# MIDTERM EXAM #1
## Physics 1B
## Instructor: Anton Bondarenko

**Friday, October 27th, 2017**
**8:00 AM – 8:50 AM**

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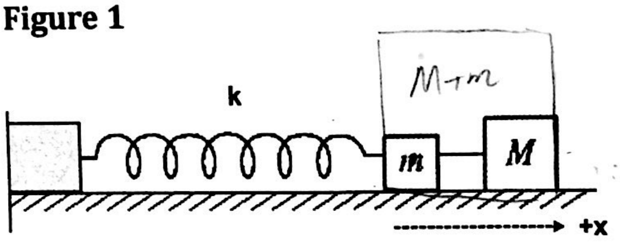
You will have **50 minutes** to complete this exam. One 3" x 5" index card and a calculator is permitted. **Notes, books, cell phones, and any other electronics are not allowed.** Please write your answer in the space below the problem. You must show your work to get full credit. Extra paper is available at the front of the room. If a problem seems confusing or ambiguous, please ask the proctor for clarifications.

# PLEASE DO NOT TURN PAGE UNTIL INSTRUCTED

**Problem 1 (30 points total):**
In **Figure 1**, two masses *M* and *m* are connected to a very light rigid bar and are attached to an ideal massless spring of spring constant *k*. Assume the mass of the rigid bar is negligible.

**Figure 1**




**Part A (10 points):** Using Newton's Second Law, write the differential equation that for *x(t)*, the system's displacement from equilibrium as a function of time, in terms of *m, M,* and *k*.

NSL: $F = (m+M)a = (m+M)\frac{d^2x}{dt^2} = -kx$

$= F_{spring}$

$\Downarrow$

$$\frac{d^2x(t)}{dt^2} = \sqrt{\frac{k}{m+M}}\,x(t)$$

**Part B (10 points):** Assume that at $t = 0$ the system is set into motion from its equilibrium position by giving the masses an initial speed $v_0$ in the $-x$ direction. Write the solution for $x(t)$ in terms of $m$, $M$, $k$, and $v_0$.

$x_0 = 0$, $V_0$, $-x$ dir, $\omega = \sqrt{\frac{k}{m+M}}$

$$A = \sqrt{x_0^2 + \frac{V_{0x}^2}{\omega^2}} = \sqrt{0^2 + \frac{V_0^2}{\left(\sqrt{\frac{k}{m+M}}\right)^2}} = \sqrt{\frac{(m+M)}{k}}\, V_0$$

$$\phi = \arctan\left(-\frac{V_0}{\omega x_0}\right) = \arctan(-\infty)$$, since $x(0)=0$ and then move left, $\phi = \frac{\pi}{2}$

Thus:

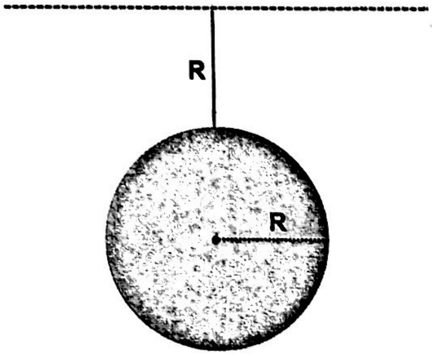
$$x(t) = A\cos(\omega t + \phi) = -\sqrt{\frac{(m+M)}{k}}\, V_0 \sin\left(\sqrt{\frac{k}{m+M}}\, t\right)$$

$t = 0 \Rightarrow 0$

$t > 0$ a bit $\Rightarrow x(t) < 0$

**Part C (10 points):** Now consider a physical pendulum consisting of a solid, uniform sphere of radius $R$ suspended on a wire also of length $R$, as shown in **Figure 2**. What must the distance $R$ be so that the period of the pendulum for small oscillations matches the period of the mass-spring system from **Figure 1**? Give an expression for $R$ in terms of $m$, $M$, $k$, and gravitational acceleration $g$. The moment of inertia of a solid, uniform sphere of mass $m_s$ and radius $R$ about an axis through its center is $(2/5)m_sR^2$. (Hint: you will need the parallel-axis theorem: $I = I_{cm} + m_sh^2$).

