Physics 1B-2 (9:00 am-9:50 am) Spring 2019 Midterm 2

Param Shah

TOTAL POINTS

72 / 100

QUESTION 1

1 Problem 1 23 / 25

+ **2 pts** dq = \lambda ds, integrate
+ **2 pts** integrate -> Q = 2R*\lambda0
+ **3 pts** dE = \lambda ds / r^2
+ **2 pts** r-hat with correct theta convention
+ **5 pts** E_x = 0 by symmetry
+ **3 pts** E_y integral
+ **5 pts** E_y = -\lambda0/(8\eps0*R) = -Q/(16\eps0*R^2); no \lambda0 in final answer
+ **3 pts** F = qE; NOTE: alternative approach using potential V to find F graded similarly: dq & Q same, dV instead of dE, r-hat and symmetry for F_x = 0 same, V integration graded like E_y integral, F = -dV/dr graded like final E_y expression;

✓ + **25 pts ALL**

+ **0 pts** 0

- 2 Point adjustment

lambda

QUESTION 2

2 Problem 2 (a) 8 / 20

+ **20 pts** Correct

✓ + **2 pts Correct relation between E and V**

✓ + **2 pts Correct Electric field for r<R1**

+ **2 pts** Correct Electric field for R1<r<R2
+ **2 pts** Correct Electric field for R2<r<R3
+ **2 pts** Correct Electric field for r>R3

✓ + **2 pts Identify metal sphere as equipotential**

+ **2 pts** Correct potential for R1<r<R2
+ **2 pts** Correct potential for r>R3
+ **2 pts** Identify metal shell as equipotential
+ **0 pts** Incorrect or unattempted

+ 2 Point adjustment

Electric field for r<R1, R1<r<R2, R2<r<R3, r>R3 to be calculated. Electric potential can be found from Electric field. Since the metal sphere and shell are conductors, the electric field in that region will be 0. And adding the electric potentials for r<R1, R1<r<R2, R2<r<R3, r>R3 will give total electric field at r<R1.Final Potential at r<R1 is kQ/6R1.

QUESTION 3

3 Problem 2 (b) 0 / 10

+ **10 pts** Correct p=12*R1
+ **3 pts** Correct potential difference inside the metal shell
+ **3 pts** Correct potential difference outside the metal shell
+ **3 pts** Correct energy relation
+ **3 pts** Correct method

✓ + **0 pts Incorrect**

QUESTION 4

4 Problem 3 (a) 11 / 15

✓ + **3 pts Correct general formula for capacitor C = k epsilon A /d**

+ **2 pts** Correct formula for capacitor 1
+ **2 pts** Correct formula for capacitor 2 and for capacitor 3

✓ + **4 pts Summing capacitors in parallel**

✓ + **4 pts Proper formula for capacitors in series**

- **1 pts** Misunderstanding of the problem that did not simplified the problem
+ **0 pts** Nothing from the above
- **1 pts** Mistake in calculations
- **2 pts** The same minor mistake (wrong area) leading to wrong capacities 1,2,3
- **4 pts** Wrong definition which capacitors in parallel

and which in series (or wrong usage of the formulas)

- **- 3 pts** Formulas are written but not applied
- **- 2 pts** Mistake in calculations involving wrong units
- **+ 2 pts** Correct plan

QUESTION 5

5 Problem 3 (b) **10 / 10**

- **+ 2 pts** Q = VC
- **+ 2 pts** C = epsilon A / d (or expression from the part a that is wrong but I can't punish for this again here)
- **+ 3 pts** F = E sigma A = E Q
- **+ 3 pts** E = V/(2 d)
- **- 1.5 pts** Missing 2 in E = V/(2 d) (or in the U = 1/2 C V^2 for energy approach)
- ✓ **+ 10 pts Correct**
- **+ 0 pts** Nothing
- **- 1.5 pts** Didn't take a derivative (or did it wrong) in the energy approach.
- **- 1 pts** The answer uses not only A,d,V
- **+ 3 pts** U = 1/2 C V^2 (energy approach)
- **+ 1.5 pts** F = -dU /dx (energy approach)
- **- 2 pts** Final result is not obtained (or wrong)
- **- 1.5 pts** Mistake involving wrong units
- **- 1 pts** Minor mistake

QUESTION 6

6 Problem 4 **20 / 20**

- **+ 5 pts** Know the relation between flux and electric field
- **+ 5 pts** Give correct magnitude or symmetry of the electric field
- **+ 5 pts** Write down correct integral
- **+ 7 pts** Know the flux is 1/24 of total flux
- ✓ **+ 20 pts Correct**
- **+ 0 pts** Incorrect

Physics 1B– Spring 2019: Midterm 2

Name Param Shah UID 205143347 Lecture Time 9am

PLEASE READ:

- This exam is closed book and closed notes. You may use one calculator; **no other electronics are permitted.**
- Please show your full solution in the boxes provided (where the scanners can pick them up). The size of the boxes provided do not indicate the difficulty of the problem.
- Indicate final answers by circling them.
- Your solutions will be graded on correctness and coherence; results given with no details will receive zero credit.
- There is additional scratch paper attached so you can collect your thoughts first.
- Academic dishonesty is reported to the Office of the Dean of Students.

