

# MT1 Physics 1B-3, S15

**Full Name (Printed)** \_\_\_\_\_

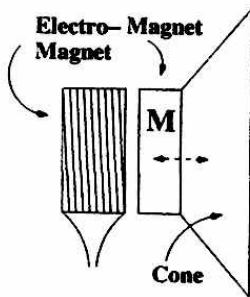
**Full Name (Signature)** \_\_\_\_\_

**Student ID Number** \_\_\_\_\_

**Seat Number** \_\_\_\_\_ 1

Problem	Grade
1	18 /30
2	16 /30
3	14 /30
Total	48 /90

- Do not peek at the exam until you are told to begin. You will have approximately 50 minutes to complete the exam.
- Don't spend too much time on any one problem. Solve 'easy' problems first. Go for partial credit!
- **HINT:** Focus on the concepts involved in the problem, the tools to be used, and the set-up. If you get these right, all that's left is algebra.
- **Have Fun!**



The diagram above shows a very simple loudspeaker. The main part consists of a magnet (mass  $M$ ) attached to a flexible cone. The speaker is designed so that, in proper use, as the magnet is displaced slightly (left or right) from its equilibrium position, the cone will push or pull on it with a linear restoring force.

On the left side of the diagram is an electromagnet. When electric current is driven through the electromagnet, it creates a magnetic field that can be used to drive the magnet and cone back and forth (creating sound). When the current is shut off, the electromagnet does nothing.

Initially, the current is shut off. The magnet is displaced slightly, and released, and the magnet/cone system is observed to oscillate back and forth with a frequency  $f$  (cycles/second). The energy of the magnet cone system is observed to drop from  $E_0$  to  $FE_0$  ( $F < 1$ ) in  $N$  complete cycles.

- 1a) (10 points) What is the natural frequency of the mass/spring system? [You may answer this in terms of frequency,  $f_0$ , or angular frequency,  $\omega_0$ , - just be clear.]



Damped osc.

$$\omega = \sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2} \quad \checkmark \quad b = ?$$

$$E_0 = \frac{1}{2} k A^2$$

$$\text{after } t = NT = \left(\frac{N}{f}\right), \quad E = FE_0$$

$$F \frac{1}{2} k A^2 = \frac{1}{2} k x^2 + \frac{1}{2} m v^2$$

$$\begin{aligned} \Sigma F &= ma \\ -kx - bv &= ma \\ ma + kx + bv &= 0 \end{aligned}$$

$$\frac{d^2x}{dt^2} + \frac{b}{m} \frac{dx}{dt} + \frac{k}{m} x = 0$$

$$x(t) = A_0 e^{-\beta t} \cos(\omega t + \phi)$$

$$F \frac{1}{2} k A_0^2 = \frac{1}{2} k \left( A_0 e^{-\beta \frac{N}{f}} \right)^2$$

$$F = e^{-\beta \frac{2N}{f}}$$

$$\ln F = -\beta \frac{2N}{f} \ln e$$

$$-\frac{f}{2N} \ln F = \beta \quad \checkmark$$

$$\omega = \sqrt{\omega_0^2 - \beta^2}$$

$$2\pi f = \sqrt{\omega_0^2 - \frac{f^2}{4N^2} (\ln F)^2}$$

$$\omega_0 = \sqrt{4\pi^2 f^2 + \frac{f^2}{4N^2} (\ln F)^2}$$

+10

- 1b) (5 points) What is the effective "spring constant" of the cone?

$$\omega_0 = \sqrt{\frac{k_{eff}}{m}} \quad \omega_0^2 m = k_{eff}$$

$$k_{eff} = M \left( 4\pi^2 f^2 + \frac{f^2}{4N^2} (\ln F)^2 \right) + 5$$

- 1c) (10 points) Suppose we connect the electromagnet to a source of sinusoidally-varying electric current. Assuming the electromagnet then exerts a sinusoidally-varying force (of amplitude  $F_0$ , and same frequency as the varying electric current) on the magnet, at what frequency of the driving current will the speaker take on the largest-amplitude vibrations?

