MT1 Physics 1B S16

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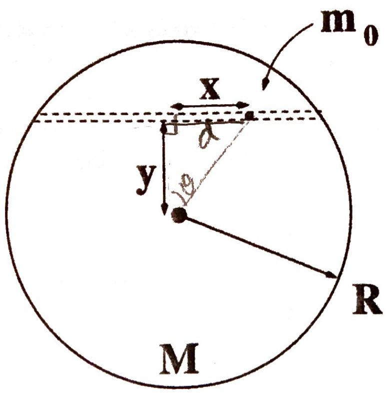
Seat Number ____________

Problem	Grade
1	23 /30
2	26 /30
3	12 /30
Total	61 /90

- Do not peek at the exam until you are told to begin. You will have approximately 50 minutes to complete the exam.
- Don't spend too much time on any one problem. Solve 'easy' problems first. Go for partial credit!
- **HINT:** Focus on the concepts involved in the problem, the tools to be used, and the set-up. If you get these right, all that's left is algebra.
- **Have Fun!** ◡̈ thanks...



$\tan\theta = \frac{d}{y}$

$y\tan\theta = d$

Inside a spherically-symmetric distribution of gravitational charge (mass), at at distance r from the center of the distribution, the gravitational field has a magnitude $g(r) = \frac{G\,m(r)}{r^2}$, where $m(r)$ is the amount of charge (mass) contained in a concentric sphere of radius r. The field itself points radially-inward, towards the center of the distribution.

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- 1a) (5 points) Consider a uniform spherical distribution of mass M, radius R. Find $m(r)$ and $g(r)$ for all $r \leq R$.

$$m(r) = \frac{\frac{4}{3}\pi r^3}{\frac{4}{3}\pi R^3} M = \frac{r^3}{R^3} M$$

$$g(r) = \frac{G\left(\frac{r^3}{R^3}M\right)}{r^2} = \frac{GMr}{R^3}$$

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- 1b) (15 points) Now suppose a thin tunnel is drilled through the sphere, such that, at closest approach, an object in the tunnel would be a distance y from the center of the distribution (as shown). Show that if a point-charge (mass) m_0 is placed in the tunnel a distance x from that point of closest approach and released, it will execute simple-harmonic motion about the point of closest approach. Find the angular frequency associated with that motion.

$\sum F_x = m a_x$ $\quad\left(\tan\theta = \frac{x}{y}\right)$

$-g(y)\tan\theta = m a_x$

$-g(y)\tan\theta = m_0 \frac{d^2x}{dt^2}$

$$\frac{d^2x}{dt^2} + \frac{g(y)}{m_0}\tan\theta = 0$$

$$\frac{d^2x}{dt^2} + \frac{g(y)}{m_0}\cdot\frac{x}{y} = 0$$

$$\frac{d^2x}{dt^2} + \frac{GMy}{R^3 m_0}\cdot\frac{x}{y} = 0$$

$$\frac{d^2x}{dt^2} + \frac{GM}{m_0 R^3}x = 0 \quad \text{(SHO!)}$$

$$\omega_0 = \sqrt{\frac{GM}{m_0 R^3}}$$

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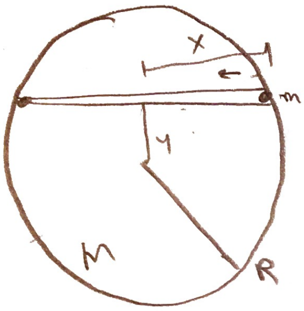
- 1c) (10 points) Suppose the sphere is the planet Earth (neglect the fact that it is rotating) and the tunnel is used to deliver mail from one town to another. How long will it take for mail deposited (at rest) on one side of the tunnel to arrive on the other ($R_{earth} \approx 6.4 \times 10^6$ m, and - no - you don't get to use a calculator)? How would this length of time compare to the time required for mail, similarly deposited, to travel the length of a tunnel through the center of the Earth? Explain.



