MIDTERM EXAM #1
Physics 1B
Instructor: Anton Bondarenko

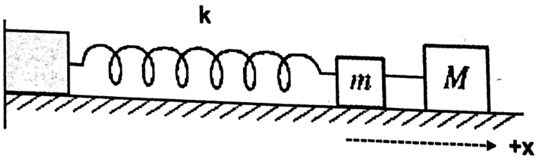
Friday, October 27th, 2017

You will have **50 minutes** to complete this exam. One 3" x 5" index card and a calculator is permitted. **Notes, books, cell phones, and any other electronics are not allowed.** Please write your answer in the space below the problem. You must show your work to get full credit. Extra paper is available at the front of the room. If a problem seems confusing or ambiguous, please ask the proctor for clarifications.

PLEASE DO NOT TURN PAGE UNTIL INSTRUCTED

Problem 1 (30 points total):
In **Figure 1**, two masses M and m are connected to a very light rigid bar and are attached to an ideal massless spring of spring constant k. Assume the mass of the rigid bar is negligible.

Figure 1



Part A (10 points): Using Newton's Second Law, write the differential equation that for $x(t)$, the system's displacement from equilibrium as a function of time, in terms of m, M, and k.

$F = (m+M)\,a(t)$

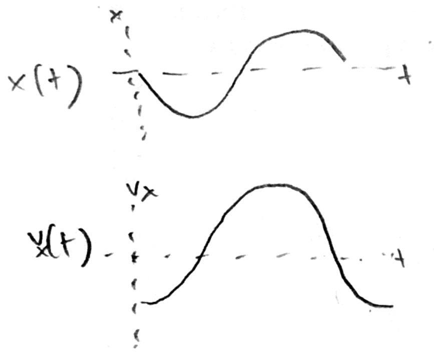
$-kx = (m+M)\,a(t)$

$\frac{d^2x}{dt^2} = a(t)$

$\boxed{x(t) = \sqrt{\frac{m+M}{k}}\,\frac{d^2x}{dt^2}}$

Part B (10 points): Assume that at $t = 0$ the system is set into motion from its equilibrium position by giving the masses an initial speed v_0 in the $-x$ direction. Write the solution for $x(t)$ in terms of m, M, k, and v_0.

$V_0 = -x$ direction



$$x(t) = -\sin(\omega t + \phi) \quad \omega = \sqrt{\frac{k}{m+M}}$$

$$\boxed{x(t) = -\sin\left(\sqrt{\frac{k}{(M+m)}}\; t\right)}$$

what amplitude?

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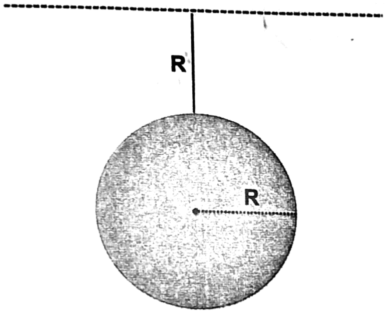
Part C (10 points): Now consider a physical pendulum consisting of a solid, uniform sphere of radius R suspended on a wire also of length R, as shown in **Figure 2**. What must the distance R be so that the period of the pendulum for small oscillations matches the period of the mass-spring system from **Figure 1**? Give an expression for R in terms of m, M, k, and gravitational acceleration g. The moment of inertia of a solid, uniform sphere of mass m_s and radius R about an axis through its center is $(2/5)m_sR^2$. (Hint: you will need the parallel-axis theorem: $I = I_{cm} + m_sh^2$).

