Problem 1

(a) What is true about the acceleration of a simple harmonic oscillation?

A) The acceleration is a maximum when the displacement is a maximum.
B) The acceleration is a maximum when the displacement is zero.
C) The acceleration is a maximum when the speed is a maximum.
D) The acceleration is zero when the object is instantaneously at rest.
E) None of the above.

(b) An object is attached to a vertical spring and bobs up and down between the two points A and B. When the kinetic energy is a minimum, the object is located:

A) midway between A and B.
B) 1/2 of the distance from A to B.
C) $1/\sqrt{2}$ times the distance from A to B.
D) at either A or B.
E) None of the above.

(c) A wave is traveling along a string. We can double the wave power by

A) increasing the amplitude of the wave by a factor of 4.
B) increasing the amplitude of the wave by a factor of 2.
C) increasing the amplitude of the wave by a factor of $\sqrt{2}$.
D) reducing the amplitude of the wave by a factor of 2.
E) None of the above.

(d) Consider the wave on a vibrating guitar string and the sound wave the guitar produces in the air. The string wave and the sound wave must have the same

A) wavelength.
B) frequency.
C) velocity.
D) amplitude.
E) More than one of the above is true.

(e) Observer A is a distance r away from a light bulb and observer B is $4r$ away from the same bulb. If observer B sees a light intensity I, observer A will see a light intensity of:

A) I.
B) $4I$.
C) $16I$.
D) $I/4$.
E) $I/16$.

$$\frac{I_B}{I_A} = \frac{r_a^2}{r_b^2} \qquad I_A = \frac{r_b^2}{r_a^2} I_b = \frac{16r^2}{r^2} I_b = 16 I_b$$

(f) A stopped pipe (with one-end open) is 1 m long and has a fundamental frequency 10 Hz. What is the sound wave speed in it?:

1m

A) 10 ms^{-1}.
B) 20 ms^{-1}.
C) 30 ms^{-1}.
D) 40 ms^{-1}.
E) Not enough information to compute.

$\lambda = 4L$ $\quad f = \frac{nv}{4L}$

$f_n \cdot 4L = V$

(g) Which one of the following is true about the sound intensity level β and intensity I?

A) Both of them obey inverse-square distance laws.
B) Both of them can be negative.
C) Both of them can never be negative.
D) β obeys the inverse-square distance law but I does not.
E) I can never be negative but β can be negative.

(h) A 1 m long pipe can produce sound of wavelengths 0.8 m, 4/3 m, 4 m (no wavelengths longer than these). This pipe is

1m $\quad \lambda = \frac{2L}{n} = \frac{2}{n}$

A) both ends open.
B) both ends closed.
C) one end open.
D) We cannot judge since the speed is unknown.
E) None of the above.

$\lambda = \frac{4L}{n}$

(i) Two pure tones are sounded together and a beat frequency f_{beat} is heard. What happens to f_{beat} if the frequency of one of the tones is increased?

$f_{beat} = f_a - f_b$

A) It increases.
B) It decreases.
C) It remains unchanged.
D) It vanishes.
E) Not enough information to judge.

(j) A simple harmonic oscillator has a maximum amplitude A and a maximum speed of v. When the displacement is $A/2$, the speed becomes?

$X = \frac{1}{2} A.$

$V_{max} = wA_{max}$

$v = wA\sin(wt)$ $\quad \cos(wt) = \frac{1}{2}$

A) $2v$.
B) $v/2$.
C) $\sqrt{3}v/2$.
D) $\sqrt{2}v/3$.
E) $\sqrt{2}v$.

cos

wt

Problem 2

A transverse string wave is traveling along the x-axis (towards +v.e. x), with speed v, amplitude A and wavelength λ. At $x = t = 0$, the displacement is upward, i.e. $y(x = t = 0) = A$. Express your answers in terms of v, A, λ. (a) What are the wave number k and angular frequency ω? (b) Write down the wave function $y(x, t)$. (c) What is the maximum magnitudes of transverse velocity and acceleration? (d) When $|y| = A/3$, what is the transverse acceleration magnitude? (e) What is the conditions for x and t at which $y(x, t) = A$? (f) If the wave reverses its propagation direction, which of the above answers (a-e) remain(s) unchanged? (g) If the initial condition is changed to $y(x = t = 0) = 0$ instead, which of the above answers (a-e) remain(s) unchanged?

Problem 3

A simple harmonic oscillator is characterized by mass m, spring constant k and amplitude A. Suppose we have an initial displacement $y(t = 0) = A$. (a) Write down the expressions for the kinetic energy $E_{KE}(t)$ and potential energy $E_{PE}(t)$. Plot them as a function of time. (b) At $t = t_0$, $E_{KE}(t_0) = E_{PE}(t_0)$. Find the smallest t_0. What is corresponding displacement magnitude? (c) When $y(t) = A/2$, what is the ratio of $E_{KE}(t)$ to $E_{PE}(t)$?

