MIDTERM EXAM #1
Physics 1B
Instructor: Anton Bondarenko

Friday, October 27th, 2017
8:00 AM – 8:50 AM

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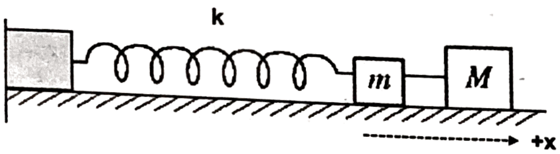
You will have **50 minutes** to complete this exam. One 3" x 5" index card and a calculator is permitted. **Notes, books, cell phones, and any other electronics are not allowed.** Please write your answer in the space below the problem. You must show your work to get full credit. Extra paper is available at the front of the room. If a problem seems confusing or ambiguous, please ask the proctor for clarifications.

PLEASE DO NOT TURN PAGE UNTIL INSTRUCTED

Problem 1 (30 points total):
In **Figure 1**, two masses M and m are connected to a very light rigid bar and are attached to an ideal massless spring of spring constant k. Assume the mass of the rigid bar is negligible.

Figure 1



Part A (10 points): Using Newton's Second Law, write the differential equation that for $x(t)$, the system's displacement from equilibrium as a function of time, in terms of m, M, and k.

$$F_s = (m+M)a_x = -kx$$

$$x(t) = A\cos(\omega t + \phi)$$

$$a_x(t) = \frac{-kx(t)}{(m+M)} \qquad \frac{d^2x(t)}{dt^2} = a_x(t)$$

$$\frac{d^2x}{dt^2} = \frac{-kx(t)}{(m+M)}$$

$$\boxed{x(t) = \frac{-(m+M)}{k}\frac{d^2x}{dt^2}}$$

Part B (10 points): Assume that at $t = 0$ the system is set into motion from its equilibrium position by giving the masses an initial speed v_0 in the $-x$ direction. Write the solution for $x(t)$ in terms of m, M, k, and v_0.

at $t=0$, $x=0$, moving $-x$

$x(t) = -A\sin(\omega t)$ $\quad V_0 \quad x_0 = 0 \quad \omega = \sqrt{\frac{k}{m+M}}$

$$A = \sqrt{x_0^2 + \frac{V_0^2}{\omega^2}} = \sqrt{0 + \frac{V_0^2}{\frac{k}{m+M}}} = \sqrt{V_0^2 \cdot \frac{m+M}{k}} = V_0\sqrt{\frac{m+M}{k}}$$

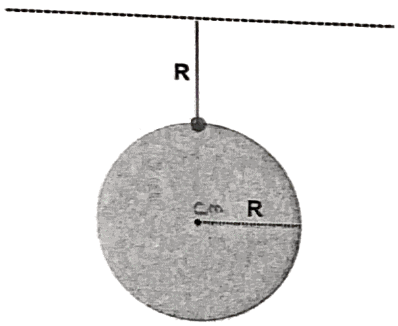
$$\boxed{x(t) = -V_0\sqrt{\frac{m+M}{k}}\sin\left(\sqrt{\frac{k}{m+M}}\cdot t\right)}$$

Part C (10 points): Now consider a physical pendulum consisting of a solid, uniform sphere of radius R suspended on a wire also of length R, as shown in **Figure 2**. What must the distance R be so that the period of the pendulum for small oscillations matches the period of the mass-spring system from **Figure 1**? Give an expression for R in terms of m, M, k, and gravitational acceleration g. The moment of inertia of a solid, uniform sphere of mass m_s and radius R about an axis through its center is $(2/5)m_sR^2$. (Hint: you will need the parallel-axis theorem: $I = I_{cm} + m_sh^2$).

