MT2 Physics 1B S16

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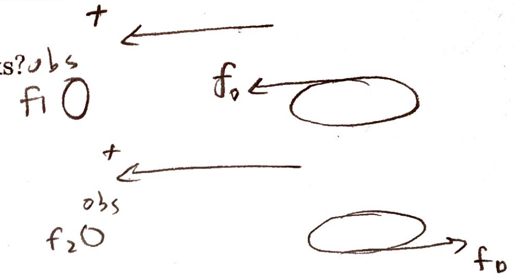
Seat Number ____________

Problem	Grade
1	12 /30
2	26 /30
3	28 /30
Total	66 /90

- Do not peek at the exam until you are told to begin. You will have approximately 50 minutes to complete the exam.
- Don't spend too much time on any one problem. Solve 'easy' problems first. Go for partial credit!
- **HINT:** Focus on the concepts involved in the problem, the tools to be used, and the set-up. If you get these right, all that's left is algebra.
- **Have Fun!**

1) You're standing on the edge of a cliff. Out in the distance, at roughly the same elevation as you, you see a vulture flying in a horizontal circle, looking for lunch. When the vulture is headed directly at you, you hear its shrieks at a frequency f_1. When it is headed directly away, you hear its shrieks at a frequency f_2. It is not unreasonable to assume that all the shrieks are emitted exactly alike.

- 1a) (10 points) At what frequency will the vulture hear its shrieks?



$$\frac{f_1}{f_0} = \frac{V_{snd} - |V_{obs}|}{V_{snd} - |V_{src}|} = \frac{V_{snd}}{V_{snd} - |V_{vul}|}$$

$$\frac{f_2}{f_0} = \frac{V_{snd}}{V_{snd} + |V_{vul}|}$$

$$f_1 = f_0 \frac{V_{snd}}{V_{snd} - |V_{vul}|} \qquad f_2 = \frac{V_{snd}}{V_{snd} + |V_{vul}|}$$

$$f_1 = f_0 \frac{f_2\, V_{snd} + |V_{vul}|}{V_{snd} - V_{vul})}$$

$$f_0 = \frac{f_1}{f_2} \left(\frac{V_{snd} - |V_{vul}|}{V_{snd} + |V_{vul}|} \right)$$

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- 1b) (10 points) How fast is the vulture moving?

$$\frac{f_1}{f_0} = \frac{V_{snd}}{V_{snd} - |V_{vul}|} \qquad \frac{f_2}{f_0} = \frac{V_{snd}}{V_{snd} + |V_{vul}|}$$

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$$\frac{f_1}{f_2} = \frac{\dfrac{1}{V_{snd} - |V_{vul}|}}{\dfrac{1}{V_{snd} + |V_{vul}|}}$$

$$\frac{f_1}{f_2} = \frac{V_{snd} + |V_{vul}|}{V_{snd} - |V_{vul}|}$$

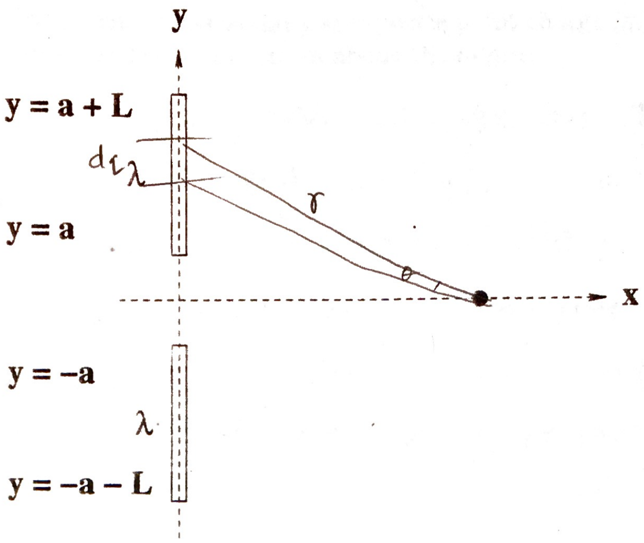
(isolate, solve for V_{vul})

- 1c) (10 points) You begin to walk away from the vulture with a speed V_x. What frequencies will you hear now, when the vulture is flying straight towards you (f_1') and straight away (f_2')? How does the ratio f_1'/f_2' compare to f_1/f_2?

$$\frac{f}{f_0} = \frac{V_{snd} \overset{(-)}{+} |V_{obs}|}{V_{snd} + |V_{src}|}$$

$$f_1 = f_0 \frac{V_{snd} \overset{(-)}{+} |V_{obs}|}{V_{snd} - |V_{src}|}$$

1/10



2) Two identical non-conducting rods of length L and linear charge density λ are placed along the y-axis so that the origin lies at the center of a gap of width $2a$ that sits between them, as shown.

