Q_1}{C_2}$$

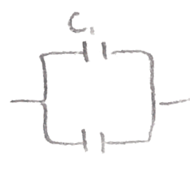
$$Q_1\left(1 + \frac{C_1}{C_2}\right) = \frac{C_1^2 V_0}{C_2}$$

$$\boxed{Q_1 = \frac{C_1^2 V_0}{C_1 + C_2}}$$ ✓

$$\boxed{Q_2 = \frac{C_1 C_2 V_0}{C_1 + C_2}}$$ ✓

b. $Q_1 = C_1 V_1 \quad Q_2 = C_2 V_2$ ✓

(6).



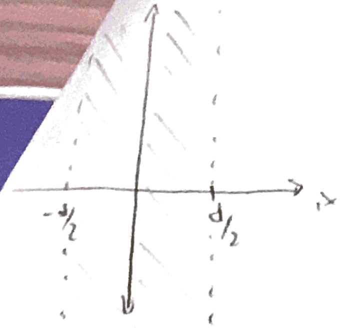
$C' = C_1 + C_2$

$Q' = C'(V_1 + V_2)$

$\boxed{Q_1' = C_1(V_1 + V_2)}$ $\quad$ $\boxed{Q_2' = C_2(V_1 + V_2)}$

The charges on both sides double

if $V_1 = V_2 = V$ ✗



a. $x = 0$
equal electric field from both sides
so $\boxed{E(0) = 0}$ 2

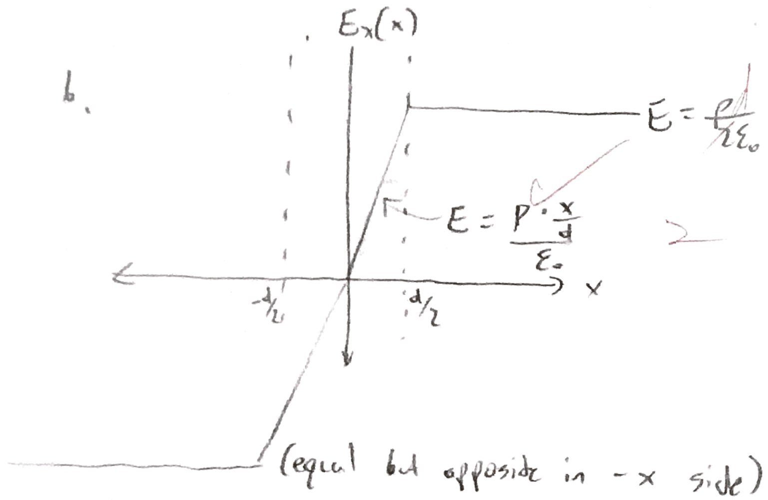
$x = d$
$E = \frac{\sigma}{2\varepsilon_0}$ for sheets 1
integrate over thickness of slab

$$\int_{-d/2}^{d/2} \frac{\sigma}{2\varepsilon_0}\,dx = \frac{\sigma d}{2\varepsilon_0} = \frac{\rho}{2\varepsilon_0}$$

$\boxed{E(d) = \frac{\rho d}{2\varepsilon_0}}$ 1.5

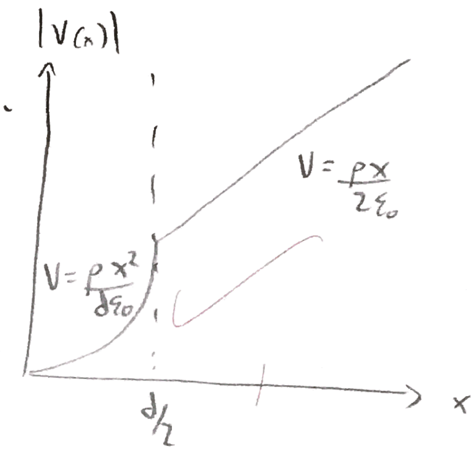
$x = 2d$
for sheets, or slabs, distance does not affect E
so $\boxed{E(2d) = \frac{\rho d}{2\varepsilon_0}}$ 0.5

b.