Problem 4

(a) Four identical sound sources are placed along the x-axis at $x = 0,\ x_0,\ 2x_0,\ 3x_0$ and each of them produces unidirectional sound with amplitude A and wavelength λ. What is the net wave amplitude if the separation (i) $x_0 = 2\lambda$, (ii) $x_0 = \lambda$, (iii) $x_0 = \lambda/2$, (iv) $x_0 = \lambda/4$?

(b) Now remove the sound source at $x = 3x_0$. What is the net wave amplitude if (i) $x_0 = 2\lambda$, (ii) $x_0 = \lambda$, (iii) $x_0 = \lambda/2$, (iv) $x_0 = \lambda/4$?

Problem 5

You are driving at velocity $v_{me} = v/5$, where v is the sound speed. A police car is approaching you from behind and you hear a siren frequency f_1. You are then relieved as the police car continues past you, after which you hear another frequency $f_2 = f_1/2$. Assuming that all velocities are constant. (a) What is the speed of the police car v_p (in terms of v)? (b) What is the siren frequency f_p heard by the police (in terms of f_1)?

2a. $\boxed{k = \frac{2\pi}{\lambda}}$ $\omega = vk$

$\boxed{\omega = \frac{2\pi v}{\lambda}}$

b. $y(x,t) = A\cos(kx - \omega t)$

$y(x,t) = A\cos\left(\frac{2\pi}{\lambda}x - \frac{2\pi v}{\lambda}t\right)$

$\boxed{y(x,t) = A\cos\left(\frac{2\pi}{\lambda}(x - vt)\right)}$

c. $v(x,t) = \frac{dy}{dt} = +\frac{2\pi v}{\lambda}A\sin\left(\frac{2\pi}{\lambda}(x - vt)\right)$

V_{max} is when $\sin(\,) = 1$ $\boxed{V_{max} = \frac{2\pi v}{\lambda}A}$

$a(x,t) = \frac{dv}{dt} = \frac{d}{dt}\,\omega A\sin(kx - \omega t)$

$= -\omega^2 A\cos(kx - \omega t)$

$a_{max} = -\omega^2 A$ replace ω

$\boxed{a_{max} = \left(\frac{2\pi v}{\lambda}\right)^2 A}$ or $\frac{4\pi^2 v^2}{\lambda^2}A$

d. $y = \frac{A}{3}$ $\cos(kx - \omega t) = \frac{1}{3}$

$a = a_{max} \cdot \frac{1}{3} = \frac{1}{3} \cdot \frac{4\pi^2 v^2}{\lambda^2}A$

$a = \boxed{\frac{4}{3} \cdot \frac{\pi^2 v^2}{\lambda^2} \cdot A}$

2

e. $\cos(kx - \omega t) = 1$

i.e. $kx - \omega t = n 2\pi$ (some multiple of 2π)

$\frac{2\pi}{\lambda}(x - vt) = n2\pi$

$x - vt = \lambda n$ 2

$\frac{x - vt}{\lambda} = n$ (some integer)

i.e. $x = vt + \lambda n$

some integer

f. answers (a), (c), (d) and (e) remain unchanged

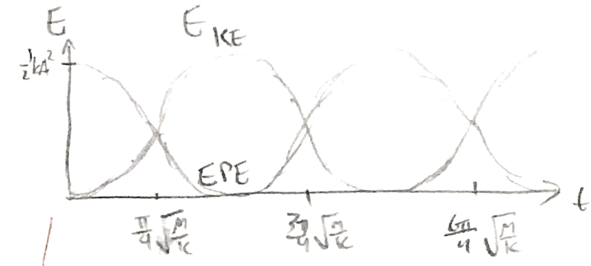
g. answers (a), (c), (d) remain unchanged

3. a. $E_{KE}(t) = \frac{1}{2}mv^2$ $\quad\quad \omega = \sqrt{\frac{k}{m}} \quad \omega^2 = \frac{k}{m}$

$v = -\omega A \sin(\omega t)$

$v^2 = \omega^2 A^2 \sin^2(\omega t)$

$$\boxed{E_{KE}(t) = \frac{1}{2} k A^2 \sin^2\left(\sqrt{\frac{k}{m}}\, t\right)}$$



$E_{PE}(t) = \frac{1}{2}kx^2 \quad\quad x = A\cos(\omega t)$

$$\boxed{E_{PE}(t) = \frac{1}{2} k A^2 \cos^2\left(\sqrt{\frac{k}{m}}\, t\right)}$$

b. $\sin^2\left(\sqrt{\frac{k}{m}}\, t\right) = \cos^2\left(\sqrt{\frac{k}{m}}\, t\right)$ $\quad\quad \boxed{t_0 = \frac{\pi}{4}\sqrt{\frac{m}{k}}}$ 2

