MT1 Physics 1B F18

Full Name (Pri

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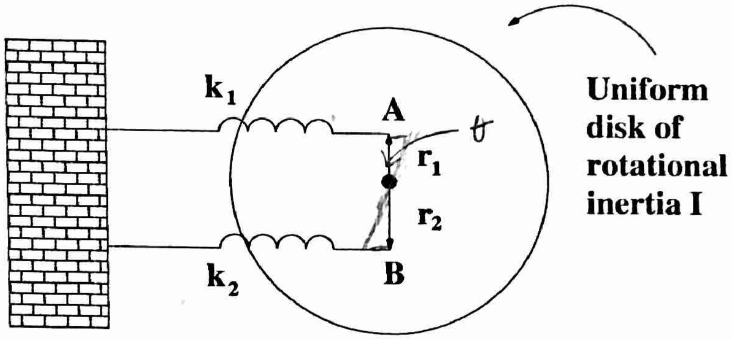
Student ID Nu

Seat Number ____

Problem	Grade
1	24 /30
2	26 /30
3	30 /30
Total	80 /90

80

- Do not peek at the exam until you are told to begin. You will have approximately 50 minutes to complete the exam.
- Don't spend too much time on any one problem. Solve 'easy' problems first. Go for partial credit!
- **HINT:** Focus on the concepts involved in the problem, the tools to be used, and the set-up. If you get these right, all that's left is algebra.
- **Have Fun!**



A uniform wheel of rotational inertia I (about its symmetry axis), free to spin (about that same symmetry axis), is attached to a wall by a pair of springs of spring constants k_1 and k_2, respectively. The springs connect to the wheel at points A and B as shown. Neither spring is stretched or squished when A is directly above B.

- a) (10 points) Suppose the wheel is rotated a small amount so that A is no longer directly over B and released. Obtain the differential equation that describes the subsequent motion of the wheel (you may assume the springs remain essentially horizontal).

8/10

$$I\alpha = -k_1 r_1 \sin\theta \, r_1 \cos\theta$$
$$+ k_2 r_2 \sin\theta \, r_2 \cos\theta$$

Spring force is negative! -2

for small θs, $\sin\theta \sim \theta$ & $\cos\theta \sim 1$

$$\therefore I\alpha = k_1 r_1^2 \theta + k_2 r_2^2 \theta$$

$$\boxed{\frac{d^2\theta}{dt^2} = \frac{k_1 r_1^2 \theta + k_2 r_2^2 \theta}{I}}$$

• b) (5 points) At what angular frequency will the system oscillate?

5

$$\omega^2\theta = (k_1 r_1^2 \theta + k_2 r_2^2 \theta)\frac{1}{I}$$

$$\omega^2 = (k_1 r_1^2 + k_2 r_2^2)\frac{1}{I}$$

$$\boxed{\omega = \sqrt{\frac{k_1 r_1^2 + k_2 r_2^2}{I}}}$$

• c) (15 points) Suppose we find that, upon releasing the wheel at rest from some small initial displacement θ_0, its amplitude falls to a fraction (F) of its initial value after it has made a large number (N) of complete cycles. Given ω_0, find the actual frequency at which the system is oscillating.

damped

11/15

$$\omega^2 = \omega_0^2 - \left(\frac{b}{2m}\right)^2$$

in this case, $\omega = \omega_0$ & find ω

$$\therefore \omega_0^2 = \omega^2 - \left(\frac{b}{2m}\right)^2$$

$$A = A_0 e^{-bt/2m}$$

$$F = e^{-bt/2m}$$

$NT = t$

$$2\pi \frac{1}{\sqrt{\left(\frac{k_1 r_1^2 + k_2 r_2^2}{I}\right)^2 \left(\frac{b}{2m}\right)^2}}$$ damped so $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\omega_0^2 - \left(\frac{b}{2m}\right)^2}}$ $\quad T = 2\pi \cancel{\sqrt{\frac{I}{k_1 r_1^2 + k_2 r_2^2}}}$

-4

$$-\ln F = \frac{b}{2m} N 2\pi \sqrt{\left(\frac{2m}{b}\right)^2 \left(\frac{k_1 r_1^2 + k_2 r_2^2}{I}\right) - 1}$$

$$-\ln F = \frac{b}{2m}\left(N 2\pi \sqrt{\frac{I}{k_1 r_1^2 + k_2 r_2^2}}\right)$$