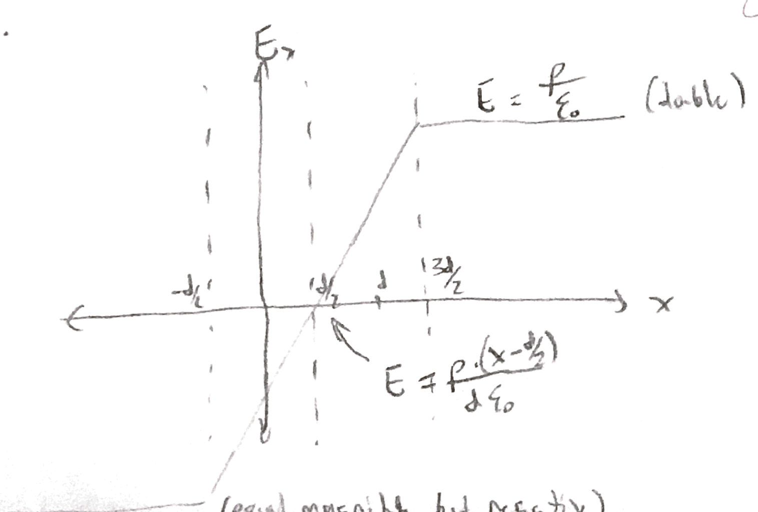
2

c.

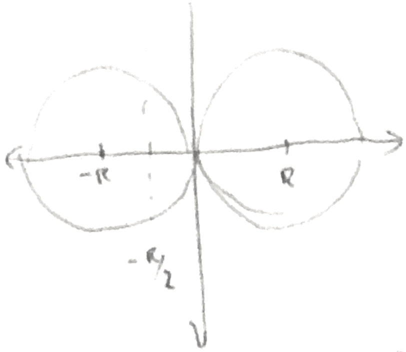


1

d.



5.



a. electric field due to sphere 1

$$\vec{E}_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{\rho \cdot 4\pi r^3}{3\ r^2} = \frac{\rho_1 r}{3\varepsilon_0} = \frac{\rho_1 R}{6\varepsilon_0}$$

$$q_1 = \rho \cdot V = \rho_1 \cdot \frac{4}{3}\pi r^3$$

$$\vec{E}_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{\rho_2 \cdot \frac{4}{3}\pi R^3}{\frac{9}{4}R^2} = \frac{\rho_2}{\varepsilon_0} \cdot \frac{4R}{27}$$

$$q_2 = \rho \cdot V = \rho_2 \cdot \frac{4}{3}\pi R^3$$

$$\frac{\rho_1 R}{6\varepsilon_0} = \frac{4\rho_2 R}{27\varepsilon_0}$$

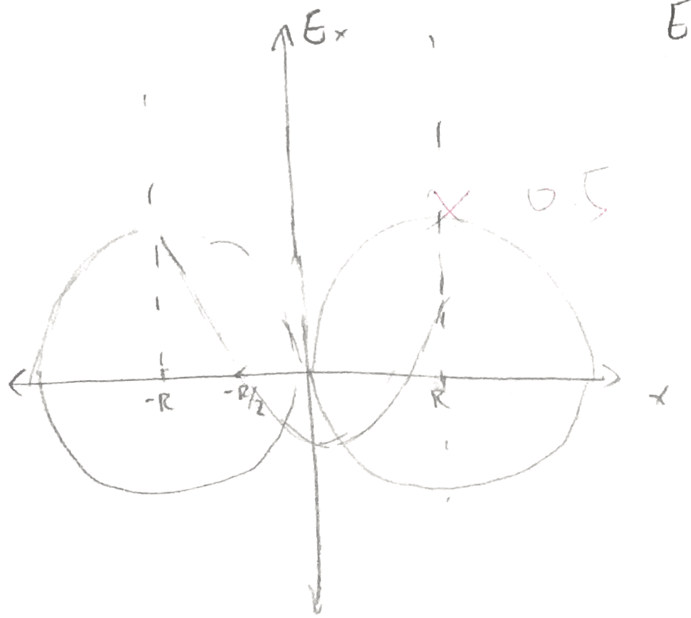
$$\boxed{\frac{\rho_1}{\rho_2} = \frac{24}{27}}$$

3

3

3

b.



$$E_x = E_1 - E_2$$

$$= \frac{R}{\varepsilon_0}\left(\frac{1}{6}\rho_1 - \frac{4}{27}\rho_2\right)$$