Potentially useful relations:

$\hat{r} = \frac{\vec{r}}{|r|}$

$2\cos^2 x = 1 + \cos 2x$

$\int \cos x \, dx = \sin x$

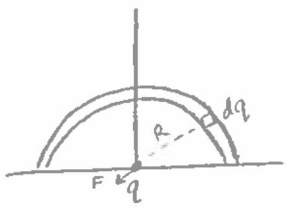
$\int \sin x \cos x \, dx = -\cos^2 x/2$

$\int \frac{dx}{(x^2+a^2)^{3/2}} = \frac{x}{a^2(x^2+a^2)^{1/2}}$

$\int \frac{dx}{(x^2+a^2)(x^2+b^2)^{1/2}} = \frac{1}{a(b^2-a^2)^{1/2}} \tan^{-1}(x\sqrt{\frac{(b^2-a^2)}{a^2(x^2+b^2)}}), \; b^2 > a^2$

Problem 1 (25 pts). A positively charged wire of total charge Q is bent into a semicircle of radius R:

The charge per unit length along the semicircle is given by $\lambda = \lambda_0 \cos\theta$. If a particle with a charge q is placed at the origin, what is the total force on the particle in terms of the total charge Q (not λ_0), q, R and any necessary universal constants?.



$$dF = \frac{kq\,dQ}{R^2}$$

$$dF = \frac{kq\cdot(\lambda_0 \cos^2\theta\, d\theta)}{R^2}$$

$$F = \int dF$$

$$= \int_0^{\pi} \frac{kq\lambda}{R^2} \cos^2\theta \, d\theta$$

$$= \frac{kq\lambda}{R^2} \int_0^{\pi} \frac{1 - \cos 2\theta}{2} \, d\theta$$

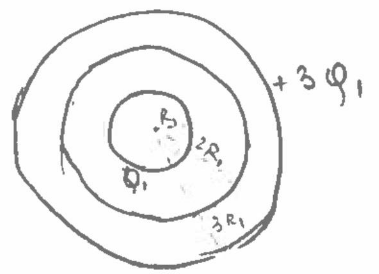
$$= \frac{kq\lambda}{R^2} \left[\frac{\theta}{2} - \frac{\sin 2\theta}{4} \right]_0^{\pi}$$

$$= \frac{kq\lambda}{R^2} \left[\frac{\pi}{2} - 0 \right]$$

$$\boxed{F = \frac{\pi \cdot kq\lambda}{2R^2}}$$

Problem 2. A solid metal sphere of radius R_1 carries a charge $-Q_1$, where $Q_1 > 0$. Surrounding this sphere is a metal shell of inner radius $R_2 = 2R_1$ and outer radius $R_3 = 3R_1$ that carries a total charge of $Q_2 = +3Q_1$.

a. What is the potential for $r < R_1$ assuming the potential is zero at infinity? (20pts)



$V_\infty = 0$

$V_r = -\int_a^b E \cdot dl$

$V = -\int_a^b E \cdot dl$

→ since E for $r < R_1$ is zero,

$V =$ constant for $r < R_1$

since V is constant,

⇒ $V_{R_1} = V_{r<R_1}$

→ $V_{R_1} = \frac{KQ_1}{R_1}$

⇒ V at $r < R_1$ $\boxed{V_r = \frac{KQ_1}{R_1}}$

b. An electron is emitted from the surface of R_1 with negligible velocity. It is accelerated and escapes through a small hole in the metal shell. It comes to rest at point P. How far is P from the center of the metal sphere? (10 pts)

from R_1 to R_2, the electron will experience a force of $F = qE$

$= e \cdot \frac{KQ_1}{r^2}$ where $R_1 < r < R_2$

$\Rightarrow a = \frac{F}{m}$

$= \frac{e}{m} \cdot \frac{KQ_1}{r^2}$

force on e from R_2 to R_3 is zero

Force on e for $r > R_3$

$\int E \cdot dA = \frac{2Q_1}{\varepsilon_0}$

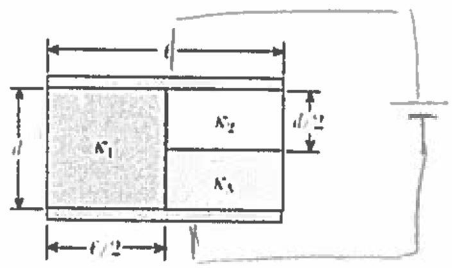
$\Rightarrow E = \frac{2Q_1}{\varepsilon_0 \cdot 4\pi r_2^2} = \frac{2KQ_1}{r_2^2}$