$$\omega_0 = \sqrt{\frac{GM}{m_0 R^3}}$$ (taken from 1b)

time to get from one side is $\frac{T}{2}$ (half a period)

$$\omega_0 = \frac{2\pi}{T} \Rightarrow T = \frac{2\pi}{\omega_0}$$

$$\frac{1}{2}T = \frac{\pi}{\omega_0} = \frac{\pi}{\sqrt{\frac{GmM}{m_0 R_3}}} =$$

$$\frac{1}{2}T = \pi\sqrt{\frac{m_0 R_3}{GM}}$$

It would take quite a ~~long~~ short amount of time, as $\sqrt{R^3} \approx \sqrt{6.4}^3 \times 10^9$ and GM is far larger than that.

Through center $\approx$ the same as through tunnel.

The motion of a driven oscillator is described by the differential equation

$$\frac{d^2x}{dt^2} + 2\beta\,\frac{dx}{dt} + \omega_0^2\,x = a\,\cos(\Omega t)$$

- 2a) (10 points) Write the equation of motion ($x(t)$) in as much detail as you can. The more correct detail you supply, the more points you will receive.

$$x(t) = A_0 e^{-\beta t} \cos(\omega_0 t + \phi) + A(\Omega) \cos(\Omega t + \delta)$$

$$A_0 = \sqrt{x_0^2 + \left(\frac{V_0}{\omega_0}\right)^2} \qquad \text{where} \quad A(\Omega) = \frac{a}{\sqrt{(\omega_0^2-\beta^2)^2 + (\beta\Omega)^2}}$$

$$x(t) = \sqrt{x_0^2 + \left(\frac{V_0}{\omega_0}\right)^2}\, e^{-\beta t} \cos(\cancel{\omega_0} t + \phi) + \frac{a}{\sqrt{(\omega_0^2-\beta^2)^2 + (2\beta\Omega)^2}} \cos(\Omega t + \delta)$$

$\sqrt{\omega_0^2-\beta^2}$

$\delta = ?$

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- 2b) (5 points) What angular frequency will the system oscillate at if $a = 0$?

$$\omega = \sqrt{\omega_0^2 - \beta^2}$$

If $a=0$, then the system behaves like a damped oscillator.

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$$2\beta = \frac{b}{m}$$

$$\frac{b}{2m} = \frac{1}{2}(2\beta) = \beta$$

- 2c) (5 points) What (angular) driving frequency will maximize the amplitude of the system after it has reached steady-state?

$$\Omega_{res} = \sqrt{\omega_0^2 - 2\beta^2}$$

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- 2d) (10 points) Find the maximum amplitude attainable by the system in steady-state and show that it has a simple relationship to the quantity you calculated in part *b*.

$$A = \frac{a}{\sqrt{(\omega_0^2-\beta^2)^2 + (\beta\Omega)^2}}$$

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$$= \frac{a}{\sqrt{(\omega_0^2-\beta^2)^2 + \beta^2(\omega_0^2 - 2\beta^2)}}$$

$$= \frac{a}{\sqrt{(\omega_0^2-\beta^2)\left[\beta^2 + \omega_0^2 - \beta^2\right] - \beta^4}}$$

$$= \frac{a}{\omega_0\sqrt{\omega_0^2 - \beta^2} - \beta_4}$$

The maximum amplitude attainable by the system in steady state is inversely proportional to the quantity calculated in part b.

3) Consider a champagne glass driven at resonance...

- 3a) (5 points) Use the concept of phase to describe the boundary condition for resonant waves on the rim of the glass.

In order for the waves to resonate, the standing waves need to be equal, yet oppossite direction. They also need to superpose in order to cause large amplitude. The ends of the glass must be nodes because after 1 complete phase, the ~~$~~ wave will superpose because the amplitudes will be synchronized.

