MIDTERM EXAM #1
Physics 1B Lecture 3
Instructor: Anton Bondarenko

Tuesday, May 1st, 2018
1:00 PM – 1:50 PM

Last name: _

First name: _

University ID:

You will have **50 minutes** to complete this exam. One standard 3" x 5" index card and a calculator are permitted. Notes, books, cell phones, and any other electronics are not allowed. **You must complete the exam using a blue or black ink pen. Exams completed in pencil will not receive credit.**

Please write your answer in the space below the problem. **You must write legibly and demonstrate your reasoning to get full credit.** For quantitative problems, always express your final answer in terms of the variables given in the problem unless otherwise stated. For clarity, please draw a box around your final answer.

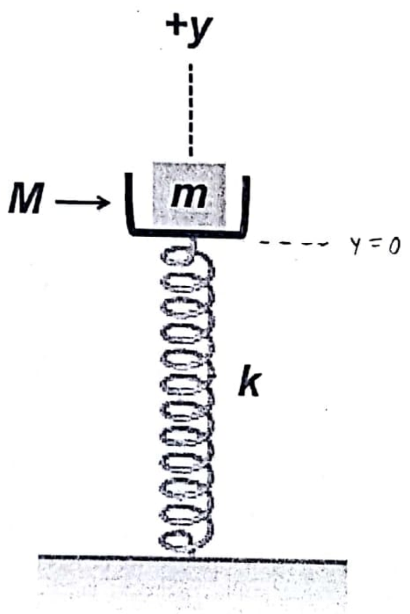
Extra paper is attached at the back of the exam and more is available in the front of the room. If a problem seems confusing or ambiguous, please ask the proctor for clarifications.

PLEASE DO NOT TURN PAGE UNTIL INSTRUCTED

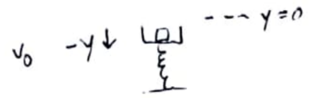
Problem 1 (30 points total):

As shown in **Figure 1**, a tray of mass M is connected to the top of a vertical ideal spring of force constant k. A block of mass m is placed inside the tray. In the coordinate system for this problem, $y = 0$ corresponds to the equilibrium position and the force of gravity acts in the $-y$ direction. The acceleration magnitude due to gravity is g.

Figure 1



Part A (10 points): Assume that at time $t = 0$, the system is set into motion from its equilibrium position by giving the tray and block an initial speed v_0 in the $-y$ direction. Determine $y(t)$, the position of the tray and block as a function of time, in terms of k, m, M, and v_0. (For this part of the problem, you can assume that the amplitude of oscillations is sufficiently small so that the block remains in contact with the tray at all times.)



since system $V_y = V_0$ at equilibrium,

V_0 is max $= \omega A$

$$Y(t) = A\cos\left(\sqrt{\frac{K}{M+m}}\,t + \phi\right)$$

$$Y(0) = 0 = A\cos(\phi)$$

$$\phi = \frac{\pi}{2}$$

$$V_y(t) = -\sqrt{\frac{K}{M+m}}\,A\sin\left(\sqrt{\frac{K}{M+m}}\,t + \frac{\pi}{2}\right)$$

$$V_y(0) = -\sqrt{\frac{K}{M+m}}\,A\sin\left(\frac{\pi}{2}\right) = V_0$$

$$-\sqrt{\frac{K}{M+m}}\,A = V_0$$

$$A = -V_0\sqrt{\frac{M+m}{K}}$$

$$\boxed{Y(t) = -V_0\sqrt{\frac{M+m}{k}}\cos\left(\sqrt{\frac{K}{M+m}}\,t + \frac{\pi}{2}\right)}$$

+8

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Part B (10 points): What is the maximum initial speed v_0 that can be imparted to the block and tray so that the block remains in contact with tray at all times during the oscillation? Express your answer in terms of k, m, M, and g.