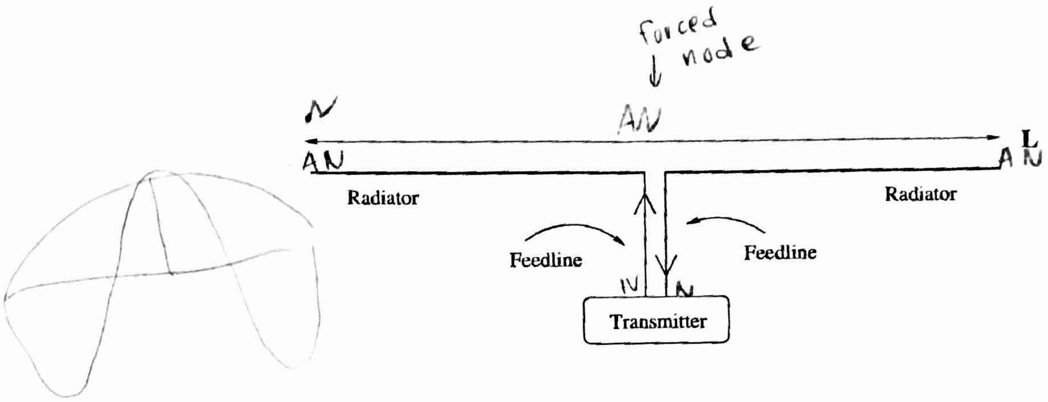
$$\left(\frac{2m}{b}\right)^2 \left(\frac{k_1 r_1^2 + k_2 r_2^2}{I}\right) \left(\frac{-N 2\pi}{\ln F}\right)^2 + 1$$

$$-\ln F\left(\frac{1}{N 2\pi}\sqrt{\frac{k_1 r_1^2 + k_2 r_2^2}{I}}\right) = \frac{b}{2m}$$

$$\frac{\left(\frac{k_1 r_1^2 + k_2 r_2^2}{I}\right)^2}{\left(\frac{-N 2\pi}{\ln F}\right)^2 + 1} = \left(\frac{b}{2m}\right)^2$$

$$\omega_0^2 = \frac{1}{I}(k_1 r_1^2 + k_2 r_2^2) - (\ln F)^2 \left(\frac{k r_1^2 + k_2 r_2^2}{N^2 4\pi^2 I}\right)$$

$$\boxed{\omega_0 = \sqrt{\left(\frac{k_1 r_1^2 + k_2 r_2^2}{I}\right)\left(1 - \frac{(\ln F)^2}{N^2 4\pi^2}\right)}}$$



A dipole antenna usually consists of two long horizontal radiating elements that are connected back to a transmitter by means of a feedline. Electrical current flows into and out of the radiators at the points where they are connected to the feedline. Electrical current has to stop at the outer end of each radiator, as there is no place for it to go.

- 2a) (5 points) If we feed the radiating elements with sinusoidally-varying electric current from the feedline, we may establish standing waves in the radiators. Will the boundary conditions on a radiator at resonance be "like" or "mixed"? On the diagram above, **clearly** label the ends of each radiator with N (**node**) or AN (**antinode**) where appropriate.

Boundary conditions on a radiator ~~will~~ at resonance will be mixed.

+3

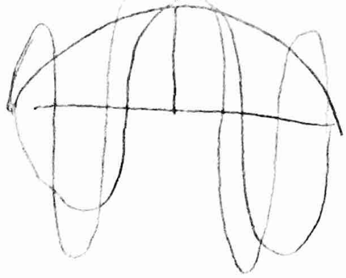
- 2b) (5 points) What is the total length of the dipole (L in the diagram) relative to the wavelength of the fundamental mode in the dipole?

$\lambda = 2L$

in fundamental mode

+5

- 2c) (5 points) If the fundamental frequency for the dipole is given by f_{fund}, what other frequencies will the dipole resonate at?



f_{fund}, $3f_{fund}$, $5f_{fund}$ ...

freq. resonate at $= (2N+1) f_{fund}$ where $N \geq 0$

+5

$vf \quad m = \frac{v_{act}}{c}$

- 2d) (5 points) In an ideal antenna, traveling waves will propagate along the radiating elements at the same speed light propagates with through a vacuum (C). In real antennas, traveling waves travel slower. The *velocity factor* for an antenna is the speed at which traveling waves propagate through it divided by the speed of light in vacuum. If a wave traveling through vacuum has a wavelength λ_{vac} when its frequency is equal to the fundamental frequency of the antenna, what is the velocity factor for this antenna?

λ_{vac} & f_{fund}

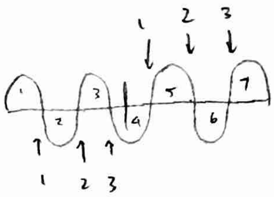
$v \cdot f \lambda$ +3

$v_{act} = f_{fund} \lambda_{vac}$

$$\text{velocity factor} = \frac{f_{fund} \lambda_{vac}}{C}$$

express c!

- 2e) (5 points) When driven at 28 MHz, three intermediate nodes are observed on each radiating element of a dipole antenna. What is the fundamental frequency of the dipole antenna? Which harmonic are we looking at?



$28 MHz / 7$

$f_{fund} = 4 MHz$

7th harmonic

+5

- 2f) (5 points) When tuning across the so-called high-frequency portion of the radio spectrum, it is noted that the dipole antenna connected to the receiver shows a greater sensitivity to signals at around 10.5, 17.5 and 24.5 MHz. What frequency is the antenna cut for (that is, what is its fundamental frequency)? What other frequencies below 50 MHz would it be expected to work well at?

