MT2 Physics 1B F18

Full Name (Printed) ______

Full Name (Signature ______

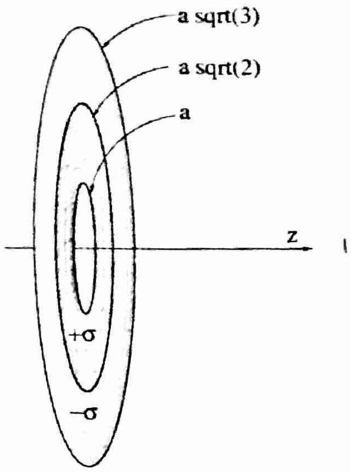
Student ID Number ______

Seat Number ______

Problem	Grade
1	30 /30
2	29 /30
3	30 /30
Total	89 /90

89

- Do not peek at the exam until you are told to begin. You will have approximately 50 minutes to complete the exam.
- Don't spend too much time on any one problem. Solve 'easy' problems first. Go for partial credit!
- **HINT:** Focus on the concepts involved in the problem, the tools to be used, and the set-up. If you get these right, all that's left is algebra.
- **Have Fun!**



$2k\sigma\pi\sqrt{z^2+a^2}$

$k_1\ k\sigma\pi\dfrac{2z}{\sqrt{z^2+a^2}}$

1) The washer shown above lies in the x, y-plane, centered on the origin. From $r = a$ to $r = \sqrt{2}\,a$, it carries a uniform surface charge density σ. From $r = \sqrt{2}\,a$ to $r = \sqrt{3}\,a$, it carries a uniform surface charge density $-\sigma$.

- 1a) (15 points) Find the electric potential (with respect to infinity) at all points on the positive z-axis.

$V = V_{in} + V_{out}$

$$V_{in} = \int_a^{\sqrt{2}a} \frac{k\sigma\, 2\pi r\, dr}{\sqrt{z^2+r^2}} \qquad z^2+r^2 = u \qquad 2r\,dr = du$$

$$\int_{z^2+a^2}^{z^2+2a^2} \frac{k\sigma\pi\, du}{u^{1/2}}$$

$$k\sigma\pi\left(2u^{1/2}\right]_{z^2+a^2}^{z^2+2a^2}$$

$$2k\sigma\pi\left[\sqrt{z^2+2a^2} - \sqrt{z^2+a^2}\right]$$

15

$$V_{out} = \int_{z^2+2a^2}^{z^2+3a^2} \frac{-k\sigma\pi\, du}{u^{1/2}}$$

$$= -k\sigma\pi\left(2u^{1/2}\right]_{z^2+2a^2}^{z^2+3a^2}$$

$$= 2k\sigma\pi\left(\sqrt{z^2+2a^2} - \sqrt{z^2+3a^2}\right)$$

$$\boxed{V_{total} = 2k\sigma\pi\left(2\sqrt{z^2+2a^2} - \sqrt{z^2+a^2} - \sqrt{z^2+3a^2}\right)}$$

- 1b) (10 points) Find the (vector) electric field at all points on the positive z-axis (be very clear how you obtain each component!)

by symmetry, $E_x = E_y = 0$

$$E_z = -\frac{dV}{dz} = -2k\sigma\pi\left(2z(z^2+2a^2)^{-1/2} - z(z^2+a^2)^{-1/2} - z(z^2+3a^2)^{-1/2}\right)$$

$$\boxed{\vec{E} = 2k\sigma\pi z\left(\frac{1}{\sqrt{z^2+a^2}} + \frac{1}{\sqrt{z^2+3a^2}} - \frac{2}{\sqrt{z^2+2a^2}}\right)\hat{k}}$$

10

- 1c) (5 points) Find the electric potential (with respect to infinity) at points along the positive z axis that are very distant from the washer [**Hint:** $(1+x)^n \approx 1 + nx + \frac{1}{2}n(n-1)x^2$]. Discuss the monopole, dipole and quadrapole moments for this charge distribution.

$z >> a$ $\quad z/a$

$$V = 2k\sigma\pi\left(2\sqrt{z^2+2a^2} - \sqrt{z^2+a^2} - \sqrt{z^2+3a^2}\right)$$

$$V = 2k\sigma\pi z\left(2\sqrt{1+\frac{2a^2}{z^2}} - \sqrt{1+\frac{a^2}{z^2}} - \sqrt{1+\frac{3a^2}{z^2}}\right)$$

$$\approx 2k\sigma\pi z\Big(2\left(1+\frac{2}{2}\frac{a^2}{z^2} + \frac{1}{2}\frac{1}{2}\left(-\frac{1}{2}\right)\frac{4a^4}{z^4}\right)$$

$$-\left(1+\frac{1}{2}\frac{a^2}{z^2} + \frac{1}{2}\frac{1}{2}\left(-\frac{1}{2}\right)\frac{a^4}{z^4}\right)$$

$$-\left(1+\frac{1}{2}\frac{3a^2}{z^2} + \frac{1}{2}\frac{1}{2}\left(-\frac{1}{2}\right)\frac{9a^4}{z^4}\right)\Big)$$

5

$$\boxed{V = 2k\sigma\pi z\left(0 + 0\,\frac{a^2}{z^2} + \frac{3}{4}\frac{a^4}{z^4}\right)}$$

monopole & dipole moments are not existant for this charge distribution. However, the quadrapole moment is contributing for this charge distribution as seen above.

Consider a very long uniform cylindrical distribution of charge, of volume charge-density ρ and radius R.

- 2a) (10 points) Find the electric field (vector) for all points inside and outside the cylinder.