Force of spring on system $\Sigma F = (M+m)a_y$

$$-ky + (m+M)g = (m+M)a_y$$

when does $F_{sp} > F_g$

mg ← keeping block on tray

effort $\times 3$

Part C (10 points): For the maximum allowed initial speed v_0 found in Part B, use conservation of mechanical energy to find the distance from the equilibrium position at which the block and tray have one-half the maximum speed. Express your answer in terms of k, m, M, and g.

$$E_i = E_f$$

$$\frac{1}{2}k\cancel{y_i^2}^{\,0} + mg\cancel{(y_i)}^{\,0} + \frac{1}{2}mv_0^2 = \frac{1}{2}ky_f^2 + mgy_f + \frac{1}{2}mv_{yf}^2$$

where $v_{yf} = \frac{1}{2}v_0$

$$-\frac{1}{2}(M+m)\left(\frac{v_0}{2}\right)^2 + \frac{1}{2}(M+m)v_0^2 = \frac{1}{2}ky_f^2 + (M+m)gy_f$$

$$\frac{1}{2}(M+m)\left(v_0^2 - \frac{v_0^2}{4}\right) = \frac{1}{2}ky_f^2 + (M+m)gy_f$$

$$y_f = \frac{-(M+m)g \pm \sqrt{((M+m)g)^2 - k(M+m)\left(v_0^2 - \frac{v_0^2}{4}\right)}}{k}$$

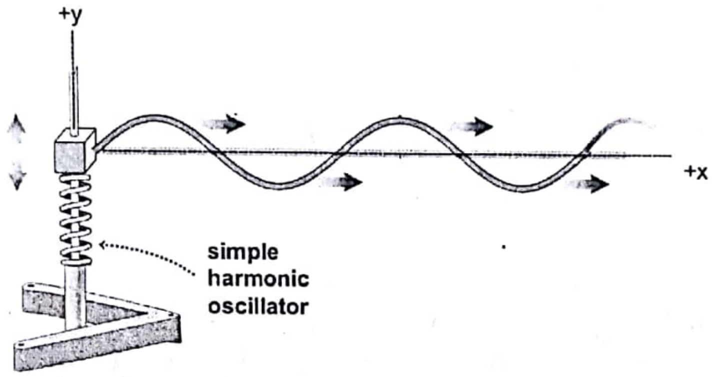
$$|y_f| = \left|\frac{-(M+m)g + \sqrt{(M+m)\left[(M+m)g - k\left(\frac{3v_0^2}{4}\right)\right.}}{k}\right|$$

effort +6

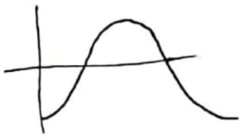
Problem 2 (40 points total):

As shown in **Figure 2**, one end of a rope is attached to a simple harmonic oscillator at $x = 0$ that oscillates along the y-axis. The oscillation generates a transverse sinusoidal wave that propagates along the rope in the $+x$ direction. The oscillator operates at frequency f and amplitude A and has its most *negative* y-displacement at $t = 0$. The rope has a uniform linear mass density μ and is stretched to a tension force of magnitude T_s.

Figure 2



Part A (10 points): Determine the expression for $y(x,t)$, the transverse displacement of the rope as a function of x-position and time, in terms of A, f, μ, and T_s.



$$y(x,t) = -A\cos(kx - \omega t + \phi)$$
$$y(0,0) = -A\cos(\phi) = -A$$
$$\cos\phi = 1$$
$$\phi = 0$$

$$\frac{\omega}{k} = v = \sqrt{\frac{T_s}{\mu}}$$

$$\omega = 2\pi f$$

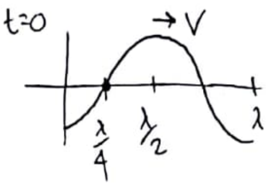
$$k = \frac{\omega}{v} = 2\pi f\sqrt{\frac{\mu}{T_s}}$$

$$y(x,t) = -A\cos\left(2\pi f\sqrt{\frac{\mu}{T_s}}\,x - 2\pi f t\right)$$
$$= \boxed{-A\cos\left(2\pi f\left(\sqrt{\frac{\mu}{T_s}}\,x - t\right)\right)}$$

Check $y(0,0) = -A\cos(0) = -A$

$$y(0,1) = -A\cos(-2\pi f)$$

Part B (10 points): Consider a particle on the rope located at $x = \lambda/4$, where λ is the wavelength of the transverse sinusoidal wave. Derive an expression for the times t at which this particle has its maximum *upward* acceleration in terms of f. (Note that there are an infinite number of solutions).