Figure 2



$$T_s = 2\pi\sqrt{\frac{m+M}{k}}$$

$$T_p = 2\pi\sqrt{\frac{I}{mgd}}$$

$$I = \frac{2}{5}M_sR^2 + M_s(2R)^2$$

$$= \frac{2}{5}M_sR^2 + 4M_sR^2$$

$$= \frac{22}{5}M_sR^2$$

$$T_p = 2\pi\sqrt{\frac{\frac{22}{5}\cancel{M_s}R^{\cancel{2}}}{\cancel{M_s}\,g\,2\cancel{R}}} = 2\pi\sqrt{\frac{\frac{22}{5}R}{2g}}$$

$$= 2\pi\sqrt{\frac{22R}{10g}} = 2\pi\sqrt{\frac{11R}{5g}}$$

$$\cancel{2\pi}\sqrt{\frac{m+M}{k}} = \cancel{2\pi}\sqrt{\frac{11R}{5g}}$$

$$\frac{m+M}{k} = \frac{11R}{5g}$$

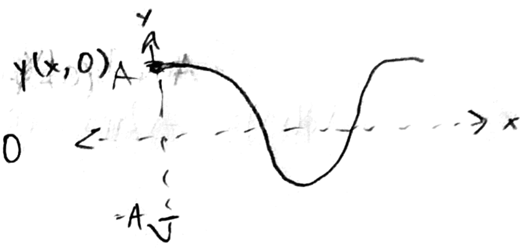
$$\boxed{\frac{5g}{11}\left(\frac{m+M}{k}\right) = R}$$

Problem 2 (30 points total):
A simple harmonic oscillator at the point $x = 0$ oscillates along the y-axis and generates a transverse wave on a rope that propagates in the $+x$ direction. The oscillator operates at frequency f and amplitude A. The rope has a linear mass density μ and is stretched to a tension force of magnitude T_s.

Part A (10 points): Write an equation for $y(x,t)$, the transverse displacement of the rope, in terms of x, t, μ, T_s, A, and f. Assume that the oscillator creating the wave has its maximum upward displacement at time $t = 0$.

$$y(x,t) = A\cos(kx - \omega t + \phi)$$

$$k = \frac{2\pi}{\lambda} \qquad \omega = 2\pi f \qquad \phi = 0$$



$$v = \lambda f$$

$$\lambda = \frac{v}{f} = \frac{\sqrt{\frac{T_s}{\mu}}}{f}$$

$$\boxed{y(x,t) = A\cos\left(\frac{2\pi}{\left(\frac{\sqrt{\frac{T_s}{\mu}}}{f}\right)} x - 2\pi f + \right)}$$

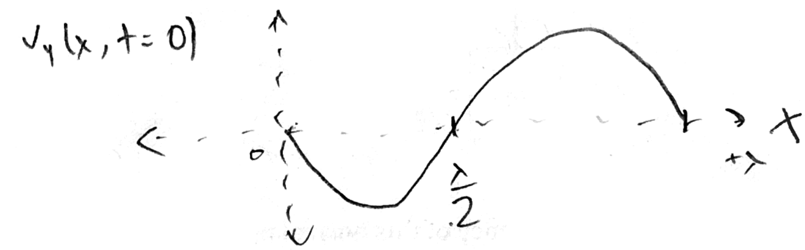
Part B (10 points): Obtain an equation for $v_y(x,t)$, the transverse velocity of the rope, in terms of x, t, μ, T_s, A, and f.

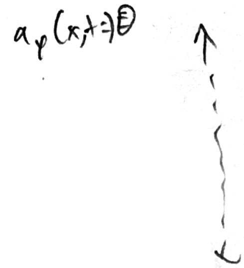
$$v_y(x,t) = \frac{\partial^2 y}{\partial t^2}$$

$$y(x,t) = A\cos\left(\frac{2\pi}{\left(\frac{\sqrt{\frac{T_s}{\mu}}}{f}\right)}x - 2\pi f t\right)$$

$$\boxed{v_y(x,t) = A 2\pi f \sin\left(\frac{2\pi}{\left(\frac{\sqrt{\frac{T_s}{\mu}}}{f}\right)}x - 2\pi f t\right)}$$

Part C (10 points): Consider the segment of the rope between $x = 0$ and $x = +\lambda$, where λ is the wavelength of the transverse wave. At $t = 0$, where on this segment are both the transverse acceleration and transverse velocity positive (upward)? Express your answer as a range of positions (x_{min}, x_{max}), where x_{min} and x_{max} are given in terms of f, T_s, and μ.