$$\frac{d^2 x}{dt^2} + \frac{b}{m} \frac{dx}{dt} + \frac{k}{m} = \frac{F_0}{m} \cos(\omega t)$$

$$A(\omega) = \frac{F_0/m}{\sqrt{(\omega_0^2 - \omega^2)^2 + \left(\frac{b\omega}{m}\right)^2}}$$

largest amp vibration  
when  $\frac{dA(\omega)}{d\omega} = 0$

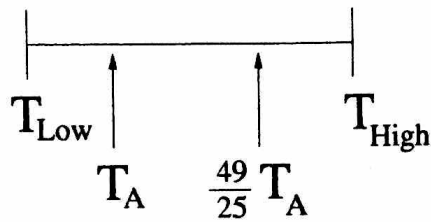
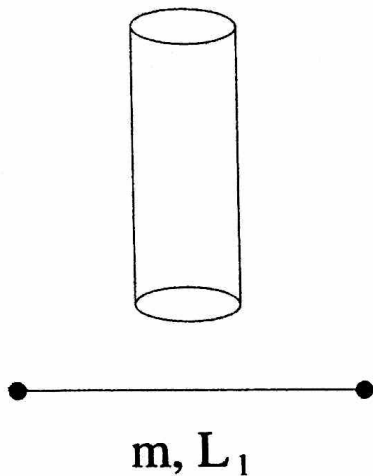
$$\text{or } \frac{d}{d\omega} \left( (\omega_0^2 - \omega^2)^2 + \left(\frac{b\omega}{m}\right)^2 \right) = 0$$

+ 2

- 1d) (5 points) Under normal circumstances,  $F_0$  has to be set large enough so that you can actually hear the sound coming out of the speaker. Describe a couple of (detrimental) things that might happen if the driver is set for "normal" values of  $F_0$ , at the frequency determined in part c.

Resonance may happen and  
the amplitude will increase dramatically.

+ 1



16/30

In the apparatus shown, the string (fixed at both ends and driven in its fundamental mode) is used to search for resonant frequencies in the neighboring tube. As the tension in the string is varied from  $T_{low}$  to  $T_{high}$ , the tube is found to resonate at precisely two settings:  $T_A$  and  $\frac{49}{25}T_A$ . For the following, you may assume that the string has a mass  $m$  and length  $L_1$ , and you may take the speed of sound in air to be  $v_{snd}$ ...

- 2a) (10 points) Is the tube open on both ends, or just one? [your answer will be judged by the quality of the argument you use to support it].

$f$  open at both  $f_{N, tube} = \frac{N v_{snd}}{2L_1}$  open at one:  $f_{N, tube} = (2N-1) \frac{v_{snd}}{4L_1}$

$$v = \sqrt{\frac{T}{\mu}} = \sqrt{\frac{TL}{m}} \quad f_{string} = \frac{v}{2L} = \frac{v}{2L} \sqrt{\frac{TL}{m}} = \frac{v}{2} \sqrt{\frac{T}{mL}}$$

$f$  resonance when  $f_{string} = f_{tube}$

$$f_1(T_A) = \frac{v}{2} \sqrt{\frac{T_A}{mL}} \quad f_1\left(\frac{49}{25}T_A\right) = \frac{v}{2} \sqrt{\frac{49}{25} \frac{T_A}{mL}} = \frac{7}{5} f_1(T_A)$$

~~for the tube to resonate,~~

The tube is open at one end.

the frequency of the string is

This factor is closest to

for a tube open at one end,

At  $\frac{49}{25}T_A$ ,

$\frac{7}{5}$  times the frequency at  $T_A$ .

$\frac{(2N-1)}{4}$  the factor

- 2b) (10 points) What is the frequency of the fundamental mode in the tube? Which harmonic does the string excite in the tube when  $T = T_A$ ?

$$f_{\text{tube},1} = \frac{v}{4L}$$

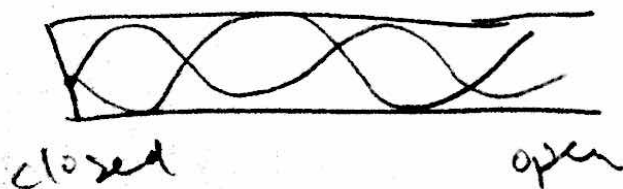
X-9

- 2c) (5 points) How long is the tube?

$$L = \frac{v}{4f}$$

X-4

- 2d) (5 points) Sketch the waveform corresponding to the lower of the two resonant frequencies in the tube. Label the ends as open and/or closed, and be clear whether the waveform you sketch describes displacement of the medium or pressure.

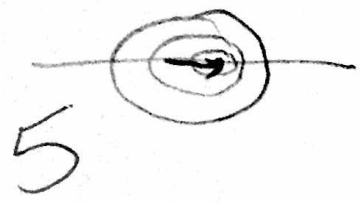
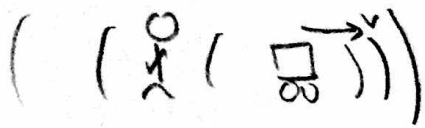
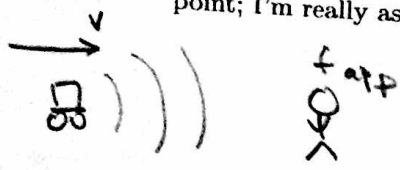


displacement  
of pressure.