$T = 1/f$

$K = 2\pi f\sqrt{\frac{\mu}{T_s}}$

$K = \frac{\cancel{2\pi}}{\lambda} = \cancel{2\pi} f\sqrt{\frac{\mu}{T_s}}$

$\lambda = T\sqrt{\frac{T_s}{\mu}} = Tv$

$$y(x=\tfrac{\lambda}{4}, t) = -A\cos\left(2\pi f\left(\sqrt{\frac{\mu}{T_s}}\left[\frac{\lambda}{4}\right] - t\right)\right)$$

$$= -A\cos\left(2\pi f\left(\sqrt{\frac{\mu}{T_s}}\left[\frac{1}{4f}\sqrt{\frac{T_s}{\mu}}\right] - t\right)\right)$$

$$= -A\cos\left(2\pi f\left(\frac{1}{4f} - t\right)\right)$$

$$= -A\cos\left(\frac{\pi}{2} - 2\pi f t\right)$$

$$v_y(\tfrac{\lambda}{4}, t) = -2\pi f A\sin\left(\frac{\pi}{2} - 2\pi f t\right)$$

$$a_y(\tfrac{\lambda}{4}, t) = 4\pi^2 f^2 A\cos\left(\frac{\pi}{2} - 2\pi f t\right)$$

at what t is $\cos\left(\frac{\pi}{2} - 2\pi f t\right) = 1$?

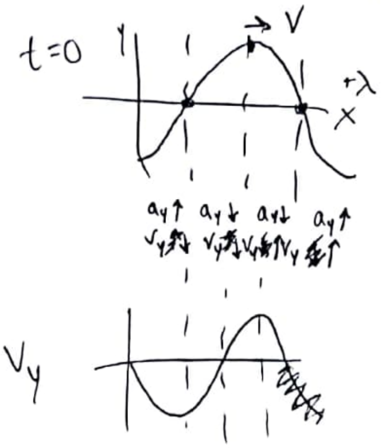
$$\frac{\pi}{2} - 2\pi f t = n \cdot 2\pi$$

$$t = \frac{2\cancel{\pi} n - \frac{\cancel{\pi}}{2}}{-2\cancel{\pi} f}$$

$$= \frac{\frac{1}{2} - 2n}{2f} = \frac{\frac{1}{2} - \frac{4n}{2}}{\frac{2f}{1}}$$

$$= \boxed{\frac{1-4n}{4f}}$$

Part C (10 points): Consider the segment of the rope between $x = 0$ and $x = +\lambda$, where λ is the wavelength of the transverse sinusoidal wave. At $t = 0$, where on this segment are both the transverse acceleration and transverse velocity positive (upward)? Express your answer as a range of positions (x_{min}, x_{max}), where x_{min} and x_{max} are expressed in terms of f, μ, and T_s.



transverse velocity positive when slope is negative since wave moving in +x-dir

$$y(x,t) = -A\cos\left(2\pi f\left(\sqrt{\frac{\mu}{T_s}}\,x - t\right)\right)$$

$$V_y(x,t) = -2\pi f A\sin\left(2\pi f\left(\sqrt{\frac{\mu}{T_s}}\,x - t\right)\right)$$

$$a_y(x,t) = 2^2\pi^2 f^2 A\cos\left(2\pi f\left(\sqrt{\frac{\mu}{T_s}}\,x - t\right)\right)$$

When $y(x,t) < 0$, $a_y(x,t) > 0$

V_y

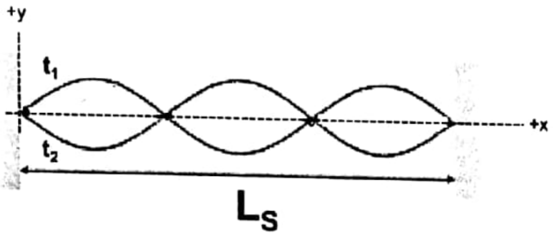
x_{min} is $\frac{3\lambda}{4}$ $\quad$ x_{max} is λ $\quad$ $\lambda = \frac{1}{f}\sqrt{\frac{T_s}{\mu}}$

$$\left(\frac{3}{4}\left[\frac{1}{f}\sqrt{\frac{T_s}{\mu}}\right],\ \left[\frac{1}{f}\sqrt{\frac{T_s}{\mu}}\right]\right)$$

$$\boxed{\left(\frac{3}{4f}\sqrt{\frac{T_s}{\mu}},\ \frac{1}{f}\sqrt{\frac{T_s}{\mu}}\right)}$$

Part D (10 points): A segment of length L_s is now cut from the same rope (linear mass density μ) and tied between two walls at the same tension magnitude T_s. A normal mode with amplitude A_{sw} is excited on the string. Two successive photographs of the string are taken at times t_1 and t_2, as shown in **Figure 3** below. Determine the expression for $y(x,t)$, the transverse displacement of the rope as a function of x-position and time, in terms of A_{sw}, L_s, μ, and T_s. Assume that the left and right ends of the string are at $x = 0$ and $x = L_s$, respectively, and that every element of the string is at $y = 0$ (i.e., the string is horizontal) at $t = 0$.