**Figure 2**




For figure 1,

$$T = \frac{1}{f} = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\frac{k}{M+m}}}$$

For Figure 2, we know that

$$I = I_{CM} + m_s \cdot d^2 \quad , \quad d = R+R = 2R$$

$$= I_{CM} + m_s(2R)^2$$

$$= \frac{2}{5}m_s R^2 + 4m_s R^2$$

$$= \frac{22}{5}m_s R^2,$$

Thus $\omega_s = \sqrt{\frac{m_s g d}{I}} = \sqrt{\frac{m_s g \cdot 2R}{\frac{22}{5}m_s R^2}} = \sqrt{\frac{5g}{11R}}$

We want $\omega_s = \omega$ such that $2\pi f_s = 2\pi f$. $f_s = f$. $T_s = T$.

So, $\sqrt{\frac{5g}{11R}} = \sqrt{\frac{k}{M+m}}$

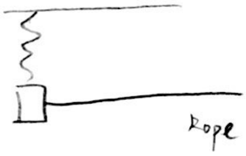
$$\frac{5g}{11R} = \frac{k}{M+m}$$

$$11R = \left(\frac{M+m}{k}\right)5g$$

$$R = \frac{5}{11}g\left(\frac{M+m}{k}\right)$$

**Problem 2 (30 points total):**
A simple harmonic oscillator at the point $x = 0$ oscillates along the y-axis and generates a transverse wave on a rope that propagates in the $+x$ direction. The oscillator operates at frequency $f$ and amplitude $A$. The rope has a linear mass density $\mu$ and is stretched to a tension force of magnitude $T_s$.

**Part A (10 points):** Write an equation for $y(x,t)$, the transverse displacement of the rope, in terms of $x$, $t$, $\mu$, $T_s$, $A$, and $f$. Assume that the oscillator creating the wave has its maximum upward displacement at time $t = 0$.




We know for $y(x,t)$ must of shape

$$y(x,t) = A_y \cos(kx - \omega t + \phi)$$

$$\omega = 2\pi f, \quad k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v}, \quad \left(\lambda = \frac{v}{f}\right)$$

$$v = \sqrt{\frac{T_s}{\mu}}, \quad \text{thus} \quad k = \frac{2\pi f}{\sqrt{\frac{T_s}{\mu}}} = 2\pi f \sqrt{\frac{\mu}{T_s}}$$

Since $t=0$ we have max up, $\cos(kx - \omega t + \phi) = 1$ on $x=0, t=0 \Rightarrow \phi$

Thus $y(x,t) = A\cos(kx - \omega t)$ ✓

$$= A\cos\left(2\pi f\sqrt{\frac{\mu}{T_s}}\, x - 2\pi f t\right)$$

**Part B (10 points):** Obtain an equation for $v_y(x,t)$, the transverse velocity of the rope, in terms of $x$, $t$, $\mu$, $T_s$, $A$, and $f$.

$$V_y(x,t) = \frac{d}{dt} y(x,t),$$

$$y(x,t) = A\cos\left(2\pi f\sqrt{\frac{\mu}{T_s}}\,x - 2\pi f t\right)$$

Thus $$v_y(x,t) = -A\omega \sin(kx - \omega t)$$

$$= -A\cdot 2\pi f\cdot \sin\left(2\pi f\sqrt{\frac{\mu}{T_s}}\,x - 2\pi f t\right)$$

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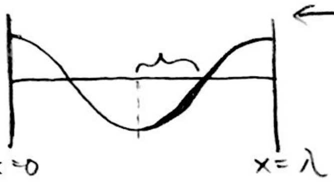
**Part C (10 points):** Consider the segment of the rope between $x = 0$ and $x = +\lambda$, where $\lambda$ is the wavelength of the transverse wave. At $t = 0$, where on this segment are both the transverse acceleration and transverse velocity positive (upward)? Express your answer as a range of positions ($x_{min}$, $x_{max}$), where $x_{min}$ and $x_{max}$ are given in terms of $f$, $T_s$, and $\mu$.