X-1

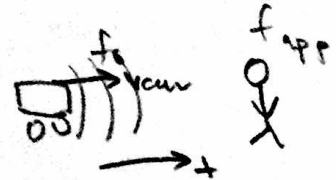
Having nothing better to do on a beautiful Saturday afternoon, you find yourself at a racetrack watching one of the cars doing test laps around the oval. As the car approaches you, you note that the whine you hear from the engine has a frequency of  $(F_{app})$ , after it has passed, you note the frequency has changed to  $(F_{dep})$ .

- 4a) (5 points) Which of the given frequencies is larger? Why? Explain how the sound coming from the engine came to be heard at two different frequencies. Note, the name of the effect may buy you a point; I'm really asking you to explain how the effect works in this case.



The frequency  $F_{app}$  is higher because of the Doppler effect. As the car approaches, the wavelengths emitted become closer together, increasing frequency. As the car departs, the wavelengths become further apart, decreasing frequency heard.

- 4b) (15 points) How fast is the car moving? (You may take the speed of sound in air to be  $V_{snd}$ )



$$\frac{F_{obs}}{F_{emit}} = \frac{V_{snd} - V_{obs}}{V_{snd} - V_{emit}} + 3$$



$$\frac{F_{app}}{f_0} = \frac{V_{snd} - 0}{V_{snd} - V_{car}} \quad f_0 = \frac{F_{app} (V_{snd} - V_{car})}{V_{snd}}$$

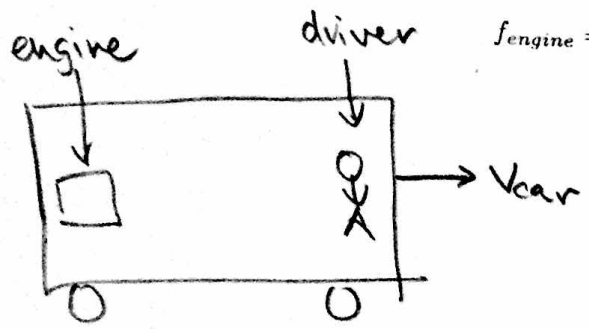
$$\frac{F_{dep}}{f_0} = \frac{V_{snd}}{V_{snd} + V_{car}} + 3$$

$$V_{snd} + V_{car} = \frac{V_{snd} f_0}{F_{dep}} = \frac{V_{snd}}{F_{dep}} \cdot \frac{F_{app} (V_{snd} - V_{car})}{V_{snd}}$$

↓ solving for this

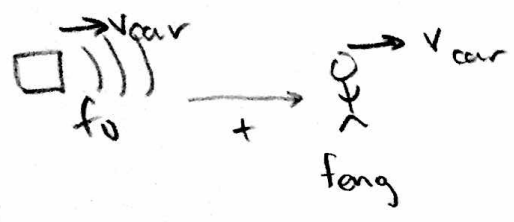
$$V_{car} = \frac{F_{app} (V_{snd} - V_{car})}{F_{dep}} - V_{snd} + 3$$

- 4c) (10 points) Show that the frequency emitted by the engine, as heard by the driver is given by...



$$f_{engine} = f_{avg} \left(1 - \frac{v_{car}^2}{v_{snd}^2}\right) \quad \text{where} \quad f_{avg} = \frac{f_{app} + f_{dep}}{2}$$

$$\frac{f_{obs}}{f_{emit}} = \frac{v_{snd} - v_{obs}}{v_{snd} - v_{emit}}$$



$$\frac{f_{app}}{f_0} = \frac{v_{snd} - 0}{v_{snd} - v_{car}} \quad \left| \quad f_0 = \frac{f_{app}(v_{snd} - v_{car})}{v_{snd}}$$

$$\frac{f_{eng}}{f_0} = \frac{v_{snd} - v_{car}}{v_{snd} - v_{car}}$$

$$\frac{f_{dep}}{f_0} = \frac{v_{snd}}{v_{snd} + v_{car}}$$

$$f_0 = \frac{f_{dep}(v_{snd} + v_{car})}{v_{snd}}$$

$$f_{eng} = f_0 \frac{v_{snd} - v_{car}}{v_{snd} - v_{car}}$$

2

0