Figure 3



$$f_3 = \frac{v}{\lambda_n} = \frac{nv}{2L_s} = \frac{3\sqrt{\frac{T_s}{\mu}}}{2L_s}$$

$$\omega = 2\pi f$$
$$k = 2\pi f\sqrt{\frac{\mu}{T_s}}$$

$$y(x,t) = A_w \sin(kx + \phi_1)\sin(\omega t + \phi_2)$$

$$y(x,t) = A_w \sin\left(2\pi\left(\frac{3}{2L_s}\sqrt{\frac{T_s}{\mu}}\right)\sqrt{\frac{\mu}{T_s}}\,x + \phi_1\right)\sin\left(2\pi\left(\frac{3}{2L_s}\sqrt{\frac{T_s}{\mu}}\right)t + \phi_2\right)$$

$$= A_w \sin\left(\frac{3\pi}{L_s}x + \phi_1\right)\sin\left(\frac{3\pi}{L_s}\sqrt{\frac{T_s}{\mu}}\,t + \phi_2\right)$$

$$y(x,0) = A_w \sin\left(\frac{3\pi}{L_s}x + \phi_1\right)\underbrace{\sin\left(\cancel{\frac{3\pi}{L_s}\sqrt{\frac{T_s}{\mu}}\,t} + \phi_2\right)} = 0$$

$$\cancel{\frac{3\pi}{L_s}\sqrt{\frac{T_s}{\mu}}\,t} + \phi_2 = 0$$

$$y(0,t) = 0 \Rightarrow \sin\left(\cancel{\frac{3\pi}{L_s}x} + \phi_1\right) = 0$$

$$\phi_1 = 0$$

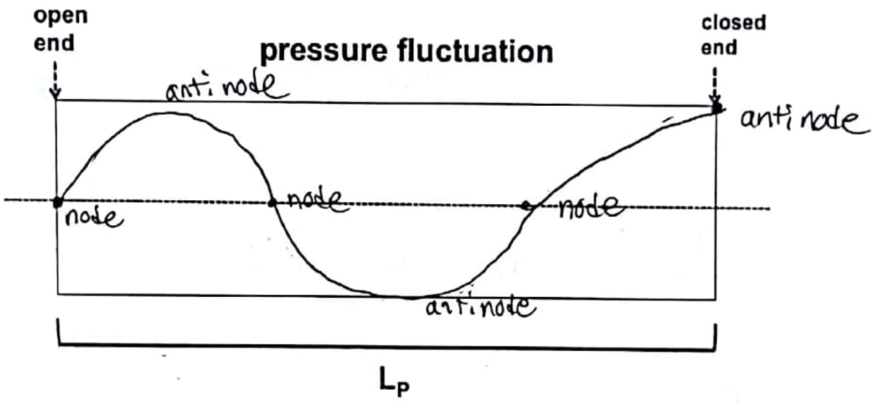
$$y(x,t) = \boxed{A_w \sin\left(\frac{3\pi}{L_s}x\right)\sin\left(\frac{3\pi}{L_s}\sqrt{\frac{T_s}{\mu}}\,t\right)}$$