$\omega = \frac{2\pi}{T_1} = \sqrt{\frac{k}{m+M}}$

Figure 2

$T_1 = 2\pi\sqrt{\frac{m+M}{k}}$



m_s $\quad I_{cm} = \frac{2}{5} m_s R^2$ $\quad h = R$

$I_{new} = I_{cm} + m_s h^2$

$I_{new} = \frac{2}{5} m_s R^2 + m_s R^2 = \frac{7}{5} m_s R^2$

$T_2 = 2\pi\sqrt{\frac{I}{Mgd}} = 2\pi\sqrt{\frac{I_{new}}{M_s g R}} = 2\pi\sqrt{\frac{\frac{7}{5}\cancel{M_s}R^{\cancel{2}}}{g\cancel{M_s}\cancel{R}}}$

$T_2 = 2\pi\sqrt{\frac{7R}{5g}}$

$T_1 = T_2$

$\cancel{2\pi}\sqrt{\frac{m+M}{k}} = \cancel{2\pi}\sqrt{\frac{7R}{5g}}$

$\frac{m+M}{k} = \frac{7R}{5g}$

$\boxed{R = \frac{5g(m+M)}{7k}}$

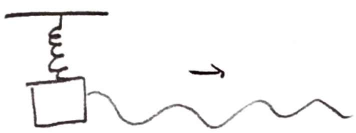
−2

Problem 2 (30 points total):
A simple harmonic oscillator at the point $x = 0$ oscillates along the y-axis and generates a transverse wave on a rope that propagates in the $+x$ direction. The oscillator operates at frequency f and amplitude A. The rope has a linear mass density μ and is stretched to a tension force of magnitude T_s.

Part A (10 points): Write an equation for $y(x,t)$, the transverse displacement of the rope, in terms of x, t, μ, T_s, A, and f. Assume that the oscillator creating the wave has its maximum upward displacement at time $t = 0$.

→ cos

$V = \sqrt{\frac{T_s}{\mu}}$



$f, A \quad \mu \quad T_s$

$v = \lambda f \qquad \lambda = \frac{1}{f}\sqrt{\frac{T_s}{\mu}}$

$y(x,t) = A\cos(kx - \omega t)$

$k = \frac{2\pi}{\lambda} = 2\pi \cdot f \cdot \sqrt{\frac{\mu}{T_s}}$

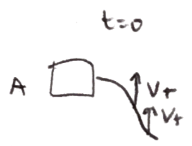
$y(x,t) = A\cos\left(2\pi f\sqrt{\frac{\mu}{T_s}}\,x - 2\pi f\, t\right)$

$\omega = 2\pi f$

$$\boxed{y(x,t) = A\cos\left(2\pi f\left(\sqrt{\frac{\mu}{T_s}}\,x - t\right)\right)}$$

Part B (10 points): Obtain an equation for $v_y(x,t)$, the transverse velocity of the rope, in terms of x, t, μ, T_s, A, and f.

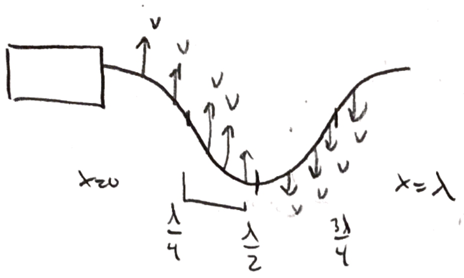
$$v_y(x,t) = \frac{\partial y(x,t)}{\partial t} \qquad y(x,t) = A\cos\left(2\pi f\sqrt{\frac{\mu}{T_s}}\,x - 2\pi f t\right)$$

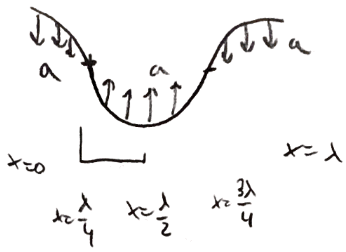


$$= (-2\pi f)\cdot -\left(A\sin\left(2\pi f\sqrt{\frac{\mu}{T_s}}\,x - 2\pi f t\right)\right)$$

$$\boxed{v_y(x,t) = 2\pi f A \sin\left(2\pi f\sqrt{\frac{\mu}{T_s}}\,x - 2\pi f t\right)}$$

Part C (10 points): Consider the segment of the rope between $x = 0$ and $x = +\lambda$, where λ is the wavelength of the transverse wave. At $t = 0$, where on this segment are both the transverse acceleration and transverse velocity positive (upward)? Express your answer as a range of positions (x_{min}, x_{max}), where x_{min} and x_{max} are given in terms of f, T_s, and μ.