$\Rightarrow F = e \cdot \frac{2KQ_1}{r^2}$

$\vec{F} = - e \cdot \frac{2kQ_1}{r_2^2}$

Problem 3.

a. A parallel-plate capacitor is constructed by filling the space between two square plates with blocks of three dielectric materials, as in the figure below. You may assume that $l >> d$. Find an expression for the capacitance of the device in terms of the plate area A, d, κ_1, κ_2, and κ_3. (15 pts)



we can assume that k_1, k_2 and k_3 will act as 3 capacitors.

for k_2: $C_2 = \dfrac{\varepsilon_0 A}{d - \frac{d}{2} + \frac{d}{2k_2}} = \dfrac{\varepsilon_0 A}{\frac{d}{2}(1 + \frac{1}{k_2})} = \dfrac{2\varepsilon_0 A}{d(1 + 1/k_3)}$

for k_3: $C_3 = C_2 = \dfrac{2\varepsilon_0 A}{d(1 + 1/k_3)}$

for k_1: $C_1 = K_1 \cdot \dfrac{\varepsilon_0 A}{d}$

k_2 and k_3 are in series

$\Rightarrow C' = \dfrac{C_2 \cdot C_3}{C_2 + C_3} = \dfrac{\left(\frac{2\varepsilon_0 A}{d}\right)^2 \cdot \left[\frac{k_3}{k_3+1} \cdot \frac{k_2}{k_2+1}\right]}{\frac{2\varepsilon_0 A}{d}\left[\frac{k_3}{k_3+1} + \frac{k_2}{k_2+1}\right]} = \dfrac{2\varepsilon_0 A}{d}\left[\dfrac{k_3 k_2}{k_3 + k_2 + 2k_3 k_2}\right]$

k_1 and combination of k_2 k_3 is in parallell

$$\Rightarrow \boxed{C_{eq} = K_1 \cdot \frac{\varepsilon_0 A}{d} + \frac{2\varepsilon_0 A}{d}\left[\frac{k_3 \cdot k_2}{k_3 + k_2 + 2k_3 k_2}\right]}$$

b. Consider the capacitor above with $\kappa_1 = \kappa_2 = \kappa_3 = 1$. Find the magnitude of the force each plate experiences due to the other plate as a function of V, the potential drop across the capacitor. Your answer should be in terms of A, d, V, and any necessary universal constants. (10 pts)

If $K_1 = k_2 = k_3 = 1$

$$C_{eq} = \frac{\varepsilon_0 A}{d} + \frac{2\varepsilon_0 A}{d}\left[\frac{1}{4\,2}\right]$$

$$= \frac{3}{2}\frac{\varepsilon_0 A}{d} = \frac{\varepsilon A}{d} \quad \text{where } \varepsilon = \frac{3}{2}\varepsilon_0$$

$$E \text{ due to one plate} = \frac{\sigma}{2\varepsilon} = \frac{Q/A}{2\cdot\frac{3}{2}\varepsilon_0} = \frac{CV/A}{3\varepsilon_0}$$

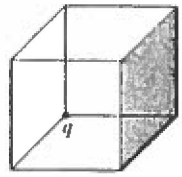
$$F = q \cdot E$$

$$= Q \cdot \frac{CV}{3\varepsilon_0 A} = \frac{C^2V^2}{3\varepsilon_0 A}$$

$$= \frac{\left(\frac{3}{2}\frac{\varepsilon_0 A}{d}\right)^2 \cdot V^2}{3\varepsilon_0 A} = \frac{3}{4}\frac{\varepsilon_0 A V^2}{d^2}$$

$$\boxed{F = \frac{3}{4}\frac{\varepsilon_0 A V^2}{d^2}}$$

Problem 4. A charge sits in the corner of a cube of side a as shown below. Find the flux through the shaded region. (There are many ways to solve this problem.) (20 pts)



We can assume the q and the cube is part of a system of 8 cubes:

Since the charge is at the centre of the bigger cube, ϕ through each face is

$$\phi_1 = \frac{q}{6\varepsilon_0}$$

Now, the flux through each smaller square of one side of the cube

$$\phi_2 = \frac{\phi_1}{4}$$

$$\boxed{\phi_2 = \frac{q}{24\varepsilon_0}}$$

Scratch paper