$r \leq R$

$$E\, 2\cancel{\pi} r\, \cancel{\ell} = \frac{\rho \cancel{\pi} r^{\cancel{2}} \cancel{\ell}}{\varepsilon_0}$$

$$\boxed{E = \frac{\rho r}{2\varepsilon_0}\hat{r}}$$

$\frac{10}{10}$

$r > R$

$$E\, 2\cancel{\pi} r\, \cancel{\ell} = \frac{\rho \cancel{\pi} R^2 \cancel{\ell}}{\varepsilon_0}$$

$$\boxed{\vec{E} = \frac{\rho R^2}{2 r \varepsilon_0}\hat{r}}$$

- 2b) (10 points) Find the electric potential for all points inside and outside the cylinder, given the potential at the center of the sphere is V_0.

↳ cylinder?

$-\int E\, dr = V$

$\frac{9}{10}$

$r \leq R$

$$-\int_0^r \frac{\rho r}{2\varepsilon_0}\, dr + V_0$$

$$-\frac{\rho r^2}{4\varepsilon_0}\Big]_0^r + V_0$$

$$\boxed{V_0 - \frac{\rho r^2}{4\varepsilon_0}}$$

limits flipped!

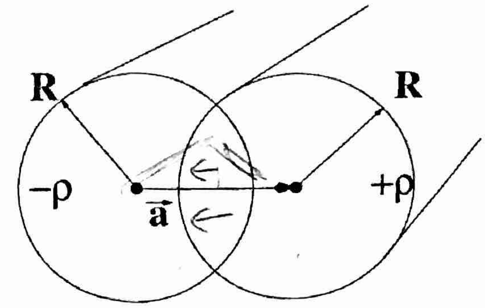
-1

$r > R$

$$V_0 - \frac{\rho R^2}{4\varepsilon_0} - \int_r^R \frac{\rho R^2}{2 r \varepsilon_0}\, dr$$

$$\boxed{V_0 - \frac{\rho R^2}{4\varepsilon_0} - \frac{\rho R^2}{2\varepsilon_0}\left(\ln|R| - \ln|r|\right)}$$

$$V_0 - \frac{\rho R^2}{4\varepsilon_0} + \frac{\rho R^2}{2\varepsilon_0} \ln\left|\frac{r}{R}\right|$$



$\langle 2R, 0\rangle - \langle R, 0\rangle$

$\vec{a} - \vec{r}$

$2R$

$2R$

- 2c) (10 points) Suppose a pair of cylinders, each of radius R, are placed so that their symmetry axes are parallel and are separated by a vector displacement $\vec{a}$ as shown. If one cylinder has a uniform volume charge density $+\rho$ and the other cylinder has a uniform volume charge density $-\rho$, find the electric field (vector) in the region where the cylinders overlap and show that it is uniform.

$$\vec{E}_{\text{left in cylinder}} = \frac{-\rho r}{2\epsilon_0}\hat{r} \qquad \vec{E}_{\text{right in cyl}} = \frac{\rho r}{2\epsilon_0}\hat{r}$$

with respect to R of left

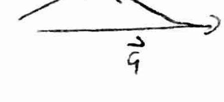
$$\vec{E}_{\text{left}} = -\frac{\rho\vec{r}}{2\epsilon_0} \qquad \vec{E}_{\text{right}} = \frac{\rho(\vec{r}-\vec{a})}{2\epsilon_0}$$

10/10

$$\vec{E}_{\text{tot}} = \frac{-\rho\cancel{\vec{r}}}{\cancel{2\epsilon_0}} + \frac{\rho\cancel{\vec{r}} - \rho\vec{a}}{\cancel{2\epsilon_0}}$$

$$\vec{E}_{\text{region}} = \frac{-\rho\vec{a}}{2\epsilon_0}$$

$$\boxed{\vec{E}_{\text{region}} = \frac{-\rho\vec{a}}{2\epsilon_0}}$$ ✓



Consider a diffuse, spherically-symmetric charge distribution of radius R and charge Q whose volume charge density varies as r^4 (r being the distance from the center of the distribution).

- 3a) (15 pts) Obtain functions that describe the volume charge density of the distribution and $q_{in}(r)$ (the total charge contained in a sphere of radius r, placed concentric with the charge distribution), in terms of Q, R and r, for all r.

$\rho \propto R^4$ $\quad Q = \int \rho \, dA$ $\quad \rho = kr^4$

$$Q = \int_0^R kr^4 \, 4\pi r^2 dr$$

$$Q = k4\pi \int_0^R r^6 dr$$

$$Q = k4\pi R^7/7$$

$$\frac{7Q}{4\pi R^7} = k$$

$$\boxed{\rho = \frac{7Q}{4\pi R^7} r^4}$$

$$\boxed{q_{in}(r) = Q\frac{r^7}{R^7}}$$

✓

+15

- 3b) (15 pts) How much work would one have to do to assemble this distribution from a reservoir of charge sitting some infinite distance away?

$$\frac{q_{in}\, dq}{4\pi\varepsilon_0 r} \quad \swarrow \rho\, dA$$

$$\int_0^R Q\frac{r^7}{R^7}\,\frac{7Q}{4\pi R^7}\,r^9\,\frac{\cancel{4\pi} r^{\cancel{2}}}{\cancel{4\pi}\varepsilon_0 \cancel{r}}\,dr$$

$$\frac{Q}{R^7\varepsilon_0}\,\frac{7Q}{4\pi R^7}\int_0^R r^{12}\,dr$$

$$\frac{7Q^2}{4\pi R^{14}\varepsilon_0}\,\frac{R^{13}}{13}$$

$$U = \boxed{\frac{7Q^2}{52\pi R\varepsilon_0}}$$

+15