$x_{min} = \frac{\lambda}{4}$ $x_{max} = \frac{\lambda}{2}$

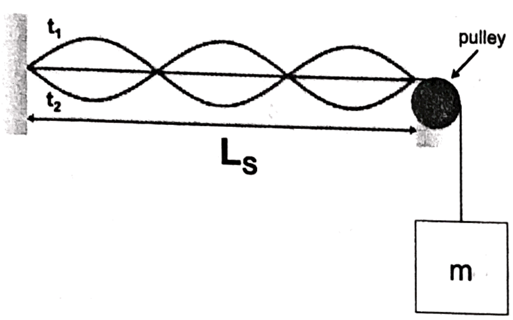
$\lambda = \frac{1}{f}\sqrt{\frac{T_s}{\mu}}$

$$\boxed{\left(\frac{1}{4f}\sqrt{\frac{T_s}{\mu}}, \frac{1}{2f}\sqrt{\frac{T_s}{\mu}}\right)}$$

Problem 3 (40 points):

A string with linear mass density μ is fixed to the wall at one end. At the other end, it is suspended over a massless, frictionless pulley and connected to block of mass m that hangs freely. The distance between the wall and pulley is L_S. A normal mode with amplitude A is excited on the string. Two successive photographs of the string are taken at times t_1 and t_2, as shown in **Figure 3** below. Assume that the string cannot move at the location of the pulley.

Figure 3



n 1
n 2
n 3

Part A (10 points): What is the frequency of this normal mode in terms of L_S, μ, m, and gravitational acceleration g?

μ, m, L_S, A

$$f_n = \frac{n}{2L}\sqrt{\frac{T_s}{\mu}}$$

$$T_s = mg$$

$n=3$

n=1
n=2
n=3

$$\boxed{f = \frac{3}{2L_S}\sqrt{\frac{mg}{\mu}}}$$

✓

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Part B (10 points): Write an equation for $y(x,t)$, the transverse displacement of the string in **Figure 3**, in terms of x, t, μ, A, L_s, m, and g. Assume that the left and right ends of the string are at $x = 0$ and $x = L_s$, respectively, and that every element of the string is at $y = 0$ (i.e., the string is horizontal) at $t = 0$.

$$y(x,t) = A_{SW} \sin(kx) \sin(\omega t)$$

$$A_{SW} = 2A \qquad k = \frac{2\pi}{\lambda}, \quad \lambda = \frac{2L_s}{3} \qquad k = 2\pi \cdot \frac{3}{2L_s} = \frac{3\pi}{L_s}$$

$$\omega = 2\pi f = 2\pi \cdot \frac{3}{2L_s}\sqrt{\frac{mg}{\mu}} = \frac{3\pi}{L_s}\sqrt{\frac{mg}{\mu}}$$

$$\boxed{y(x,t) = \cancel{2}A \sin\left(\frac{3\pi}{L_s}x\right) \sin\left(\frac{3\pi}{L_s}\sqrt{\frac{mg}{\mu}}\, t\right)}$$

-2

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Part C (10 points): Assume that the string in **Figure 3** vibrates the air around it, producing sound at the frequency from Part A. Simultaneously, you blow air into a pipe of length L_P that is closed at one end and open at the other. At what air temperature T will the fundamental frequency of the pipe match the frequency emitted by the string? Give an expression for T in terms of string L_S, L_P, μ, m, g, molar mass of air M_{mol}, adiabatic index of air γ, and gas constant R.