$$\lambda = \frac{v}{f} = \frac{\sqrt{\frac{T_s}{\mu}}}{f}$$




← Draw graph

From graph, with $\frac{d^2y}{dx^2} = \frac{1}{v^2}\frac{d^2y}{dt^2}$ (wave func) (concavity pos, acc pos)

we know it should be from $(x = \frac{\lambda}{2},\ x = \frac{3}{4}\lambda)$

Thus the range is $\left(\frac{\lambda}{2} = \frac{1}{2f}\sqrt{\frac{\mu}{T_s}},\ \frac{3}{4}\lambda = \frac{3}{4f}\sqrt{\frac{\mu}{T_s}}\right)$

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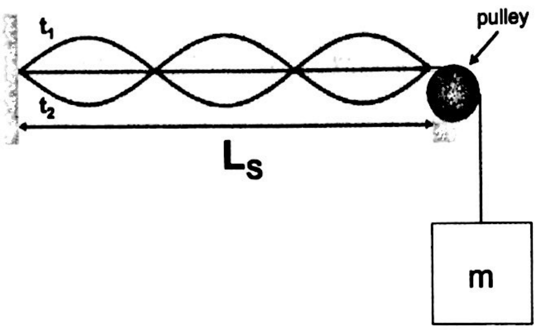
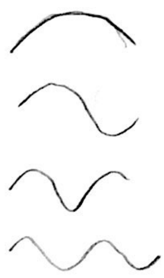
**Problem 3 (40 points):**

A string with linear mass density $\mu$ is fixed to the wall at one end. At the other end, it is suspended over a massless, frictionless pulley and connected to block of mass $m$ that hangs freely. The distance between the wall and pulley is $L_S$. A normal mode with amplitude $A$ is excited on the string. Two successive photographs of the string are taken at times $t_1$ and $t_2$, as shown in **Figure 3** below. Assume that the string cannot move at the location of the pulley.

**Figure 3**



**Part A (10 points):** What is the frequency of this normal mode in terms of $L_S$, $\mu$, $m$, and gravitational acceleration $g$?

From graph, we know

$$\lambda = \frac{2}{3} L_s$$

Since we hang the mass, $|T_s| = |W_m| = |mg|$

Thus $v = \sqrt{\frac{T_s}{\mu}} = \sqrt{\frac{mg}{\mu}}$

Thus $f = \frac{v}{\lambda} = \frac{\sqrt{\frac{mg}{\mu}}}{\frac{2}{3} L_s} = \frac{3}{2L_s}\sqrt{\frac{mg}{\mu}}$ ✓

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**Part B (10 points):** Write an equation for *y(x,t)*, the transverse displacement of the string in **Figure 3**, in terms of *x, t, μ, A, Lₛ, m,* and *g*. Assume that the left and right ends of the string are at *x = 0* and *x = Lₛ*, respectively, and that every element of the string is at *y = 0* (i.e., the string is horizontal) at *t = 0*.

We know from A, $\lambda = \frac{2}{3} L_s$, $v = \sqrt{\frac{mg}{\mu}}$, $f = \frac{3}{2L_s}\sqrt{\frac{mg}{\mu}}$

This is a standing wave, thus having formula

basic -2

$y_{sw} = A_{sw} \sin kx \sin \omega t$, $A_{sw} = \cancel{2}A$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{\frac{2}{3}L_s} = \frac{3\pi}{L_s}$ ✓, $\omega = 2\pi f = \frac{3\pi}{L_s}\sqrt{\frac{mg}{\mu}}$ ✓

We want $y(x=0, t=0) = 0$.

$y(x=L_s, t=0) = 0$,

Thus $y_{sw} = 2A \cdot \sin\left(\frac{3\pi}{L_s} x\right) \sin\left(\frac{3\pi}{L_s}\sqrt{\frac{mg}{\mu}}\, t\right)$

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**Part C (10 points):** Assume that the string in **Figure 3** vibrates the air around it, producing sound at the frequency from Part A. Simultaneously, you blow air into a pipe of length $L_P$ that is closed at one end and open at the other. At what air temperature $T$ will the fundamental frequency of the pipe match the frequency emitted by the string? Give an expression for $T$ in terms of string $L_S$, $L_P$, $\mu$, $m$, $g$, molar mass of air $M_{mol}$, adiabatic index of air $\gamma$, and gas constant $R$.