$f_{fund} = 3.5 MHz$

Other frequencies below 50Hz
31.5 MHz, 38.5 MHz, 45.5 MHz
would be expected to work well

+5

$(f = (7 MHz) N + 3.5 MHz)$

- 3a) (15 points) Spectators at an auto race note that as cars race away from them, the sound they hear from the engines drops to half the frequency they heard as the cars approached. How fast are the cars moving? (You may take the speed of sound in air to be V_{snd})

cars moving toward $\quad \dfrac{f_{ot}}{f_{src}} = \dfrac{V_{snd}}{V_{snd} - V}$

cars moving away $\quad \dfrac{f_{oa}}{f_{src}} = \dfrac{V_{snd}}{V_{snd} + V}$

15

$$f_{ot} = 2 f_{oa}$$

$$\frac{V \cancel{snd}}{V_{snd} - V} = 2 \frac{V \cancel{snd}}{V_{snd} + V}$$

$$2 = \frac{V_{snd} + V}{V_{snd} - V}$$

$$2 V_{snd} - 2V = V_{snd} + V$$

$$V_{snd} = 3V$$

$$\boxed{V_{car} = \frac{V_{snd}}{3}}$$

- 3b) (15 points) Show that the frequency emitted by the engine is given by...

$$f_{engine} = f_{avg}(1 - \frac{v_{car}^2}{v_{snd}^2}) \quad \text{where} \quad f_{avg} = \frac{f_{approach} + f_{depart}}{2}$$

$f_{src} = f_{engine}$

$$\frac{f_{ot}}{f_{src}} = \frac{V_{snd}}{V_{snd} - V_{car}} \qquad f_{ot} = \frac{V_{snd}}{V_{snd} - V_{car}} f_{src}$$

$$\frac{f_{oa}}{f_{src}} = \frac{V_{snd}}{V_{snd} + V_{car}} \qquad f_{oa} = \frac{V_{snd}}{V_{snd} + V_{car}} f_{src}$$

$$f_{avg} = \frac{\left(\frac{V_{snd}}{V_{snd} - V_{car}}\right) f_{src} + \left(\frac{V_{snd}}{V_{snd} + V_{car}}\right) f_{src}}{2}$$

15

$$= \frac{f_{src}}{2}\left(\frac{V_{snd}^2 + \cancel{V_{snd} V_{car}} + V_{snd}^2 - \cancel{V_{snd} V_{car}}}{(V_{snd} + V_{car})(V_{snd} - V_{car})}\right)$$

$$= \frac{f_{src}}{\cancel{2}}\left(\frac{\cancel{2} V_{snd}^2}{V_{snd}^2 - V_{car}^2}\right)$$

$$f_{avg} = f_{src}\left(\frac{V_{snd}^2}{V_{snd}^2 - V_{car}^2}\right)$$

$$\left(\frac{V_{snd}^2 - V_{car}^2}{V_{snd}^2}\right) f_{avg} = f_{src}$$

$$f_{engine} = f_{avg}\left(\frac{v^2_{snd} - v^2_{car}}{v^2_{snd}}\right)$$

$$\boxed{f_{engine} = f_{avg}\left(1 - \frac{V_{car}^2}{V_{snd}^2}\right)}$$

- 3b) (15 points) Show that the frequency emitted by the engine is given by...

$$f_{engine} = f_{avg}\left(1 - \frac{v_{car}^2}{v_{snd}^2}\right) \quad \text{where} \quad f_{avg} = \frac{f_{approach} + f_{depart}}{2}$$

$f_{src} = f_{engine}$

$$\frac{f_{ot}}{f_{src}} = \frac{V_{snd}}{V_{snd} - V_{car}} \qquad f_{ot} = \frac{V_{snd}}{V_{snd} - V_{car}} f_{src}$$

$$\frac{f_{oa}}{f_{src}} = \frac{V_{snd}}{V_{snd} + V_{car}} \qquad f_{oa} = \frac{V_{snd}}{V_{snd} + V_{car}} f_{src}$$

$$f_{avg} = \frac{\left(\frac{V_{snd}}{V_{snd} - V_{car}}\right) f_{src} + \left(\frac{V_{snd}}{V_{snd} + V_{car}}\right) f_{src}}{2}$$

15

$$= \frac{f_{src}}{2}\left(\frac{V_{snd}^2 + \cancel{V_{snd}V_{car}} + V_{snd}^2 - \cancel{V_{snd}V_{car}}}{(V_{snd} + V_{car})(V_{snd} - V_{car})}\right)$$

$$= \frac{f_{src}}{\cancel{2}}\left(\frac{\cancel{2}V_{snd}^2}{V_{snd}^2 - V_{car}^2}\right)$$

$$f_{avg} = f_{src}\left(\frac{V_{snd}^2}{V_{snd}^2 - V_{car}^2}\right)$$

$$\left(\frac{V_{snd}^2 - V_{car}^2}{V_{snd}^2}\right) f_{avg} = f_{src}$$

$$f_{engine} = f_{avg}\left(\frac{v^2_{snd} - v^2_{car}}{v^2_{snd}}\right)$$

$$\boxed{f_{engine} = f_{avg}\left(1 - \frac{V_{car}^2}{V_{snd}^2}\right)}$$

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