$$v = \sqrt{\frac{\gamma R T}{M_{mol}}}$$

$$f_s = \frac{3}{2L_s}\sqrt{\frac{mg}{\mu}}$$

$n = 1$

$$f_1 = \frac{1 \cdot v}{4L_p}$$

$$f_1 = \frac{1}{4L_p} \cdot \sqrt{\frac{\gamma R T}{M_{mol}}}$$

$$f_1 = f_s$$

$$\frac{3}{2L_s}\sqrt{\frac{mg}{\mu}} = \frac{1}{4L_p}\sqrt{\frac{\gamma R T}{M_{mol}}}$$

$$\frac{6L_p}{L_s}\sqrt{\frac{mg}{\mu}} = \sqrt{\frac{\gamma R T}{M_{mol}}}$$

$$\frac{36L_p^2}{L_s^2} \cdot \frac{mg}{\mu} = \frac{\gamma R T}{M_{mol}}$$

$$\boxed{\left(\frac{36L_p^2}{L_s^2} \cdot \frac{mg}{\mu}\right)\frac{M_{mol}}{\gamma R} = T}$$

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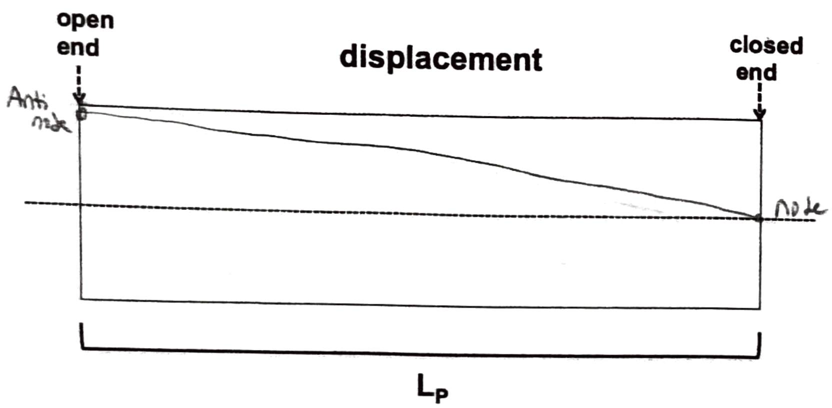
Part D (10 points): In the spaces below, draw a representation of the fundamental mode of the pipe from Part C in terms of the particle displacement and in terms of the pressure fluctuation, labeling all nodes and anti-nodes. You do not need to label the amplitudes.

$node_d = anti_p$ $node_p = anti_d$

$\lambda_1 = \frac{1v}{4L}$



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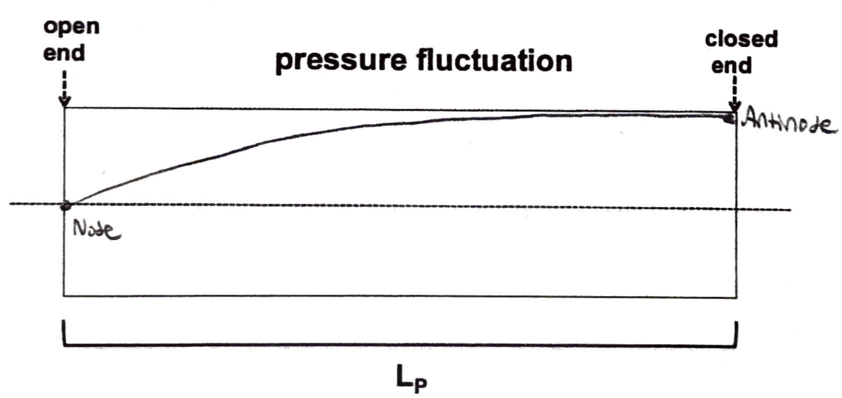

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