$$f_s = f = \frac{3}{2L_s}\sqrt{\frac{mg}{\mu}}$$

closed one end, open the other $\Rightarrow$ $\underset{\text{fundamental}}{\lambda_s} = 4L_p$

we have $V_s = \lambda_s f_s$,

$$V_s = 4L_p \cdot \frac{3}{2L_s}\sqrt{\frac{mg}{\mu}} = \frac{6L_p}{L_s}\sqrt{\frac{mg}{\mu}}$$

However, we also have

$$V_s = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma RT}{M_{mol}}}$$

Thus $\sqrt{\frac{\gamma RT}{M_{mol}}} = \lambda_s f_s$

$$\Rightarrow \sqrt{\frac{\gamma RT}{M_{mol}}} = \frac{6L_p}{L_s}\sqrt{\frac{mg}{\mu}}$$

$$\frac{\gamma RT}{M_{mol}} = \frac{36 L_p^2 mg}{L_s^2 \mu}$$

$$T = \frac{36 L_p^2 mg M_{mol}}{L_s^2 \mu \gamma R}$$ ✓

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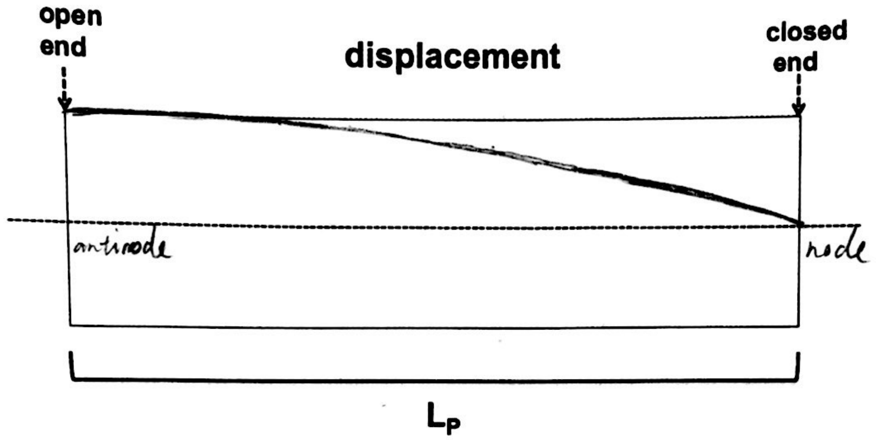
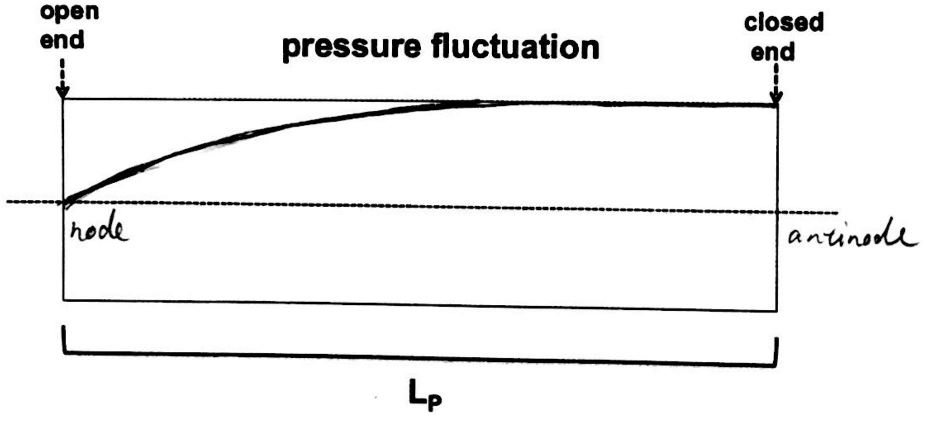
**Part D (10 points):** In the spaces below, draw a representation of the fundamental mode of the pipe from Part C in terms of the particle displacement and in terms of the pressure fluctuation, labeling all nodes and anti-nodes. You do not need to label the amplitudes.

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