Physics 1B-2 (9:00 am-9:50 am) Spring 2019 Final

Param Shah

TOTAL POINTS

91 / 100

QUESTION 1

1 Problem 1a **10 / 10**

✓ **+ 10 pts** **Correct**

+ **2 pts** Kerkhoff's law/equivalent resistors
+ **2 pts** I_1
+ **2 pts** I_2
+ **2 pts** I_3
+ **1 pts** V
+ **1 pts** Q
- **2 pts** Wrong units
- **2 pts** Arithmetic error
+ **0 pts** Incorrect/blank

QUESTION 2

2 Problem 1b **9 / 10**

+ **10 pts** Correct

✓ **+ 3 pts** **I_1**

✓ **+ 2 pts** **I_2**

✓ **+ 1 pts** **I_3**

✓ **+ 3 pts** **V_c**

+ **1 pts** Q
- **2 pts** Wrong units
- **2 pts** Arithmetic error
+ **0 pts** Incorrect/Blank

QUESTION 3

3 Problem 2a **5 / 5**

✓ **+ 5 pts** **Correct**

- **3 pts** Inverted fraction
+ **0 pts** Incorrect
- **2 pts** Arithmetic error

QUESTION 4

4 Problem 2b **6 / 6**

✓ **+ 6 pts** **Correct**

+ **0 pts** Incorrect/No explanation

QUESTION 5

5 Problem 3 **9 / 15**

+ **15 pts** Correct R=3 Ohms

✓ **+ 7 pts** **Correct, but did not derive the ratio $$R_2/R_1 = R_u/R_{var}$$**

+ **8 pts** Applied loop law, but didn't finish or has incorrect current continuity
- **5 pts** Algebraic error
- **2 pts** Numerical Error
+ **0 pts** Incorrect (did not apply loop law correctly)

+ 2 **Point adjustment**

Checking that answer gives correct result of Va=Vb

QUESTION 6

6 Problem 4a **8 / 8**

✓ **+ 8 pts** **Correct**

+ **2 pts** Identify the number of bulbs glowing
+ **4 pts** Identify that B, C and D glow with equal brightness as they are all in parallel
+ **2 pts** Bulb A glows with max brightness since it is in series with the battery
+ **0 pts** Incorrect approach

QUESTION 7

7 Problem 4b **8 / 8**

✓ **+ 8 pts** **Correct**

+ **4 pts** Identify that only bulb A glows
+ **4 pts** Identify that when the switch is closed, the current flows through the least resistance path which is the switch and hence no brightness in B, C and D
+ **0 pts** Incorrect approach

QUESTION 8

8 Problem 5 8 / 10

✓ + **1 pts** Know the formula of Electric field

✓ + **1 pts** Know the direction points to r hat

✓ + **1 pts** Know Ex=0

✓ + **2 pts** Correct dEy

✓ + **1 pts** Know the relation between theta and r

✓ + **2 pts** Correct integral

+ **2 pts** Correct final answer

+ **10 pts** Correct

QUESTION 9

9 Problem 6a 5 / 5

✓ + **5 pts** sqrt(3)kq/d^2

+ **2 pts** kq/d^2 with or without factor of 2 (answer results from incorrect application of superposition); OR incorrect trig functions OR E_x non-zero, etc. (one error)

+ **1 pts** E = any other non-zero answer OR multiple errors listed above

+ **0 pts** E = 0 or otherwise complete misunderstanding of question

QUESTION 10

10 Problem 6b 5 / 5

✓ + **5 pts** q_3 = -q/2

+ **2 pts** any other negative q_3 OR +q/2

+ **1 pts** any positive q_3 besides +q/2 OR multiple errors leading to incorrect answer

+ **0 pts** q_3 = 0 (W1 = 0, so if W2=/=0 and W3 = 0 then how can W_tot = 0?) OR no solution / not solved