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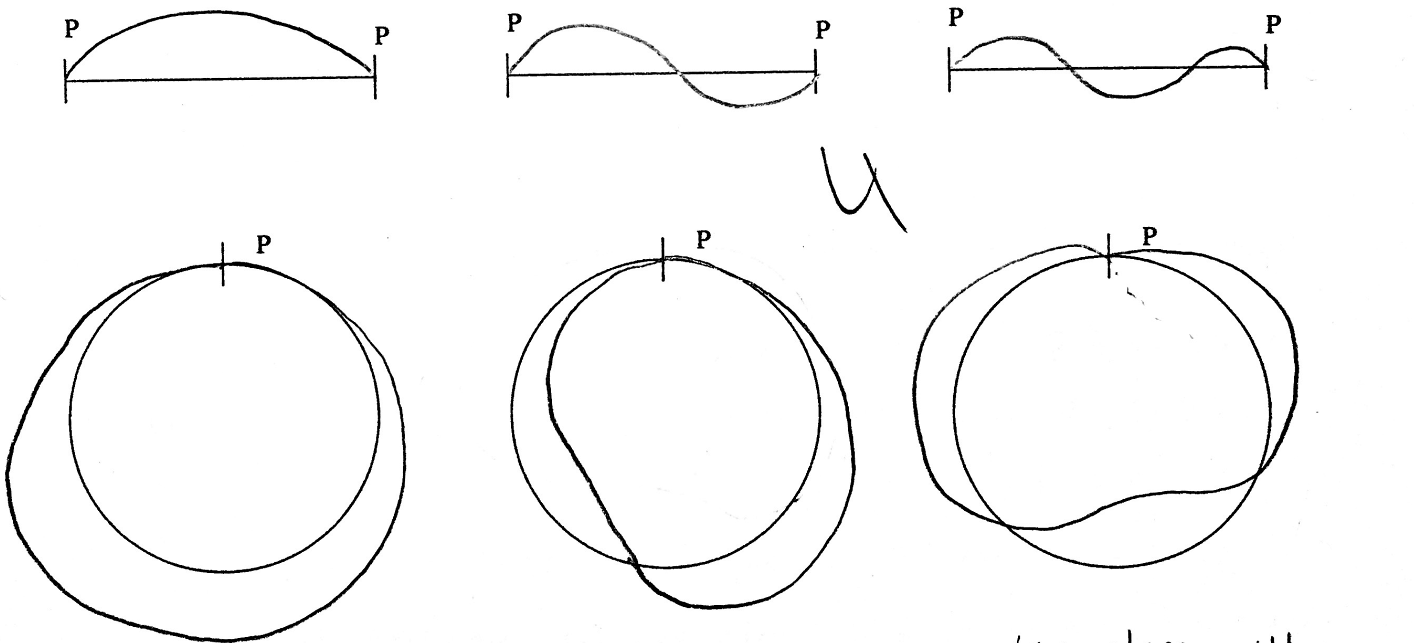
- 3b) (15 points) Use the templates below to sketch the first three resonant modes on the rim of a champagne glass (hint: it might be easier if you sketch the sine waves on the horizontal lines first). In each case, describe how the motion would look to an external observer.



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the glass will appear to vibrate slowly

the glass will appear to vibrate a little faster, possibly even looking a little deformed.

the glass will vibrate even faster, seeming to deform a little more.

- 3c) (5 points) If sound moves through the glass at a rate V_{snd} and the glass has a diameter D, what frequencies will cause the glass to vibrate with large amplitude displacements?

Length of rim $= 2\pi R = \pi D$

$f_n = n\left(\frac{V_{snd}}{2\pi D}\right)$ resonant frequencies with like boundary conditions.

$f_0 = \frac{V_{snd}}{2\pi D}$

$f = nf_0$ for $n \geq 1$.

$f = n\left(\frac{V_{snd}}{2\pi D}\right)$ for $n \geq 1$

3

- 3d) (5 points) Would the fundamental frequency or a harmonic be more effective at shattering the glass? Discuss.

A harmonic would be more effective at shattering the glass because A harmonic would cause large amplitude vibrations with a higher frequency. Because the frequency is higher, there is more energy in the glass and it would shatter.

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