$\sqrt{\frac{k}{m}}\, t = \frac{\pi}{4}$

$t = \frac{\pi}{4}\sqrt{\frac{m}{k}}$

$|y(t_0)| = A\cos\left(\sqrt{\frac{k}{m}} \cdot \sqrt{\frac{m}{k}}\, \frac{\pi}{4}\right)$

$= A\cos\left(\frac{\pi}{4}\right)$ 2

$= \frac{A}{\sqrt{2}}$ $\quad\quad \boxed{|y(t_0)| = \frac{A}{\sqrt{2}}}$

c. $y = \frac{A}{2} \quad\quad \cos(\omega t) = \frac{1}{2}$

$\omega t = \frac{\pi}{3}$

$t = \frac{\pi}{3}\sqrt{\frac{m}{k}}$ 3

$$\frac{E_{KE}}{E_{PE}} = \frac{\frac{1}{2}kA^2\sin^2\left(\frac{\pi}{3}\right)}{\frac{1}{2}kA^2\cos^2\left(\frac{\pi}{3}\right)} = \frac{\sin^2\left(\frac{\pi}{3}\right)}{\cos^2\left(\frac{\pi}{3}\right)} = \frac{0.75}{0.25}$$

$$\boxed{\begin{array}{c} E_{KE} : E_{PE} = 3:1 \\ \frac{E_{KE}}{E_{PE}} = 3 \end{array}}$$

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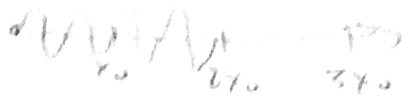
4\.

a. (i) $x_0 = 2\lambda$



constructive waves

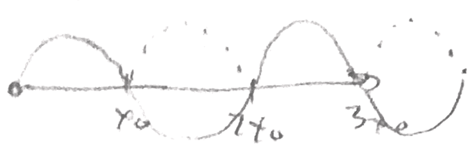
Net Wave Amplitude = $\boxed{4A}$

(ii) $x_0 = \lambda$

also constructive

Net Wave Amp. = $\boxed{4A}$

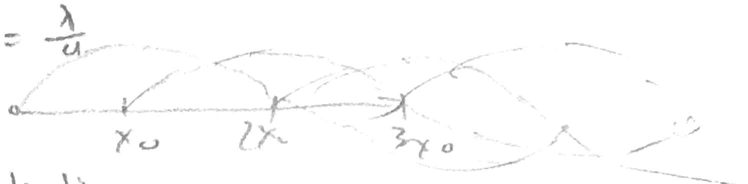
(iii) $x_0 = \frac{\lambda}{2}$



net wave amplitude = $\boxed{0}$

destructive

(iv) $x_0 = \frac{\lambda}{4}$



net wave amplitude = $\boxed{0}$

destructive

6\. (i.) constructive, minus one source net amplitude = $\boxed{3A}$

(ii.) constructive, minus one source net amplitude = $\boxed{3A}$

(iii)

first two destruct, 3rd by itself net amplitude = $\boxed{A}$

(iv).

First and third destruct, 2nd by itself net amplitude = $\boxed{A}$

5. $V_{me} = V/5 \quad f_1 = \text{behind} \quad f_2 = f_1/2$

a.

$$f_1 = \frac{V - V_{me}}{V - V_p} f_{car} \qquad f_2 = \frac{V + V_{me}}{V + V_p} f_{car}$$

10

$$\frac{V + V_{me}}{V + V_p} f_{car} = \frac{1}{2} \cdot \frac{V - V_{me}}{V - V_p} f_{car}$$

$$(V + V_{me})(V - V_p) = \frac{1}{2}(V - V_{me})(V + V_p)$$

$$\frac{6V}{5}(V - V_p) = \frac{1}{2}\left(\frac{4V}{5}\right)(V + V_p)$$

$$\frac{6V^2}{5} - \frac{6V \cdot V_p}{5} = \frac{2V^2}{5} + \frac{2V \cdot V_p}{5}$$

$$\frac{4V^2}{5} = \frac{8\, V \cdot V_p}{5}$$

$$\frac{4V^2}{8V} = V_p$$

$$\boxed{V_p = \frac{1}{2} V}$$

b.

$$f_1 = \frac{V - V_{me}}{V - V_p} f_p$$

$$= \frac{V - \frac{V}{5}}{V - \frac{1}{2}V} f_p$$

$$= \frac{\frac{4}{5}V}{\frac{1}{2}V} f_p$$

$$= \frac{8}{5} f_p$$

$$\boxed{f_p = \frac{5}{8} f_1}$$