QUESTION 11

11 Problem 7a 5 / 5

+ **1 pts** solve ODE

+ **1 pts** x=Acos(ωt+φ); Note: if your solution is not sinusoidal, max points = 1/5

+ **1 pts** x(0) = sqrt(3) ; Note: if you haven't solved for A and phi, then the ICs can't be specified

+ **1 pts** 2 of 3 Correct: A or ωt or φ ; Note: you can satisfy the above IC without having 2 of 3 correct, and vice-versa, which is why they're separate rubric items

✓ + **5 pts** x(t) = 2cos(2t + pi/6) = sqrt(3)*cos(2t) - sin(2t) ; Note the signs! If the signs are incorrect then at least one of the ICs won't be satisfied

+ **0 pts** 0

QUESTION 12

12 Problem 7b 5 / 5

+ **2 pts** Know x=0

+ **2 pts** Know ωt+φ=π/2

+ **1 pts** solve

- **1 pts** Right approach, wrong final answer.

✓ + **5 pts** Correct

+ **0 pts** 0

QUESTION 13

13 Problem 8 8 / 8

✓ - **0 pts** t = 8.4 ms

- **4 pts** t =/= 8.4 ms

- **2 pts** multiple incorrect values for: v = 2.5 m/s, \omega = 100*pi rad/s, k = 40*pi rad/m, dx = 2.1 cm, \phi = 0, etc. OR incorrect approach solution (e.g. wrong wave eq'n) OR orders of magnitude ; no final answer

- **8 pts** nothing OR zero OR multiple AND incorrect approach

Physics 1B Final Exam Spring 2019

Name Param Shah UID 205143347 Lecture Time 9am

PLEASE READ:

- This exam is closed book and closed notes. You may use a calculator; no other electronics are permitted.
- Please show your full solution in the boxes provided (where the scanners can pick them up).
- Indicate any final numerical answers by circling them.
- Your solutions will be graded on correctness and coherence; results given with no details (except where specifically noted) will receive zero credit. There is additional scratch paper attached so you can collect your thoughts first. The more easy to follow your solution is, the more partial credit you are likely to get!
- **Academic dishonesty is reported to the Office of the Dean of Students.**

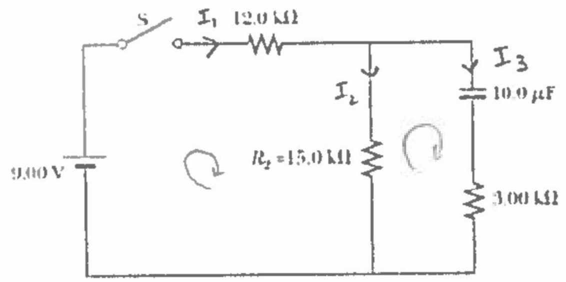
Useful integrals:

$$\int \frac{1}{\sqrt{(a^2+x^2)^3}}dx = \frac{x}{a^2\sqrt{a^2+x^2}}$$

$$\int \frac{r}{\sqrt{(x^2+r^2)^3}}dr = -\frac{1}{\sqrt{r^2+x^2}}$$

Problem 1.

For the RC circuit in the figure, R1 = 12.0 $k\Omega$ and R3 = 3.00$k\Omega$. The currents in R1, R2, and R3 are denoted as I1, I2, and I3, respectively. The charge on the capacitor is denoted as Q, and the voltage across the capacitor is denoted as Vc. Suppose that initially there is no charge on the capacitor, and the switch is open.



a. Close the switch, and find I1; I2; I3, Q, and Vc immediately after the switch is closed. (10 pts)

Immediately after switch is closed, $V_c = 0$ and $Q = 0$

R_{eq} (parallel) $= \frac{R_3 \cdot R_2}{R_2 + R_3} = \frac{15 \times 3}{15 + 3} = 2.5\ k\Omega$