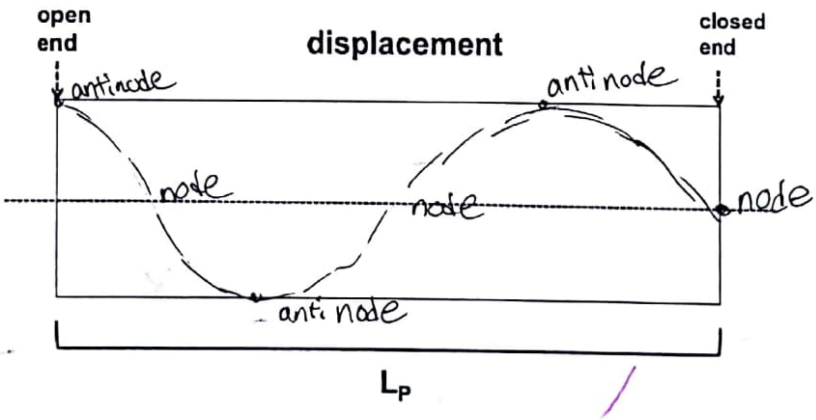
Problem 3 (30 points total):

Consider a musical pipe of length L_P that is open at one end and closed at the other. Air is blown into the pipe, exciting a longitudinal normal mode. Measurements indicate that there are three pressure fluctuation nodes associated with this normal mode.

Part A (10 points): In the spaces below, draw a representation of this normal mode in terms of the pressure fluctuation and in terms of the particle displacement, labeling all nodes and anti-nodes. You do not need to label the amplitudes.

9





Part B (10 points): Measurements indicate that the maximum pressure fluctuation is p_{max}, the maximum particle displacement is D_{max}, and the speed of sound in air is v_s. Determine the expressions for the pressure fluctuation $P_W(x,t)$ and displacement $D(x,t)$ as functions of x-position and time in terms of D_{max}, p_{max}, v_s, and L_P. Assume the open and closed ends of the pipe are at $x = 0$ and $x = L_P$, respectively, and that the entire pipe is at the equilibrium pressure at $t = 0$.

$K = \frac{\omega}{V}$

9

$$D(x,t) = D_{max} \sin(kx + \phi_1) \sin(\omega t + \phi_2)$$

$f_s = \frac{5V_s}{4L}$

$$= D_{max} \cos\left(2\pi \frac{5V_s}{4L_p} \frac{1}{V_s} x\right) \sin\left(2\pi \frac{5V_s}{4L_p} t + \phi_2\right)$$

$$\boxed{D(x,t) = D_{max} \cos\left(\pi \frac{5}{2L_p} x\right) \sin\left(\cancel{2}\pi \frac{5V_s}{2L_p} t + \frac{\pi}{2}\right)}$$

$\sin(\theta + \pi/2) \rightarrow \cos(\theta)$

$$P_\omega(x,t) = P_{max} \sin\left(2\pi \frac{5V_s}{4L} \frac{1}{V_s} x\right) \cos\left(2\pi \frac{5V_s}{4L} t + \phi_2\right)$$

$$P_\omega(x,0) = 0 = P_{max} \sin\left(\cancel{2}\pi \frac{\cancel{5V_s}}{_2\cancel{4}L} \frac{1}{\cancel{V_s}} x\right) \cos(\phi_2)$$

$$\cos(\phi_2) = 0$$

$$\phi_2 = \frac{\pi}{2}$$

$$\boxed{P_w(x,t) = P_{max} \sin\left(\pi \frac{5}{2L_p} \cancel{x}\right) \cos\left(\pi \frac{5V_s}{2L_p} + \frac{\pi}{2}\right)}$$

x

$\cos(\theta + \pi/2) \rightarrow \sin\theta$

Part C (10 points): You now take a second identical pipe of length L_P and excite a normal mode corresponding to the next allowed harmonic above the one in Parts A and B. If both pipes are played simultaneously, what is the resulting beat frequency? Express your answer in terms of v_s and L_P.

10

$$f_5 = \frac{5v_s}{4L}$$

$$f_7 = \frac{7v_s}{4L}$$

$$f_{beat} = |f_5 - f_7| = \left|\frac{5v_s}{4L_p} - \frac{7v_s}{4L_p}\right|$$

$$= \frac{2v_s}{4L_p} = \boxed{\frac{v_s}{2L_p}} \text{ Hz}$$

SCORING

Problem 1:
Part A: ______ / 10
Part B: ______ / 10
Part C: ______ / 10
Total: 17 / 30

Problem 2:
Part A: ______ / 10
Part B: ______ / 10
Part C: ______ / 10
Part D: ______ / 10
Total: 40 / 40

Problem 3:
Part A: 9 / 10
Part B: 9 / 10
Part C: 10 / 10
Total: 28 / 30

Total Midterm #1 Score ~~8~~ 85 / 100