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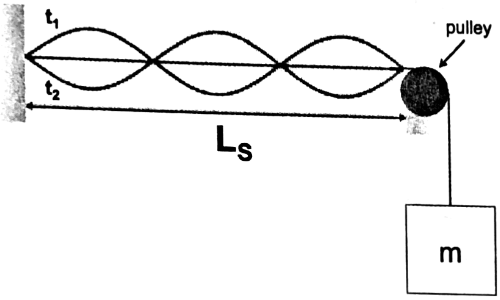
$\left(\frac{\lambda}{2}, \frac{3\lambda}{4}\right)$ λ

$$\boxed{\left(\frac{\sqrt{\frac{T_s}{\mu}}/f}{2}, \frac{3\left(\sqrt{\frac{T_s}{\mu}}/f\right)}{4}\right)}$$

Problem 3 (40 points):

A string with linear mass density μ is fixed to the wall at one end. At the other end, it is suspended over a massless, frictionless pulley and connected to block of mass m that hangs freely. The distance between the wall and pulley is L_S. A normal mode with amplitude A is excited on the string. Two successive photographs of the string are taken at times t_1 and t_2, as shown in **Figure 3** below. Assume that the string cannot move at the location of the pulley.

Figure 3



$+y$

$-y$

$\frac{2L_s}{n}$ $\quad n=3$

$\frac{4L}{3}$

Part A (10 points): What is the frequency of this normal mode in terms of L_S, μ, m, and gravitational acceleration g?

$v = f\lambda \qquad \lambda = \frac{2}{3}L_s$

$f = \frac{v}{\lambda} \qquad v = \sqrt{\frac{T_s}{\mu}} = \sqrt{\frac{mg}{\mu}} \qquad T_s - mg = 0$

$T_s = mg$

$$\boxed{f = \frac{\sqrt{\frac{mg}{\mu}}}{\frac{2L_s}{3}} = \frac{3\sqrt{\frac{mg}{\mu}}}{2L_s}}$$

✓

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Part B (10 points): Write an equation for *y(x,t)*, the transverse displacement of the string in **Figure 3**, in terms of *x, t, μ, A,* L_s*, m,* and *g*. Assume that the left and right ends of the string are at *x = 0* and *x =* L_s, respectively, and that every element of the string is at *y = 0* (i.e., the string is horizontal) at *t = 0.*

$$y(0,0) = 0$$

$$y(x,0) = 0$$

$$y(x,t) = A_{sw} \sin(kx)\sin(\omega t)$$

$$\boxed{y(x,t) = A \sin\left(\frac{3\pi}{L_s} x\right) \sin\left(\frac{6\pi\sqrt{\frac{mg}{\mu}}}{2L_s} t\right)} \checkmark$$

$$\omega = 2\pi f$$

$$k = \frac{2\pi}{\lambda}$$

$$k = \frac{2\pi}{\frac{2L}{3}} = \frac{3\pi}{L_s} \checkmark$$

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Part C (10 points): Assume that the string in **Figure 3** vibrates the air around it, producing sound at the frequency from Part A. Simultaneously, you blow air into a pipe of length L_P that is closed at one end and open at the other. At what air temperature T will the fundamental frequency of the pipe match the frequency emitted by the string? Give an expression for T in terms of string L_S, L_P, μ, m, g, molar mass of air M_{mol}, adiabatic index of air γ, and gas constant R.