$\Rightarrow R_{eq}$ (for whole circuit) $= 12 + 2.5 = 14.5\ k\Omega$

$\Rightarrow I = \frac{V}{R_{eq}} = \frac{9}{14.5 \times 10^3} = 6.2 \times 10^{-4}$

$\Rightarrow$ $I_1 = 6.2 \times 10^{-4}$ A

$I_2 = \frac{R_3}{R_2 + R_3} I_1 = \frac{3}{18} I_1$

$I_2 = 1.033 \times 10^{-4}$ A

$I_3 = I_1 - I_2$

$I_3 = 5.16 \times 10^{-4}$ A

b. After the switch is closed for a length of time sufficiently long for the capacitor to become fully charged, find I1; I2; I3, Q, and Vc. (10 pts)

After very long time,

$\boxed{I_3 = 0}$

$I_1 = I_2$

$$I_1 = \frac{V}{R_{eq}} = \frac{9}{12 + 15\ k\Omega} = \frac{9}{27 \times 10^3}$$

$\boxed{I_1 = I_2 = 3.33 \times 10^{-4}\ A}$

$$Q = C\mathcal{E}(1 - e^{-t/RC})$$

$t \to \infty$

$Q = C\mathcal{E} = 10 \times 10^{-6} \times 9 \Rightarrow \boxed{Q = 9 \times 10^{-5}\ C}$

V_c = Potential drop across $15\ k\Omega$ resistor

$= IR$

$= 3.33 \times 10^{-4} \times 15 \times 10^3$

$\boxed{V_c = 4.995\ V}$

Problem 2. A steel wire and a nichome wire have the same length and the same potential difference applied from one end to the other. Nichome has a resistivity of $100 \times 10^{-8}\Omega \cdot m$ and steel has a resistivity of $25 \times 10^{-8}\Omega \cdot m$

a. What must the ratio of the radii of nichome to steel wires be if the currents are to be the same? (5 pts)

$R = \rho \frac{L}{A}$

∵ I and V are same for both wires

$R_1 = R_2$

$\rho_1 \frac{L_1}{A_1} = \rho_2 \frac{L_2}{A_2} \Rightarrow \frac{100 \times 10^{-8}}{\pi (r_1)^2} = \frac{25 \times 10^{-8}}{\pi (r_2)^2}$

$(r_1)^2 \cdot 25 = 100 \cdot (r_2)^2$

$\Rightarrow 5 r_1 = 10 r_2$

$\Rightarrow \boxed{\frac{r_1}{r_2} = 2}$

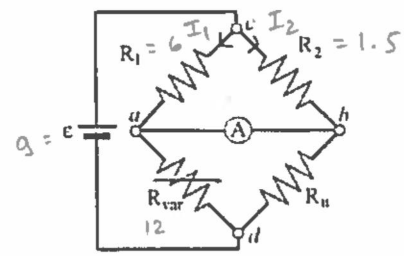
b. Can the current density be made the same by suitable choices of the radii? If so, give a pair of values. If not, explain why not. (6 pts)

$J = \frac{I}{A}$

The current density cannot be made the same because if the have equal radii, they will have different resistances and therefore will have different currents.

$\Rightarrow$ same A but different I

Problem 3. A circuit consists of two resistors with resistances R1 = 6.0Ω and R2 = 1.5 Ω, a variable resistor, the resistance R_{var} of which can be adjusted, a resistor of unknown value R_u, and 9.0 volt battery connected as shown in the figure. When R_{var} is adjusted to 12 ohms, there is zero current through the ammeter. What is the unknown resistance R_u? Justify all steps. (15 pts)



This is an example of a wheatstone's bridge

When the bridge is balanced, the current through the ammeter would be zero (because the potential at 'a' and 'b' is the same)

$\Rightarrow \frac{R_1}{R_{var}} = \frac{R_2}{R_u}$

$\Rightarrow \frac{6}{12} = \frac{1.5}{R_u}$

$\Rightarrow \boxed{R_u = 3\ \Omega}$

Justification:

$R_{eq} = \frac{18 \times 4.5}{18 + 4.5} = 3.6\ \Omega$

$\Rightarrow I = \frac{V}{R_{eq}} = 2.5\ A$

$I_1 = \frac{4.5}{18 + 4.5} \times 2.5 = 0.5\ A$

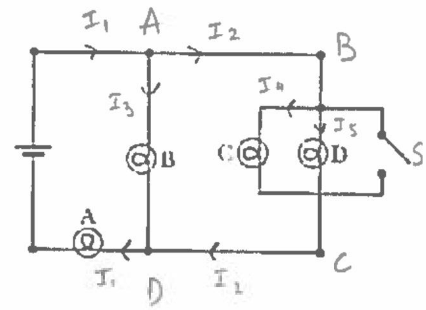
$I_2 = 2.5 - 0.5 = 2\ A$

$\Rightarrow V_{ac} = 6 \cdot I_1 = 6 \times 0.5 = 3\ V$

$\Rightarrow V_{bc} = 1.5 \times I_2 = 1.5 \times 2 = 3\ V$

$\Rightarrow V_a = V_b$

Problem 4.



a. Assume the switch is open (as shown above). Rank the bulbs according to descending order of brightness and explain your reasoning. (8 pts)

A > B = C = D

$P = I^2 R$

when R = same , $P \propto I^2$

→ The current I_1 splits into I_2 and I_3

⇒ $I_1 > I_2$ and $I_1 > I_3$

⇒ A is the brightest

→ since resistance in the left branch is twice resistance in the right branch

$I_2 = \frac{1}{3} I_1$ and $I_3 = \frac{2}{3} I_1$

but since I_3 again splits equally into 2 branches

$I_2 = I_4 = I_5 = \frac{1}{3} I_1$

⇒ Brightness of B = C = D

b. Now assume the switch is closed. Again rank the bulbs in a descending order of brightness. Explain the reasoning for your ranking. (8 pts)

A > B = C = D = 0

A is the brightest due to the same reason as part (a).

No current will flow through C and D because of the switch. All the current will flow through the switch.

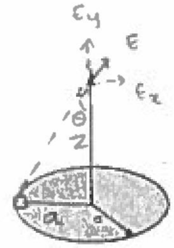
→ Now considering the loop ABSCD (marked in the figure, $\Delta V = 0$ (through Kirchoff's Law)

$\Rightarrow I_3 \cdot B + 0 = 0$

$\Rightarrow I_3 = 0$

⇒ Bulb B would not also glow

Problem 5. Find the electric field a distance z above a uniformly charged disk of radius a as shown below.



$$dE_y = \frac{K\,dq}{(r^2+z^2)^2}\cos\theta = \frac{K(2\pi r)\,\sigma z\,dr}{(r^2+z^2)^{3/2}}$$

$$dE_x = \frac{K\,dq}{(r^2+z^2)^2}\sin\theta \qquad \Sigma\, dE_x = 0 \quad \text{due to symmetry}$$

$$\Rightarrow \quad E_y = \int dE_y = \int_0^a \frac{K(2\pi r)\sigma z}{(r^2+z^2)^{3/2}}\,dr$$

$$= K\pi z\sigma \int_0^a \frac{2r}{(r^2+z^2)^{3/2}}\,dr$$

$$= K\pi a\sigma \left[\frac{2}{\sqrt{r^2+z^2}}\right]_0^a$$

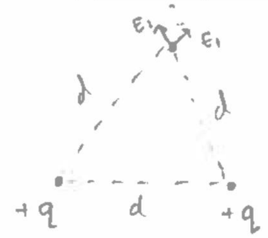
$$= \frac{2\pi K\sigma a}{\sqrt{a^2+z^2}} = \boxed{\frac{\sigma a}{2\varepsilon_0\sqrt{a^2+z^2}}}$$

when $a >> z$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

Problem 6. Three point charges, which initially are infinitely far apart, are placed at the corners of an equilateral triangle with sides d. Two of the point charges are identical and have charge +q.

