



In the apparatus shown, the string (fixed at both ends and driven in its fundamental mode) is used to search for resonant frequencies in the neighboring tube. As the tension in the string is varied from  $T_{low}$  to  $T_{high}$ , the tube is found to resonate at precisely two settings:  $T_A$  and  $T_B$ . For the following, you may assume that the string has a mass  $m$  and length  $L$ , and you may take the speed of sound in air to be  $v_{snd}$ .

- 2a) (10 points) Using the information provided above, how would you determine whether the tube was open on one both ends or just one? Your answer will be judged by the quality of the argument you use to support it.

tube that is closed on just one vs on both ends would have different consecutive frequencies. if both ends were open, then  $f_{req} = \frac{n v_x}{2L}$  and the ratio of consecutive harmonics would be  $\frac{f_n}{f_{n+1}} = \frac{n}{n+1}$  whereas if one end was open, then  $f_{req} = \frac{(2n+1) v_x}{4L}$  so  $\frac{f_n}{f_{n+1}} = \frac{2n+1}{2n+3}$ . so, we can divide  $f_A$  by  $f_B$  and compare the ratio, and since  $\frac{f_A}{f_B} = \sqrt{\frac{T_A}{T_B}}$

$$f_A = \frac{v_x}{\lambda} = \frac{1}{2L} \sqrt{\frac{T_A L}{m}} \quad f_B = \frac{1}{2L} \sqrt{\frac{T_B L}{m}} \quad \frac{f_A}{f_B} = \sqrt{\frac{T_A}{T_B}}$$

- 2b) (10 points) Suppose  $\frac{T_B}{T_A} = \frac{121}{81}$ . What is the frequency of the fundamental mode in the tube?

$$\sqrt{\frac{T_B}{T_A}} = \sqrt{\frac{121}{81}} = \frac{11}{9} \quad \frac{\sqrt{T_A}}{\sqrt{T_B}} = \frac{9}{11} = \frac{2n+1}{2n+3} \Rightarrow n=4$$

$$f_A = (2n+1) \frac{v_x}{4L} = \frac{2n+1}{4L} \sqrt{\frac{T_A L}{m}}$$

$$\frac{f_A}{f_B} = \frac{9}{11}$$

$$f_B = \frac{11}{9} f_A$$

fundamental when  $n=0$

$$\text{Fundamental freq.} = \frac{1}{4} f_A = \frac{1}{4} \left( \frac{1}{2L} \sqrt{\frac{T_A L}{m}} \right)$$

- 2c) (5 points) How long is the tube?

$$f_A = \frac{(2n+1)v_{snd}}{4L} \Rightarrow L = \frac{2n+1}{4f_A} v_{snd}$$

$$\frac{f_A}{f_B} = \frac{\sqrt{T_A}}{\sqrt{T_B}} = \sqrt{\frac{81}{121}} = \frac{9}{11}$$

$$f_B = \frac{(2n+3)v_{snd}}{4L}$$

$$f_0 = \frac{v_x}{4L} = \frac{1}{4} \sqrt{\frac{T_A}{m}} = \frac{v_{snd}}{4L} \Rightarrow \frac{1}{81} \cdot \frac{T_A}{m} = (v_{snd})^2$$

$$L = 81 \frac{(v_{snd})^2 m}{T_A}$$

- 2d) (5 points) Sketch the waveform corresponding to the lower of the two resonant frequencies in the tube. Label the ends as open and/or closed, and be clear whether the waveform you sketch describes displacement of the medium or pressure.

$n=4$

9th and 11th harmonic

