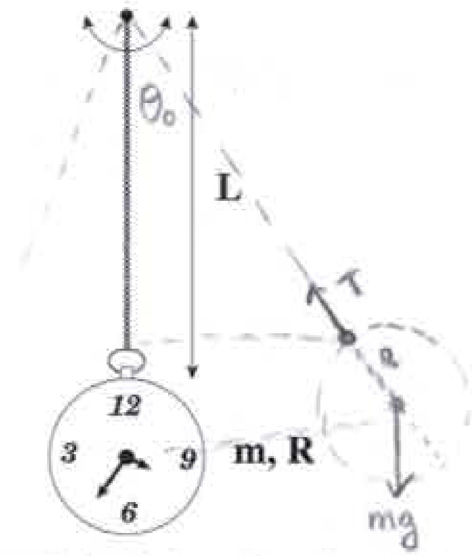


You are getting sleeeepy. . . A pocket-watch (modeled as a thin uniform disk of mass M and radius R) is attached to a very light chain of length L and released at rest from an initial small angle θ_0 (measured between the chain and the vertical) to execute the familiar periodic motion.

- 1a) (10 points) Derive the natural angular frequency of that periodic motion. Evaluate your answer in the limit $R \to 0$ and discuss.

$T_T = 0$

$T_g = -mgL\sin\theta$

$$\Sigma \tau = I\alpha$$

$$-mgL\sin\theta = I\frac{d^2\theta}{dt^2}$$

$$\frac{d^2\theta}{dt^2} + \frac{mgL\sin\theta}{I} = 0$$

$$\frac{d^2\theta}{dt^2} + \frac{\cancel{m}gL\sin\theta}{\frac{1}{2}\cancel{m}R^2 + \cancel{m}L^2} = 0$$

$$\frac{d^2\theta}{dt^2} + \frac{gL\sin\theta}{\frac{1}{2}R^2 + L^2} = 0$$

if θ is small $\sin\theta \approx \theta$

$$\frac{d^2\theta}{dt^2} + \frac{gL}{\frac{1}{2}R^2 + L^2}\theta = 0$$

$$\boxed{\omega_0 = \sqrt{\frac{gL}{\frac{1}{2}R^2 + L^2}}}$$

$$\lim_{R\to 0} \omega_0 = \sqrt{\frac{gL}{L^2}} = \sqrt{\frac{g}{L}}$$

as $\lim_{R\to 0}$, the natural frequency would be that of a simple pendulum whose ω_0 would only depend on g and L

- 1b) (10 points) If the pocket-watch swings back-and-forth with a period T, what is the magnitude of the acceleration due to gravity at the location of the stopwatch?

$$T = \frac{2\pi}{W_0} \qquad a_x(t) = -AW_0^2\cos(W_0 t + \phi)$$

$$\cdot T = \frac{2\pi}{\sqrt{\frac{gL}{\frac{1}{2}R^2+L^2}}}$$

$$W_0 = \frac{2\pi}{T}$$

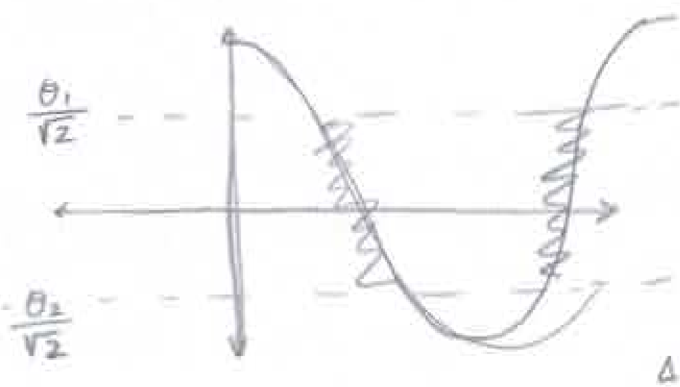
$$\theta = \theta_1 \cos\left(\frac{2\pi}{T}t + \phi\right)$$

$$a(t) = \theta_0\left(\frac{2\pi}{T}\right)^2 \cos\left(\frac{2\pi}{T}t + \phi\right)$$

$$a_{max} = \theta_0\left(\frac{2\pi}{T}\right)^2$$

when $\theta = 0$, $a = 0$

- 1c) (10 points) What fraction of time will the stopwatch spend between $\theta_0/\sqrt{2}$ and $-\theta_0/\sqrt{2}$? No guessing - you must show your work for full credit.



$$\theta = \theta_0 \cos(W_0 t + \phi)$$

$$\frac{\theta_0}{\sqrt{2}} = \theta_0 \cos(W_0 t + \phi)$$

$$\cos(W_0 t + \phi) = \frac{1}{\sqrt{2}} \qquad \cos(W_0 t + \phi) = -\frac{1}{\sqrt{2}}$$

$$W_0 t + \phi = \frac{\pi}{4} \qquad W_0 t + \phi = \frac{3\pi}{4}$$

$$\Delta\theta = \frac{3\pi}{4} - \frac{\pi}{4} = \frac{2\pi}{4} = \frac{\pi}{2}$$

$$\text{fraction of time} = \frac{2\Delta\theta}{T} = \frac{2\left(\frac{\pi}{2}\right)}{2\pi} = \boxed{\frac{1}{2}}$$

$$= \frac{2\Delta t_{1/2}}{T}$$