a. What is the electric field at the corner of the triangle opposite the two +q charges? (5 pts)



$\vec{E}_{Total} = \vec{E}_1 + \vec{E}_2$

$\vec{E}_{1y} = -\vec{E}_{2y} \Rightarrow \vec{E}_{Total\,y} = 0$

$\vec{E}_{Total\,x} = E_1 \cos 30^\circ + E_2 \cos 30^\circ$

$= \frac{kq}{d^2} \cdot \frac{\sqrt{3}}{2} + \frac{kq}{d^2} \cdot \frac{\sqrt{3}}{2}$

$\boxed{\vec{E}_{Total\,x} = \sqrt{3}\,\frac{kq}{d^2}}$

b. If zero net work is required to place the three charges at the corners of the triangle, what must the value of the third charge be? (5 pts)

Let unknown charge be q'

work done in bringing the first $+q$ charge $= 0$

work done in bringing to second $+q$ charge $= \frac{Kq^2}{d}$

work done in bringind the q' charge $= \frac{Kqq'}{d} + \frac{Kqq'}{d}$

Total work done $= 0$

$\Rightarrow \frac{Kq^2}{d} + \frac{Kqq'}{d} + \frac{Kqq'}{d} = 0$

$q^2 + 2qq' = 0$

$\Rightarrow \boxed{q' = -\frac{1}{2}q}$

Problem 7. A certain oscillator satisfies the equation $\frac{d^2x}{dt^2} + 4x = 0$. Initially the particle is at the point $x = \sqrt{3}$ when it is projected towards the origin with speed $v_0 = 2$.

a. Find x(t). (5 pts)

$\omega^2 = 4$

$\Rightarrow \omega = 2$

$A = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}} = \sqrt{3 + \frac{4}{4}} = 2$

$\Rightarrow \boxed{A = 2}$

$\Rightarrow \boxed{x(t) = 2\cos(2t + \phi)}$

$\tan\phi = \frac{1}{\sqrt{3}} \quad \phi = \tan^{-1}\frac{1}{\sqrt{3}}$

$= 0.523$

$\Rightarrow \boxed{x(t) = 2\cos(2t + 0.523)}$

b. How long does it take for the particle to first reach the origin? (5 pts)

$x(t) = 2\cos(2t + 0.523)$

$\Rightarrow \cos(2t + 0.523) = 0 \qquad (\because x(t) = 0)$

$2t + 0.523 = 1.57$

$t = \frac{1.57 - 0.523}{2}$

$\boxed{t = 0.523 \text{ secs}}$

Problem 8. A sinusoidal wave moving to the left has a wavelength of 5.0 cm and a frequency of 50 Hz. At t = 0 s, the wave has a crest at x = 0 cm. What is the earliest time after t = 0 s at which there is a crest at the position x = 2.9 cm?(8 pts)

$\lambda = 5 \times 10^{-2}$ m

$f = 50$ Hz $\Rightarrow \omega = 2\pi f = 314.16$ rad/sec

$\Rightarrow v = \lambda f = 5 \times 10^{-2} \times 50$

$v = 2.5$ m/s

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{5 \times 10^{-2}} = 125.66\ m^{-1}$

$y(x,t) = A\cos(kx + \omega t)$

$y(x,t) = A\cos(125.66x + 314.16t)$

$y(2.9, t) = A\cos(125.66 \times 2.9 \times 10^{-2} + 314.16 \times t) = A$

$\Rightarrow 125.66 \times 2.9 \times 10^{-2} + 314.16 \times t = 2\pi$

$t = \frac{2\pi - 125.66 \times 2.9 \times 10^{-2}}{314.16}$

$t = 0.0084$ sec

Scratch paper

Scratch paper

$\int E \cdot dA = \frac{q}{\varepsilon_0}$