15

- 2a) (15 points) Find the (vector) electric field at all points on the positive x-axis.

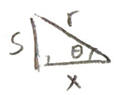
For top:

$$d\vec{E}_{top} = \frac{dq}{4\pi\epsilon_0 r^2}\hat{r}$$

$$\lambda = \frac{dq}{ds}$$

$$dq = \lambda\, ds$$

$$\vec{E}_{top} = \int \frac{\lambda x \sec^2\theta\, d\theta}{4\pi\epsilon_0 r^2}(-\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$$



$$x\tan\theta = s$$

$$x\sec^2\theta\, d\theta = ds$$

$$r = \frac{x}{\cos\theta}$$

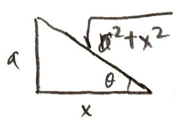
$$dq = \lambda x \sec^2\theta\, d\theta$$

$$\hat{r} = (-\cos\theta\,\hat{i} + \sin\theta\,\hat{j})$$

$$\vec{E}_{top} = \frac{\lambda x}{4\pi\epsilon_0}\int \frac{1}{x^2}(-\cos\theta\,\hat{i} + \sin\theta\,\hat{j})\, d\theta$$

$$\vec{E}_{top} = \frac{\lambda}{4\pi x\epsilon_0}\left(-\sin\theta\,\hat{i} - \cos\theta\,\hat{j}\right)\Big|$$

$$\vec{E}_{top} = \frac{-\lambda}{4\pi x\epsilon_0}\left[\left(\frac{(a+L)}{\sqrt{(a+L)^2+x^2}} - \frac{a}{\sqrt{a^2+x^2}}\right)\hat{i} + \left(\frac{x}{\sqrt{(a+L)^2+x^2}} - \frac{x}{\sqrt{a^2+x^2}}\right)\hat{j}\right]$$



$$(\text{min})\ \sin\theta = \frac{a}{\sqrt{a^2+x^2}}$$

$$(\text{min})\ \cos\theta = \frac{x}{\sqrt{a^2+x^2}}$$

$$(\text{max})\ \sin\theta = \frac{a+L}{\sqrt{(a+L)^2+x^2}}$$

$$(\text{max})\ \cos\theta = \frac{x}{\sqrt{(a+L)^2+x^2}}$$

For bottom:

$$\vec{E}_{bot} = \frac{\lambda}{4\pi x\epsilon_0}\int(-\cos\theta\,\hat{i} - \sin\theta\,\hat{j})\, d\theta$$

$$\vec{E}_{bot} = \frac{\lambda}{4\pi x\epsilon_0}\left(-\sin\theta\,\hat{i} + \cos\theta\,\hat{j}\right)$$

$$\left[\vec{E}_{tot} = \vec{E}_{top} + \vec{E}_{bot} = \frac{\lambda}{2\pi x\epsilon_0}\left(\frac{a+L}{\sqrt{(a+L)^2+x^2}} - \frac{a}{\sqrt{a^2+x^2}}\right)\hat{i}\right]$$

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- 2b) (5 points) Under what conditions would you expect a point charge (q, m) released at rest on the $x - axis$, to undergo simple harmonic oscillation about the origin?

If the charge of the point charge on the x-axis was oppositely charged as the two bars ~~theear~~ then there would be S.H.O. The electric field and electric force would point in opposite directions and would ~~flip~~ flip when the particle went to the other side. Thus, it would be a linear, restoring force.

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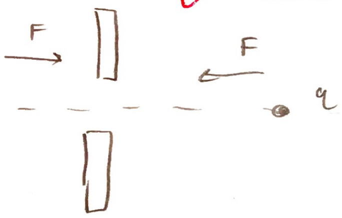
- 2c) (10 points) Suppose a point charge was released under the conditions you stated above. Show that it will indeed execute simple harmonic motion and find the angular frequency of that motion.



$$\vec{F} = q\vec{E} = \frac{\lambda q}{2\pi x \epsilon_0}\left(\frac{a+L}{\sqrt{(a+L)^2+x^2}} - \frac{a}{\sqrt{a^2+x^2}}\right)\hat{i}$$

$$\frac{\lambda q}{2\pi x \epsilon_0}\left(\frac{a+L}{\sqrt{(a+L)^2+x^2}} - \frac{a}{\sqrt{a^2+x^2}}\right) = m\frac{d^2x}{dt^2}$$

$$\frac{d^2x}{dt^2} - \frac{\lambda q}{2\pi x m \epsilon_0}\left(\frac{a+L}{\sqrt{(a+L)^2+x^2}} - \frac{a}{\sqrt{a^2+x^2}}\right) = 0$$

if $x \ll a$:

?

$$\frac{d^2x}{dt^2} + \frac{\lambda q}{2\pi x m \epsilon_0}\left(\frac{a+L}{a+L} + \frac{a}{a}\right)$$

$$\frac{d^2x}{dt^2} + \frac{\lambda q}{\pi m \epsilon_0} = 0$$

$$\omega_0 = \sqrt{\frac{\lambda q}{\pi \epsilon_0}}$$

A very long cylindrical distribution of radius R has a volume charge density given by:

$$\rho(\vec{r}) = \frac{6\lambda_0}{\pi R^4}\, r\,(R-r)$$

where r is measured from the longitudinal symmetry axis of the distribution.

- 3a) (10 points) How much charge is contained in a coaxial cylinder of radius r and length x? Consider both the case where $r < R$ *and* $r > R$.

for $r < R$: Volume of coaxial $= \pi r^2 x$, $dV = 2\pi r x\, dr$

$$dq = \rho\, dV = \frac{6\lambda_0}{\pi R^4} r(R-r)(2\pi r x)\,dr$$

$$\rho(\vec{r}) = \frac{6\lambda_0}{\pi R^4} r(R-r) = \frac{Q}{V}$$

$$Q = \frac{12x\lambda_0}{R^4}\int_0^r r^2(R-r)\,dr = \frac{12x\lambda_0}{R^4}\left(\frac{r^3R}{3} - \frac{r^4}{4}\right)$$

$$Q = \frac{6\lambda_0}{\pi R^4}(r(R-r))(\pi r^2 x)$$

$$Q = \frac{12x\lambda_0 r^3}{R^4}\left(\frac{R}{3} - \frac{r}{4}\right)$$

for $r \geq R$: $$Q = \frac{12x\lambda_0}{R^4}\int_0^R r^2(R-r)\,dr$$

$$= \frac{12x\lambda_0}{R^4}\left(\frac{r^3R}{3} - \frac{r^4}{4}\right)\Big|_0^R$$

$$= \frac{12x\lambda_0}{R^4}\left(\frac{R^4}{3} - \frac{R^4}{4}\right)$$

$$= 12x\lambda_0\left(\frac{1}{3} - \frac{1}{4}\right)$$

$$= x\lambda_0$$

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- 3b) (10 points) Find the electric field at all points inside and outside the cylinder.

for $r < R$:

$$\int \vec{E}\cdot dA = \frac{q_{in}}{\epsilon_0}$$

$$\vec{E}(2\pi r x) = \frac{12x\lambda_0 r^3\left(\frac{R}{3} - \frac{r}{4}\right)}{R^4\epsilon_0}$$

$$\vec{E} = \frac{6\pi\lambda_0 r^2\left(\frac{R}{3} - \frac{r}{4}\right)}{R^4\pi\epsilon_0}$$

for $r \geq R$:

$$\int \vec{E}\cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

$$\vec{E}(2\pi r x) = \frac{x\lambda_0}{\epsilon_0}$$

$$\vec{E} = \frac{\lambda_0}{2\pi r\epsilon_0}\ \hat{r}$$

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- 3c) (10 points) Use $r = R$ as a reference and find the potential for all points inside and outside the cylinder.

$$\Delta V = -\int_{R_{ref}}^{r} \vec{E} \cdot d\vec{r}$$

$R_{ref} = R$

for $r < R$:

$$\Delta V = -\frac{6\pi\lambda_0}{R^4 \pi \epsilon_0} \int_R^r r^2 \left(\frac{R}{3} - \frac{r}{4} \right) dr$$

$$\Delta V = \frac{-6\pi\lambda_0}{R^4 \pi \epsilon_0} \left(\frac{r^3 R}{12} - \frac{r^4}{16} \right) \Big|_R^r$$

$$\Delta V = \frac{6\pi\lambda_0}{R^4 \pi \epsilon_0} \left[\frac{R^4}{12} - \frac{R^4}{16} - \frac{r^4}{12} + \frac{r^4}{16} \right]$$

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for $r \geq R$: $\Delta V = -\int_{R_{ref}}^{r} \vec{E} \cdot dr$

$$\Delta V = -\int_R^r \frac{\lambda_0}{2\pi r \epsilon_0} dr$$

$$\Delta V = \frac{-\lambda_0}{2\pi\epsilon_0} \int_R^r \frac{1}{r} dr$$

$$\Delta V = \frac{\lambda_0}{2\pi\epsilon_0} \left(\ln(R) - \ln(r) \right)$$

$$\Delta V = \frac{\lambda_0}{2\pi\epsilon_0} \ln\left(\frac{R}{r} \right)$$