$$v = \sqrt{\frac{\gamma R T}{M_{mol}}}$$

$$v = \lambda f$$

$$\lambda = \frac{2}{3} L_s \qquad f = \frac{3\sqrt{\frac{mg}{\mu}}}{2L_s}$$

maybe this

$$\cancel{\lambda f = \sqrt{\frac{\gamma R T}{M_{mol}}}}$$

$$\cancel{\lambda f = \sqrt{\frac{mg}{\mu}} = \sqrt{\frac{\gamma R T}{M_{mol}}}}$$

$$\cancel{\frac{mg}{\mu} = \frac{\gamma R T}{M_{mol}}}$$

$$\cancel{\boxed{\left(\frac{mg}{\mu}\right)\left(\frac{M_{mol}}{\gamma R}\right) = T}}$$

$L_s \neq L_p$

-2

$$f = \frac{\sqrt{\frac{mg}{\mu}}}{\frac{2}{3}L_s} = \frac{\sqrt{\frac{\gamma R T}{M_{mol}}}}{\frac{4}{3}L_p}$$

$\lambda_1 = 4L_p$

-2

$$f = \frac{4}{3 L_s}\sqrt{\frac{mg}{\mu}} = \frac{2}{3L_s}\sqrt{\frac{\gamma R T}{M_{mol}}}$$

$$16\frac{mg}{\mu} = 4\frac{\gamma R T}{M_{mol}}$$

$$\boxed{\frac{4mg}{\mu}\left(\frac{M_{mol}}{\gamma R}\right) = T}$$

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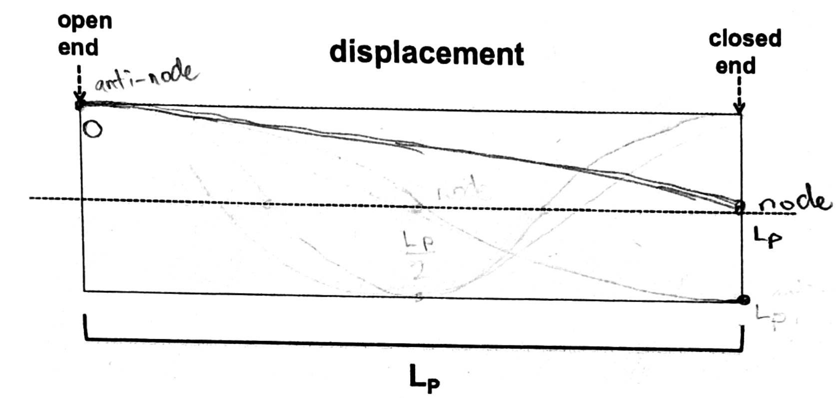
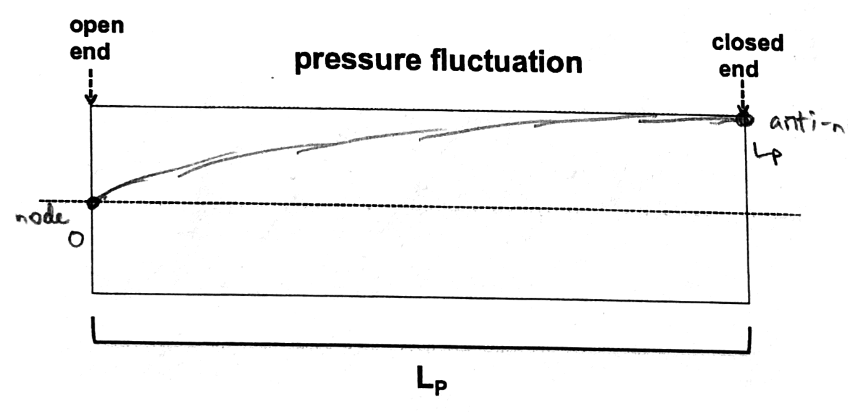
Part D (10 points): In the spaces below, draw a representation of the fundamental mode of the pipe from Part C in terms of the particle displacement and in terms of the pressure fluctuation, labeling all nodes and anti-nodes. You do not need to label the amplitudes.

$L_S =$





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